Temperature's Effect on the Speed of Light

PHYS 350 - Group 5

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Date: April 13, 2021

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Abstract

In this experiment, our group investigates the effect of temperature on the speed of light by experimentally measuring the change in the angle of refraction through common glass over a 100°C range in temperature while simultaneously running simulations on Comsol with silica glass to compare results. Both results from the experiments and simulations confirmed the linear relationship between temperature and the speed of light travelling through glass. Although the simulation agreed with our predication where the speed of light shows an inverse linear relationship to temperature, the experimental findings show a linearly proportional relation. The differing results are due to the fact that the experiments used common glass and the simulations used silica glass. Both results are consistent with theoretical values as temperature's relation to speed of light through glass can be linearly proportional or inversely proportional, depending on the type of glass used.

Introduction and Theory

Light exists all around us in the physical world and is a necessary component in our everyday lives. Light exists in our pockets in the form of smartphones and cameras. In medicine, light and other optical equipment are used as a tool in the diagnosis of ailments, therapy for patients, and when operating surgery. [5] As our dependence on technology in society grows so does the field of photonics, which takes advantage of optics and the behaviour of light. Optical advancements have given us an ability to better communicate and stay connected through a wide range of services such as residential and mobile services. An example of this is fiber optics, which is able to transmit and receive information at much faster speeds than cable. [2]

The understanding of light and its behaviour in changing conditions is thus crucial as we continue to develop more complex optical systems. Experiments have shown that temperature can influence the refractive index of a material since temperature can vary the bond strength between a material's molecules. [4] However, little research has been conducted to determine the relation between the speed of light travelling through material and the temperature of that material. This project aims to understand the how the speed of light is influenced as it travels through glass at varying temperatures.

To determine the refractive index of a material, we can use Snell's Law (Eq. 1) to relate a known refractive index with the incident and refractive angles.

$$n_1 sin\theta_1 = n_2 sin\theta_2 \qquad (1)$$

Where n_1 and n_2 are the refractive indices and θ_1 and $\theta_2$are the angles to the normal line that the ray enters and exits the medium.

Once the refractive index of a material is known, the speed of light travelling through the material can easily be found using the definition of the index of refraction (Eq. 2).

$$n = \frac{c}{v} \qquad (2)$$

Where n is the refractive index of the material, c is the speed of light through vacuum with units m/s , and v is the speed of light travelling through the material with units m/s. The speed of light decreases when it travels from a medium with a lower index of refraction to a higher index of refraction. This can be derived from Snell's Law and the definition for the index of refraction as shown in Appendix (1).

In order to measure the angle of refraction, the testing material (in this case glass) must be in the shape of a semi-circle. The glass is required to be this shape as any other shape would cause the light to refract as it was exiting the material again. With the semi-circle, as the light exits, it exits such that it is at a right angle and thus, does not refract again as shown in Figure 1.

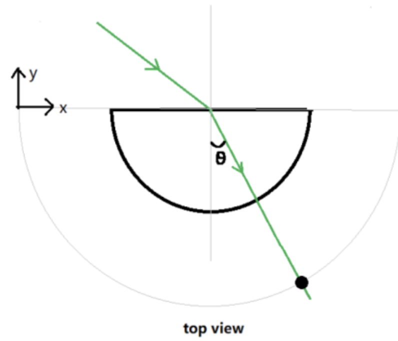


Figure 1: With a semi-circular shape, the laser light will not refract a second time as it exits the material, allowing us to measure the angle of refraction, θ.

To measure the angle of refraction, this experiment's general mathematical model relies on the tan trigonometric relation (Eq. 3).

$$tan\,\theta = \frac{x}{y} \qquad (3)$$

Where θ is the angle, x is the horizontal distance, and y is the vertical distance. (The terms horizontal and vertical distance are being used as the setup will be aligned in such as way that the coordinate plane aligns with the rotated Cartesian plane.) In the experimental setup, the glass semi-circle will be turned 45°. In Figure 2, since the incident angle is 45°, we know that $\theta + \emptyset = 45°$. By measuring the X and Y distances, we can calculate $\emptyset$ using Equation 3 and subtract $45° - \emptyset = \theta$ in order to find the angle of refraction (See Fig 2).

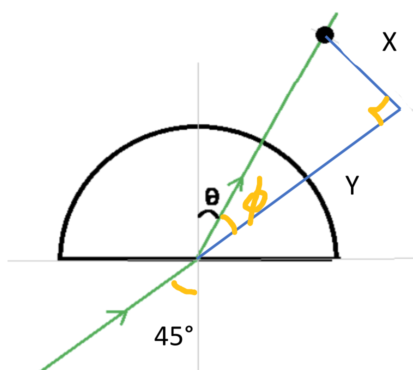


Figure 2: Mathematical model of experimental setup. Using this model and trigonometry, we can calculate the angle of refraction by measuring the distances X and Y.

