ESSAYS IN MONETARY POLICY AND BANKING

by

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Abstract

This dissertation investigates the impact of central banks’ asset purchase programs on the economy and the role of frictions in the corporate loan markets. It builds a series of models with trading and information frictions in goods market and credit market. Chapter 1 introduces the main idea in this thesis and presents a review on central banks’ asset purchase programs and unconventional monetary policies.

Chapter 2 constructs a model of the monetary economy with multiple nominal assets. Assets differ in terms of the liquidity services they provide. I show that the central bank can control the overall liquidity and welfare of the economy by changing the relative supply of assets. A liquidity trap exists away from the Friedman rule that has a positive real interest rate; the central bank’s asset purchase/sale programs may be ineffective in instances of low enough inflation rates. My model also enables me to study the welfare effects of a restriction on trading with government bonds.

Chapter 3 investigates the effects of open-market operations on the distributions of assets and prices. It offers a theoretical framework to incorporate multiple asset holdings in a tractable heterogeneous-agent model. This model features competitive search, which produces distributions of money and bond holdings as well as price dispersion among submarkets. At a high enough bond supply, the equilibrium shows segmentation in the asset market; only households with good income shocks participate in the bond market. Segmentation in the asset market is generated endogenously without assuming any rigidities or frictions in the asset market. Numerical exercises show that when the asset market is segmented, the central bank can improve welfare by purchasing bonds and supplying money.

Chapter 4 develops a model of loan markets in which lenders post an array of heterogeneous contracts, then borrowers tradeoff terms of loan contracts and matching probability between themselves. I show that a unique separating equilibrium exists where each type of borrower applies to a certain type of contract. Chapter 4 also provides empirical evidence of both price dispersion and credit rationing in the corporate loan market. Chapter 5 offers concluding remarks and possible extensions.
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Chapter 1

Introduction

Central bank open-market operations involve the purchase or sale of government bonds in the bond market. Conventionally, open market operations have been the means to implement monetary policies. The central banks’ involvement in the bond market impacts the prices of bonds and changes the interest rate. Conventional monetary policy is considered solely in terms of the choice of an operating target for a short-term nominal interest rate. In an environment where assets differ only in terms of risk and return, only one rate matters. As an example, assume a model environment with two types of trees. A short-term tree that yields fruit today and a long-term tree that yields fruit tomorrow. If the model is frictionless and the trees are only different in terms of risk and return, the short-term rate and the long-term rate in this model are connected through a no-arbitrage condition. A policy maker needs to control only one of these rates. Different asset purchase (or sale) schemes or other monetary policies (e.g., a helicopter drop of fiat money) by a central bank yield
the same result as long as they affect one of these rates in a similar fashion. In order to investigate a rich set of central banks’ policy options, we need a model that takes into account frictions in the asset market in which assets are not perfect substitutes.

Central banks’ asset purchase programs involve purchasing assets and paying with other assets that are different in terms of the liquidity services they provide. The liquidity characteristics of central banks’ and households’ balance sheets are affected by this practice. To investigate the liquidity effects of these policies, chapter 2 constructs a model of the monetary economy where households can trade goods for different types of assets that have different liquidity characteristics. I use a theoretical model to show the situations in which open-market operations may affect the decisions of households and the overall welfare in the economy. In these cases, a central bank can affect the amount of produced goods in the economy by adjusting the supply of illiquid assets and money. There is an optimal supply of illiquid assets that maximizes welfare in the economy. In an economy with two types of government-issued assets with different liquidity characteristics, the central bank is able to use open-market operations to change its balance sheet and therefore the liquidity characteristics of agents’ portfolios. A central bank’s asset purchase/sale programs can improve welfare by increasing (or decreasing) liquidity in periods of low (or high) liquidity.

Chapter 3 investigates the effects of central banks’ asset purchase (or sale) programs on the distribution of households’ asset holding. It constructs a model of a monetary economy with heterogeneous agents in which the central bank implements policies by changing the supply of nominal bonds and money. Chapter 3 shows that a segmented asset market arises under a specific parameters set; some households
participate in the bond market and hold positive portfolios of bond and money, while
others do not participate in the bond market and only keep money for transaction
purposes. When deciding whether to participate in the asset market, households com-
pare liquidity services provided by money with returns on bond. In an equilibrium
with a segmented asset market, open-market operations affect the participation deci-
sions of the households and, therefore, have real effects on the distribution of assets
and prices in the economy. Numerical exercises show that when the asset market is
segmented, the central bank can improve welfare by purchasing bonds and supplying
money.

On a different note, chapter 4 studies corporate loan markets. In a frictionless
credit market, firms that find it optimal to have bank debt in their capital structure
should be able to issue bank debt at a given market price. In the real world, however,
there is excess demand for loanable funds; i.e., certain firms apply for loans but fail
to receive them. There is also heterogeneity in the type of contracts that are used in
the credit market. Chapter 4 develops a model of a market with adverse selection and
search frictions, and shows that there exists a unique separating equilibrium, in which
each type of firm in need of bank credit applies to a different type of contract. In
this model, banks post the terms and conditions of credit agreements and potential
borrowers choose where to direct their search. Banks have imperfect information
about borrowers, and through the posted terms of trade, they can attract certain
types of borrowers and screen out others. The model is able to generate heterogeneity
in credit contracts along with credit rationing on the borrowers’ side. By using a novel
dataset that records successful and unsuccessful applications for bank credit as well as
the conditions of each credit agreement and firm characteristics, chapter 4 verifies the co-existence of unmatched credit seekers and the distribution in types of contracts.

The rest of this chapter reviews central banks’ asset purchase programs and unconventional monetary policies.

1.1 A review of central banks’ asset purchase programs

The key tool for implementing monetary policy is the interest rate on overnight loans between banks. In normal times, this rate is sensitive to the quantity of excess reserves. A central bank can control the rate on overnight loans by regular open-market operations. During and after the financial crisis of 2007, many central banks implemented policies that involved central banks’ participation in a variety of markets. Large-scale asset programs that change the size and the composition of central banks’ balance sheets were a major part of these policies. Traditional frictionless models of the monetary economy are not able to capture the real effects of these policies. Tobin (1969) P. 29 noticed the inability of traditional monetary models and states “there is no reason to think that the impact [of monetary policy] will be captured in any single [variable]... whether it is a monetary stock or a market interest rate”. Later Tobin and Buiter (1981) used the arguments in Tobin (1969) and state that central banks should actively participate in the private capital market to impact the economy in the long run. These policies would directly impact rates of return on capital and therefore
affect capital formation. This was the first time that economists were thinking about unconventional monetary policies. In order to investigate these policies, we need models with frictions in the asset market. Later economists started to build models that can generate real effects of central banks’ asset purchase (or sale) programs and are able to capture the real effects of unconventional monetary policies. In what follows, I review the literature on asset purchase programs with an emphasis on the models of unconventional monetary policies.

The first studied unconventional monetary policy is called “operation twist”. In 1961, in response to the recession, the Kennedy administration and the Federal Reserve decided to flatten the yield curve on treasury debt by keeping the short-term rate constant and lowering the long-term rates. Under this policy the Federal Reserve kept its federal funds rate constant and purchased long-term Treasury debt and agency-backed private debt. On the other side, the Treasury reduced its issuance of long-term debt and increased its issuance of short-term debt. These policies affected the short-term and long-term rates, agency, and the corporate bond market.

In the early 2000s, Japan suffered from problems caused by hitting the zero interest rate lower bound. In response, the Bank of Japan implemented a series of policies that would become known as quantitative easing (QE), which consisted of providing high levels of bank reserves and purchasing long-term government-issued debt.

During and after the global financial crisis of 2007, many central banks implemented a series of unconventional monetary policies in response to the financial crisis.

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1Modigliani and Sutch (1966), Modigliani and Sutch (1967) and Swanson, Reichlin, and Wright (2011) discuss these policies.
Also, the European Central Bank implemented similar policies in response to the euro area’s sovereign debt crisis in the period of 2010-2012. The term “unconventional” refers to the type of assets that were bought and the scale of these operations. During this period, an extensive amount of different types of assets, ranging from short-term and long-term government bonds to mortgage-backed securities and commercial papers, were purchased. As a result of central banks’ asset purchase programs, there have been two effects on the central banks’ balance sheets. First, the composition of the central banks’ balance sheets have shifted from short-term government bonds to a combination of long-term bonds and other types of assets. Second, the size of central banks’ balance sheets has expanded as a result of these operations. Some academics and policy makers separate the changes in central banks’ balance sheets in terms of their impact on unconventional monetary policies\(^2\): First, quantitative easing involves expanding the asset side of a central bank’s balance sheet by purchasing conventional assets and issuing reserves on the liability side\(^3\). Second, credit easing involves changing the composition of a central bank’s balance sheet by selling conventional assets and buying unconventional assets\(^4\). Others broadly define an expansion of a central bank’s balance sheet by purchasing unconventional assets as quantitative easing and do not separate changes in the composition and size. In this thesis, I separate the two channels of policy. Specifically, I will highlight changes in the composition of the central banks’ balance sheets in chapter 2 when I investigate the liquidity effects of

\(^2\) Bernanke (2009)
\(^3\) For an overview of QE see Cheung (2013).
\(^4\) Following the policy of the Federal Reserve in the early 1960s, this policy is also called “operation twist.”
credit easing.