The theoretical value for the speed of light travelling through glass at room temperature is approximately $2 \times 10^8 m/s$. [3] We predict that the speed of light travelling through glass will have an inverse relation with temperature. This means that we expect the speed of light at higher temperatures to be slower than $2 \times 10^8 m/s$.

Apparatus and Data

Experiment

The experiment was conducted in an optical room on an optical table to minimize vibrations which could alter the alignment of the setup and decrease external light from impacting our measurements. This set up consisted of three sections: Laser alignment, heating and conduction plate, and the webcam.

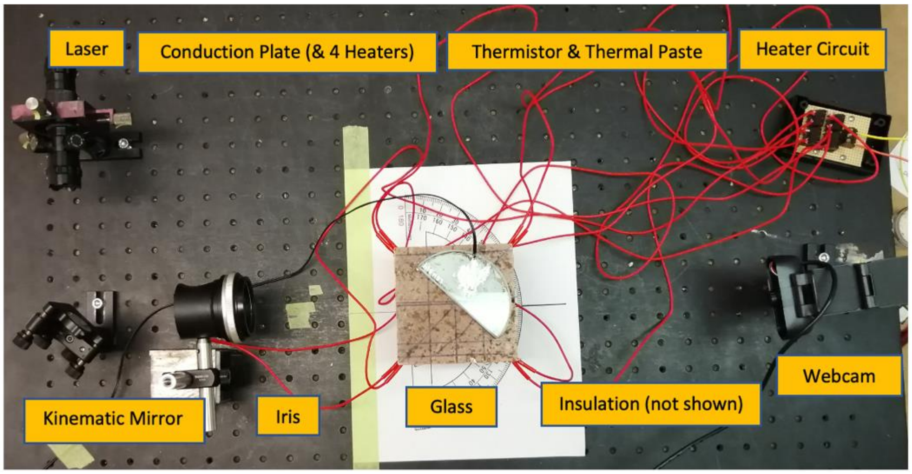


Figure 3: An overhead view of the setup on the optical table with the testing material (glass) at a 45 ° angle with respect to the laser's path.

Laser Alignment: In order to control the angle which is incident upon the material, we must be able to trace the laser's path. By adjusting the kinematic mirror, the laser's path is aligned along a straight path. To coarse tune, we mount the laser such that it's beam once reflected off the mirror is in the desired direction. We use a kinematic mirror to fine tune the alignment as the laser itself should not be disturbed after mounted. Additionally, vertical alignments of the conduction plate and webcam can easily be done using the laser since the laser simply needs to hit the side of the glass.

Once the laser alignment is straight and at the right height, the heating system must be assembled.

Heating and Conduction Plate: To heat our system, four 40W heater cartridges and a copper conduction plate are used. Four holes are drilled into each corner of the conduction plate and the heater cartridges are then placed in these holes. This design was done in order to heat the conduction plate and glass as evenly as possible to ensure that the temperature was uniform throughout the glass. To mitigate error in temperature readings, foam insulation was placed on top of the material to prevent heat loss.

To measure the temperature of the material, a thermistor was placed on top of the glass with a coat of thermal paste to more easily allow thermal conduction. This thermistor was connected to an Arduino UNO which read the temperature in degrees Celsius on the computer. Simple circuits and scripts of code were required for the heater cartridges and the thermistor. See Appendix (2) for code.

While wearing gloves, apply thermal paste to the flat face of the glass semi-circle before placing the side with thermal paste onto the copper conduction plate. The thermal paste is to help the copper plate to heat the glass.

The last part for this section is to align the material such that the laser light is incident upon the side of the glass at an angle of 45°. We do this by utilizing the symmetry of the conduction plate which is a 9.9 X 9.9 cm square. By aligning the conduction plate so that it was perpendicular to the laser, we know that the laser will hit the diagonal of the conduction plate at 45°.

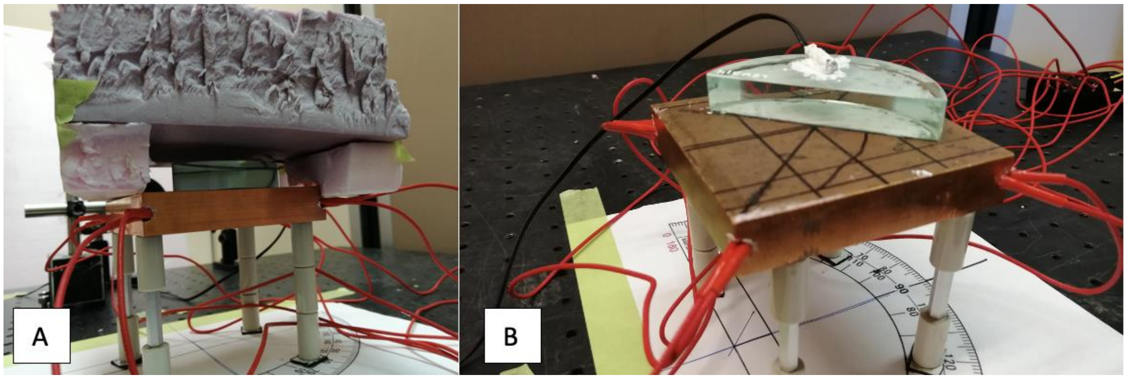


Figure 4A: Picture of the glass on heated (the red wires are connected to the heater cartridges) conduction plate held up with ceramic standoffs and covered by pink, foam insulation. Figure 4B: Picture of glass on heated conduction plate without foam insulation. The white paste is thermal paste used to assist thermal conduction from the glass to the thermistor.

Webcam: Finally, the webcam needs to be calibrated which in this case means finding its tarring or zero point. This point is just the point at which the laser impacts the webcam without any refraction (imagine if the laser goes straight through the glass). To do this, the webcam must be set up on the computer and able to capture the image of the laser. Once this was calibrated, like the rest of the system, precautions should be taken to not disturb the system as small errors can greatly influence measurements.

Once the system is set up (laser is aligned, heating system is assembled, and webcam is calibrated), experimental data collection can begin! When taking a trial:

1. While wearing proper laser safety equipment (in this case, protective laser lab goggles), turn on the laser and turn off the lights in the optical room.
2. Start Arudino code to take temperature readings and begin heating the system by setting the power source to send in 12A of current and provide 30V of voltage.
3. Keep a careful eye on the Ardiuno's serial monitor. Once the temperature reading from the thermistor reaches the desired temperature, capture the image of the using the webcam. Note the temperature reading along with the uncertainty for this measurement.
4. Continue to repeat step 3 for each temperature (30°, 40°, 50°) until the glass reaches 130°.
5. File all temperature readings, images, uncertainties, and any notable observations for data analysis.

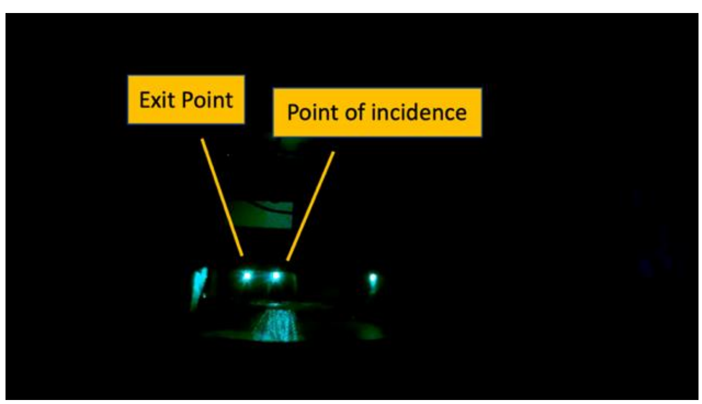


Figure 5: Image capture by webcam at 30°. Note that the unlabelled spots of light are reflections of the laser's light.

Simulations

Using the multiphysics simulation software, Comsol, we modelled a similar setup to measure the speed of light through silica glass with varying temperature. The packages used in this simulation were:

- Geometry and material: Used to obtain silica glass material and shape it into a semi-circle
- Ray optics: Used to generate a light source and model the laser's path
- Heat transfer through solids: Used to model a thermal source to heat the material

The key parameters of the simulations are the index of refraction (set when choosing the material), the angle of incidence, and the temperature of the glass. Once the key parameters of the simulation are set, in order to take a measurement:

1. Choose temperature parameter and click study
2. Comsol gives the angle of refraction
3. Export txt file with information and file away for analysis

Analysis

For experiments, once the picture at a given temperature was taken, the photo would be imported into a Python script which would give each pixel a numerical value based on its luminosity. These points were then mapped onto a luminosity vs position graph where the maximum luminosity determined the point at which the laser exited the glass. An example of this kind of graph is given for the trial with temperature 25°C (Fig. 6).

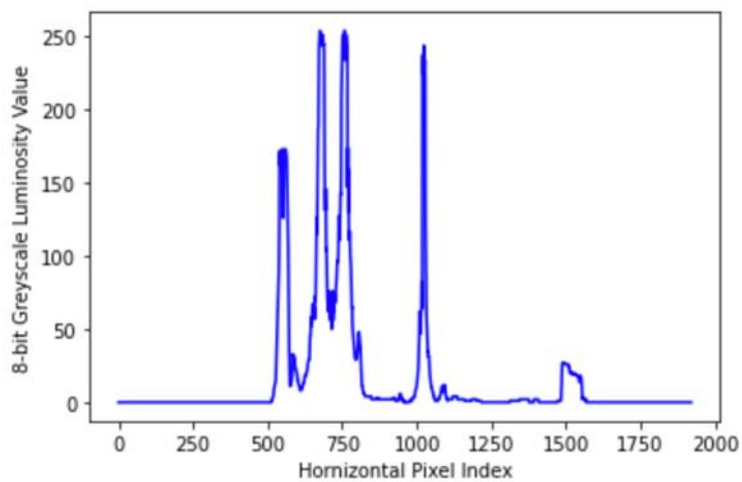


Figure 6:Luminosity vs position for the first trial taken when the temperature of the glass was measured to be 25℃.

Once the peak was determined, using trigonometric relations, the angle of refraction was calculated. A sample calculation for the trial with temperature 25° is given below.[1]

$$tan\theta = \frac{x}{y} \qquad (4)$$

$$\theta = tan^{-1}\frac{x}{y} \qquad (5)$$

Where x is the position/distance from the luminosity vs position graph (Fig. 6).

[1] Note that all calculations, error propagations, and graphs were done using QExPy on Jupyter notebooks

$$\theta = tan^{-1}\frac{1.96 \pm 0.05}{4.95 \pm 0.1} \quad (6)$$

$$\theta = 21.7° \pm 0.01° \quad (7)$$

$$\theta = 45° - 21.7° \pm 0.01° \quad (8)$$

$$\theta = 23.3° \pm 0.01° \quad (9)$$

Using Snell's law, we can now calculate the refractive index of glass at 25°C.

$$\frac{n_1}{n_2} = \frac{sin\theta_2}{sin\theta_1} \quad (10)$$

$$n_2 = n_1 \frac{sin\theta_1}{sin\theta_2} \quad (11)$$

$$n_2 = (1)\frac{\sin{(45°)}}{\sin{(23.3°)}} \quad (12)$$

$$n_2 = 1.79 \quad (13)$$

When plotting the refractive index of glass versus temperature we found the following parameters:

- Slope = -0.0014±0.0004
- Y-intercept = 1.86±0.04
- Reduced chi squared value = 0.85

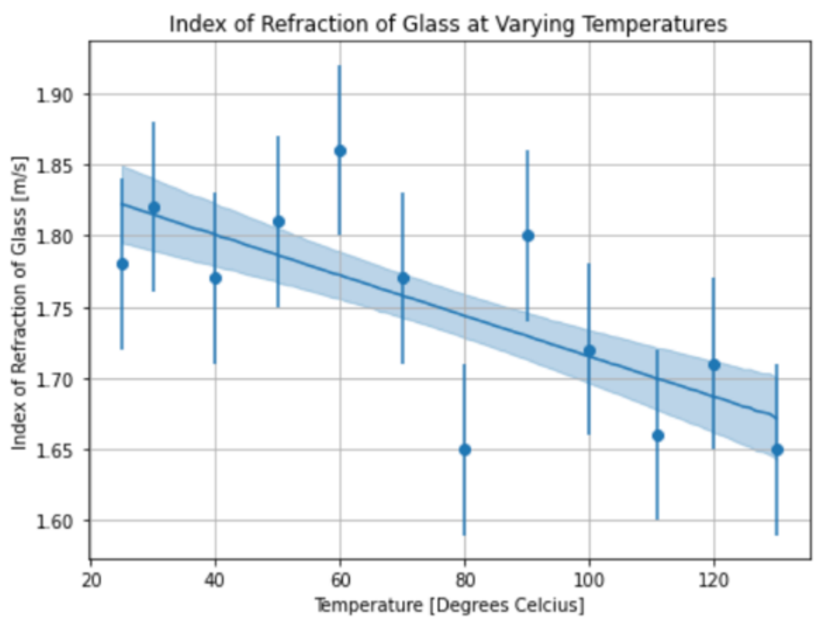


Figure 7: This graph shows the refractive index of glass versus temperature and depicts a linearly inverse relation.

From Figure 7, we can see a clear inverse relation between the refractive index and the temperature of the glass. The reduced chi square value supports that this is good fit.

Finally, using the calculated refracted index and the definition of the index of refraction, the speed of light was calculated travelling through the glass at the given temperature.

$$n = \frac{c}{v} \quad (14)$$

$$v = \frac{c}{n} \quad (15)$$

$$v = \frac{3\times 10^8\ m/s}{1.79} \quad (16)$$

$$v = 1.68 \times 10^8\ m/s \quad (17)$$

All calculations following the same procedure, and all collected raw data and analyzed data can be found in the Appendix (3) and (4). A plot of results found are given below (Fig. 8) and show a linear proportionality between the speed of light and the temperature of the glass. When plotting the graph, the following parameters were found:

- Slope = 14000 ± 4000
- Y-intercept = $1.61 \times 10^8 \pm 0.04 \times 10^8$
- Reduced chi squared value = 0.82

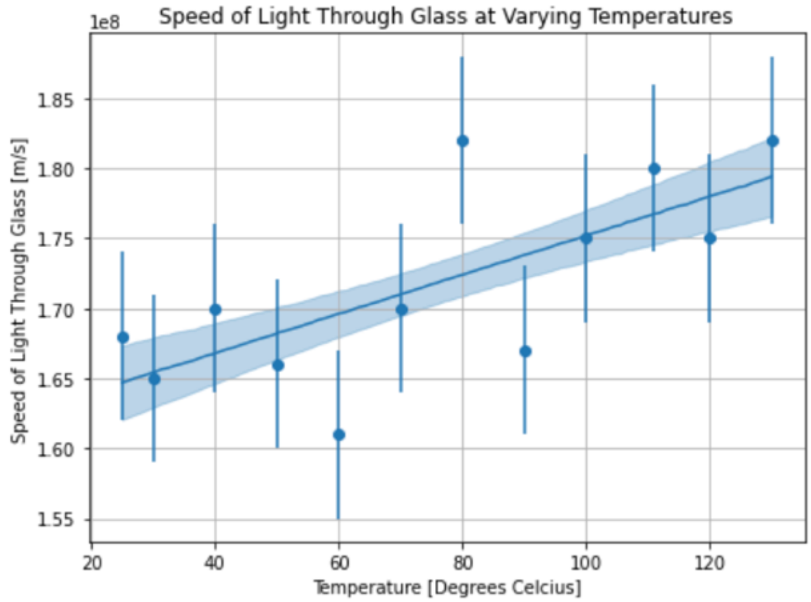
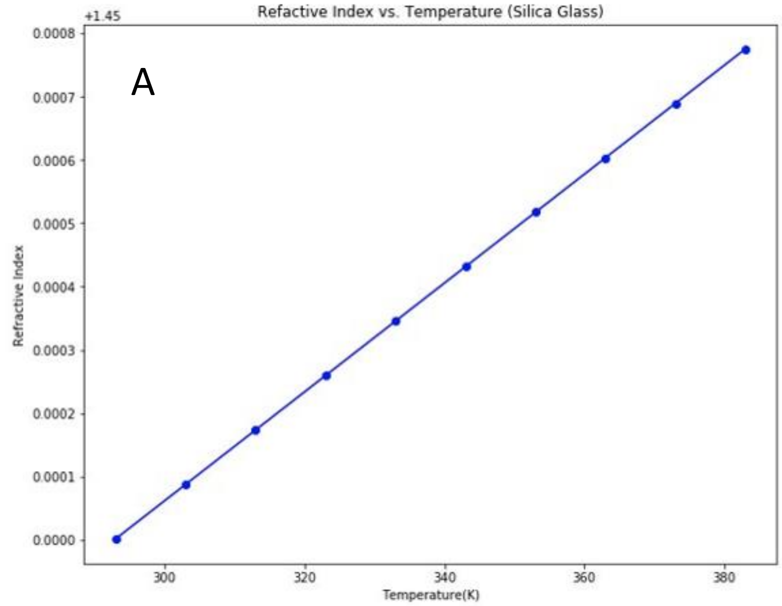


Figure 8: The speed of light travelling through glass versus the temperature of the glass. This graph shows the calculate speed of light and depicts a linear proportionality between the speed of light and the temperature of the medium it is travelling through.

The uncertainty recorded for the temperature readings was ± 0.02°C which was taken from the fluctuation in temperature read from the thermistor. This uncertainty is likely inaccurate since the temperature of the glass rose quickly. This meant that the temperature of the glass did not have time to stabilize which would have given a more accurate temperature. Additionally, the thermistor was only placed on top of the glass and so the bottom of the glass may have been a high temperature since it was in contact with the conduction plate.

There is a large uncertainty for the speed of light which can be attributed to errors in alignment (discussed below). However, the values for the speed of light in the experiment agrees with literature as none of the values surpass the theoretical value which is $2 \times 10^8 m/s$.

For simulations, the data was extracted from the simulations as txt files. See Appendix (5) for raw simulation data. Analysis for simulation data was completed in Jupyter notebooks and the plots are shown in Figures 9A and 9B. See Appendix (6) for all analyzed simulation data.



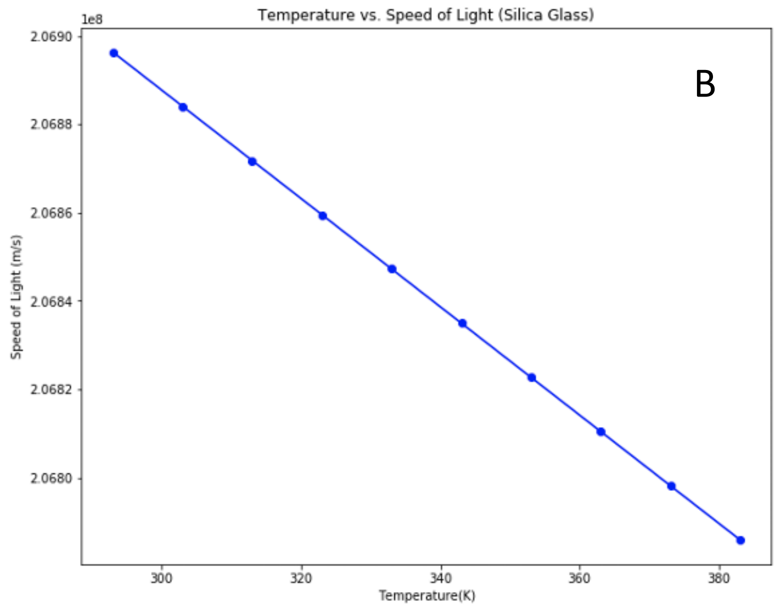


Figure 9A: A plot of refractive index of glass versus the temperature of glass. This shows a linear proportionality between the refractive index and the temperature Figure 9B: A plot of the speed of light versus the temperature of glass which depicts an inverse relationship between the speed of light and temperature.

Uncertainties in temperature and speed of light in this case are negligible as they are far too small to influence the data. The uncertainty is small for simulations as it is modelled in a perfect environment, where alignment is exact, and the temperature is uniform throughout the entire glass semi-circle.

Results and Discussion

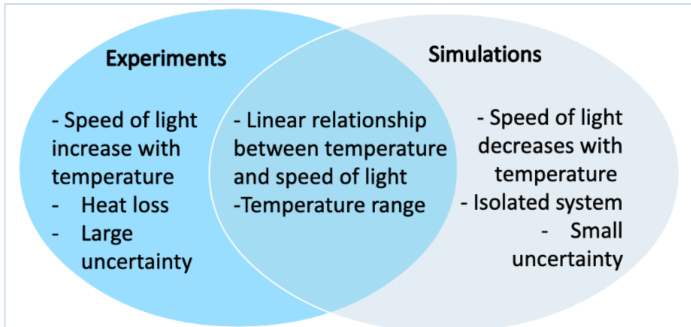


Figure 10: This Venn diagram summarizes the key similarities and differences from the results of this project's experiment and simulations.

From Figures 8 and Figure 9B, we can see that the results of the experiment and the simulations show different relations between the temperature of glass and the speed of light. The experiments show that the speed of light increases with temperature and the simulations show that the speed of light decreases as temperature increases. This discrepancy is due to the fact that the experiment team tested common glass (made of up a combination of silica, sodium carbonate, and calcium oxide) while the simulation team tested pure silica glass. However, both agree findings show that the relationship between the speed of light and temperature is linear which agrees with literature. Findings are summarized in Figure 10.

The simulation results have high confidence as measurements have low uncertainty as the system was modelled as a setup and environment that was free from real world sources of errors. However, due to the lack of real-world influences, the simulation results would be very difficult to match in an experiment. For more accurate and realistic results, the simulations should include heat loss.

The experiment, on the other hand, has many sources of errors which leads to large uncertainties in measurements and contradicts our predication. However, the experimental values for speed of light were all below the theoretical value ($2 \times 10^8 m/s$) which gives the findings some level of confidence. We can also recall that different types of glass will have varying reactions to changes in temperature. [1] When our predication was made, it was based on the assumption that the material would be pure silica (which was used in simulations) since many sources use this type of glass. However, the experimental glass was likely a combination of materials which could be a potential explanation to these different results.

Alignment was the largest contributor to error within our measurements as changes in the angle of refraction were predicted to be quite small. This is another potential explanation as to why the simulation and experimental results were different. Since even the slightest alignment issue can influence the measurement greatly, the collected measurements could have been skewed. Evidence of improper alignment are seen in the lab as there is internal reflection within the glass (Fig. 11B). Additionally, due to internal reflection, interference occurs which is observable when the diffraction pattern shows a sinusoidal pattern (Fig. 11A). A systematic alignment error occurred when aligning the material with the laser as it was difficult to ensure that the laser was incident precisely in the centre of the material at exactly 45°.

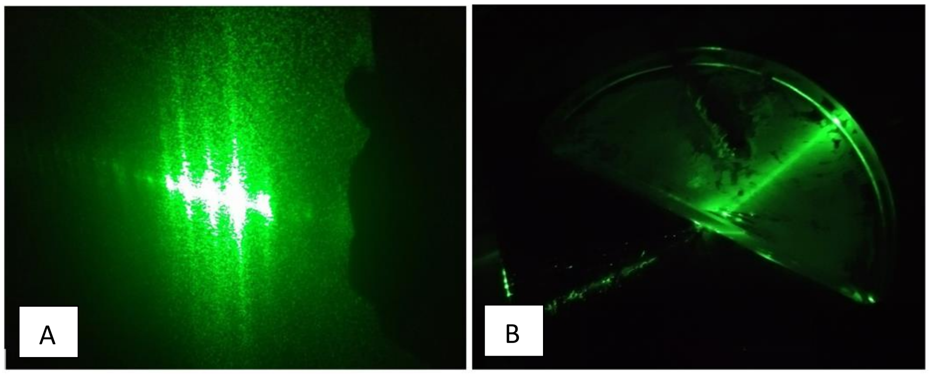


Figure 11A: Diffraction pattern after laser passes through glass material at room temperature. Figure 11B: Internal reflection observed within the glass material at room temperature after alignment.

In addition, vibrations from the external environments may have influenced the alignment as well as we only used the top of the optical table which would not provide as much damping as an actual optical table. This is a notable source of error since it was observed disturbing the system (the webcam was seen moving) when another group was heard talking loudly as they walked past our room. These vibrations most likely did not affect the laser alignment, as the laser alignment was fairly simple and so, if the vibrations had moved it enough for it to be influential on measurements, the laser would have not passed through the iris which we would have immediately noticed. It may have affected the conduction plate and the glass alignment as the standoffs supporting the conduction plate were not bolted down to the optical table, but rather given rubber feet to provide some damping. However, this was unlikely to have substantially affected measurements since the conduction plate is fairly heavy and would require much larger vibrations to actually have a noticeable effect. In order to mitigate the error from vibrations, the system was mounted as closely possible to the optical table since vibrations would have a greater effect on systems that are higher on the optical table.

Temperature readings in the experiment are low confidence values as errors were introduced from the heating system. Although the conduction plate was designed to ensure that it was heated evenly, it is difficult to ensure that the temperature of the glass itself was uniform when taking measurements. When taking measurements, we consistently heated the glass and so the temperature of the system rose in a matter of minutes and did not wait for the temperature to stabilize. This means the temperature reading from the thermistor may have been lower than the true temperature which would have been a systematic error as it would be present in each reading.

Random errors in temperature include heat loss and the contact made from the thermistor. To mitigate heat loss, we used foam to cover the glass material. Heat loss should not contribute much to the error in temperature as the temperature range was not vastly large. Additionally, after the collecting all measurements, it was noted that the thermistor had moved in such a manner that part of it was not in contact with the glass which would contribute to error as the measured temperature would be lower than the true temperature.

Another source of error could stem from assumptions made when designing the lab. Our model assumes that the system will act as a 2D ray optics model, however it was noted that during the trials, there was laser refraction in the z-axis. Since the changes in angle are so small, this small unaccounted distance may have given a smaller speed of light than its true value. Additionally, the temperature of the glass was assumed to be uniform, but that is unlikely because the temperature readings were taken without waiting for the temperature to stabilize.

5. Conclusions

In this project, the experimental results point to a linearly proportional relation between the temperature of the glass and the speed at which light travels through glass (as seen in Fig. 8). However, the simulations found that speed of light is inversely proportional to the temperature of the glass (as seen in Fig. 9B). Since the glasses were made of differing compositions, the relation between the speed of light and the temperature of the glass were different. However, both agreed that there exists a linear relation between the speed of light travelling through glass and the temperature of the glass.

This project was limited in time as we only have roughly 9 weeks to enter the lab, assemble the setup, and conduct the experiment. Additionally, the off-campus members of the team only had a few months to learn, understand, and conduct simulations. If our team had been provided a longer timeline, both on-campus and off-campus teams would have been able to collect more data which would allow for a more in-depth analysis to reinforce our findings. In addition, a larger data sample would help minimize random errors as well as more obviously highlight any systematic errors in the project.

Improvements for this project could include a more precise method of aligning the laser and material. Alignment was the largest factor in uncertainty and error in the experimental measurements so more precise and accurate alignment could give more accurate findings. One method would be to have the conduction plate on a rotational stand. By calibrating it properly, the incident angle could be set very precisely. If given a longer timeline, more data could have been collected which would help to further supporting findings. It was noted during the experiment, that vibrations did affect the webcam and so an improvement would have been to either conduct the experiment on an actual optical table (which would provide more damping and decrease the effects of vibrations) or have a larger camera which would be less susceptible to vibrations.

Extensions of this work could include expanding the materials tested such as testing glass that was coated with different materials, silicon, or even skin tissue. Additionally, expanding the temperature range could give a better understanding of the relationship between the speed of light and temperature at low or high limits of temperature. These extensions could give us a better understanding of light in varying environments which would be useful for many applications in the real world whether that be in the fast-growing field of silicon photonics or as a medical tool for doctors.

References

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Appendix for:

Temperature's Effect on the Speed of Light

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Submitted April 13, 2021

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(1) Given Snell's Law and the definition for the index of refraction we can show that light travelling from a lower index of refraction to a higher one will slow down. Consider a medium with index of refraction n_1 and a medium with index of refraction n_2 which is higher than n_1 (i.e. $n_1 < n_2$). This gives us the following equations from the definition of the index of refraction:

$$n_1 = \frac{c}{v_1}$$

$$n_2 = \frac{c}{v_2}$$

Where c is the speed of light through vacuum and v_1 and v_2 are the speed of light travelling through the medium with index of refraction n_1 and n_2, respectively. Dividing these equations, we find the following:

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

If $n_1 < n_2$, then for this equation to be true $v_1 > v_2$. This shows that as light propagates from a medium with a lower refractive index to a medium with a higher refractive index, its speed must slow down.

(2) Arduino Code: Code file was too large, but is available upon request

(3) Raw Experimental Data: The pixel in column two is the peak point on the luminosity vs position graph. To translate this to the distance from the zero point, we need to use the length of each pixel. We know the webcam has 1920 pixels in one row we calculated that each pixel was about 0.0240 cm/pixl.

Temperature (± 0.02 °C)	Pixel (± 1)	Distance (± 0.05 cm)
25	82	1.96
30	84	2.01
40	81	1.94
50	84	2.00
60	86	2.07
70	81	1.94
80	74	1.76
90	83	1.99
100	78	1.87

111	74	1.78
120	78	1.86
130	74	1.77

Table 1: Raw experimental data collected over a temperature range of 25-130 °C. The in the second column was the pixel which was the max luminosity on the luminosity vs position graph and the third column in the distance of that pixel from the zero point.

(4) Analyzed Experimental Data: Values calculated in the second and third columns follow the same procedure as demonstrated in the sample calculation Analysis section. All analysis was done in Jupyter notebooks and the code used can be made available upon request as it is far too large to fit here.

Temperature (± 0.02 °C)	Angle of Refraction (± 0.01 °)	Speed of Light (±0.06 x 10^8 m/s)
25	23.35	1.68
30	22.88	1.65
40	23.59	1.70
50	22.99	1.66
60	22.28	1.61
70	23.59	1.70
80	25.42	1.82
90	23.11	1.67
100	24.32	1.75
111	19.83	1.80
120	24.38	1.75
130	25.37	1.82

Table 2: Analyzed experimental data collected over a temperature range of 25-130 °C. Using Snell's Law and the definition of refractive index, the angel of refraction and the speed of light through glass was calculated.

(5) Simulation Raw Data: Collected simulations data from Comsol simulations quite large and unable to fit in Appendix. Can be made available upon request.

(6) Analyzed Simulation Data: Data was analyzed in Jupyter notebooks. This file was too large but can be made available upon request.