What is the logic behind unconventional monetary policies? To answer this question, one has to think about the policy tools at or near the nominal policy interest rate of zero. At this point, central banks cannot use the conventional tools for monetary policy to further stimulate the economy. Policy makers believe that at zero lower bound, central banks are able to push down interest rates at different points of yield curve by purchasing long-term assets. They also believe central banks are able to affect prices of unconventional assets that are not the target of conventional monetary policies. The increase in prices of unconventional assets is believed to cause higher consumption and demand through wealth effects and also spill over to other assets in the economy through the portfolio rebalancing channel. Another channel that is highlighted by policy makers is that unconventional monetary policies signal lower interest rates in the long run and therefore increase investment.

In late 2008, after reducing its target for the federal funds rate to nearly zero, the Federal Reserve began a series of large-scale asset purchases (LSAPs). The Federal Reserve began purchasing long-term government bonds and assets that were guaranteed by government sponsored agencies. In purchasing guaranteed mortgage-backed securities (MBSs), the belief of the policy makers was that a lower yield on MBSs translates to lower mortgage rates and would stimulate the housing market. Moreover, central bankers believed that in search for higher yields, private investors would shift

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5See for example Bernanke and Reinhart (2004).
6For a discussion on different channels of these programs see Bernanke, Reinhart, and Sack (2004).
7i.e., Fannie Mae or Freddie Mac.
their portfolios towards more corporate bonds and other securities, and the effects would spill over to other markets.

In May 2010, in response to the sovereign debt crisis in the euro area, the European Central Bank decided to start its own version of unconventional monetary policies, the Securities Markets Programme (SMP). Unlike the Federal Reserve’s large-scale asset purchases (LSAPs), SMP was not aimed at affecting different points on the yield curve. The program’s main goal was to purchase government bonds that were experiencing a big drop in demand (mainly government bonds issued by Italy, Greece, Ireland, Spain, and Portugal). As a result of the SMP announcement, markets experienced a significant and persistent change in rates\(^8\). The last asset purchase program by the ECB took place in February 2012. In 2009 and 2010, the Bank of England also set up the Asset Purchase Facility (APF) to purchase a variety of high quality assets. This program was very similar to the Federal Reserve’s set of unconventional policies in response to the financial crisis and resulted in the expansion, and changes in the composition, of assets of the Bank of England.

1.2 Literature on asset purchase programs

Following Wallace (1981), a branch of literature uses a Modigliani-Miller argument to show that the size and the composition of central banks’ balance sheets and thus open-market operations do not have any real effect on the economy. In these models, assets are perfectly substitutable in terms of liquidity services. Open-market operations do

\(^8\)For a discussion of SMP, see Eser and Schwaab (2013).
not change the liquidity characteristics of households’ asset portfolios. In order to capture real effects of asset purchase programs, we need models with frictions in the asset market in which assets are imperfect substitutes.

In cash-in-advance models, Grossman and Weiss (1983)\footnote{Rotemberg (1982) uses a similar model.} assumes that only a fixed fraction of the population can withdraw funds from their bank accounts each period. This model is a variation of earlier costly exchange models\footnote{Models that follow a Baumol (1952) and Tobin (1956) structure.}, where agents suffer transaction costs when they trade assets. Grossman and Weiss (1983) assumes that some agents suffer infinite transaction costs when they trade assets (non-traders in the asset market) and that for some agents there are no transaction costs (traders in the asset market). Only agents who are withdrawing funds from their accounts are affected by open-market operations each period. Since agents have limited access to their bank accounts, they spend their cash holdings slowly over time. This generates the delayed response to monetary shocks seen in the data.

Following Grossman and Weiss (1983), a branch of literature uses asset market segmentation to explain persistence responses to monetary shocks observed in the data\footnote{For an overview of this literature see Edmond and Weill (2008)}. Unlike the Friedman helicopter drop\footnote{In the instance of Friedman’s helicopter drop, money is uniformly distributed by the central bank.}, in models with segmented asset markets, only the fraction of agents who are active in the asset markets immediately receive the monetary shocks. As a result, it would take time for the monetary shocks to affect other agents in the economy. This literature explains the real effects of money injection and open-market operations using the generated segmentation in the
Similar to Grossman and Weiss (1983), Alvarez, Lucas, and Weber (2001) assumes that only a fixed fraction of the population can withdraw funds from banks each period. In Alvarez, Atkeson, and Kehoe (2000), agents must pay a Baumol-Tobin style\(^{13}\) fixed cost to transfer money between the asset market and the goods market. However, their assumption that agents spend all of their money balances in each period takes away a part of the sluggish movements seen in data. In a similar fashion, Khan and Thomas (2010) assumes agents pay idiosyncratic fixed costs to transfer wealth between interest-bearing assets and money.

Andres, Lopez-Salido, and Nelson (2004) study portfolio rebalancing channels by building a New Keynesian model of the monetary economy. The model features short-term bonds and long-term bonds and limited participation of agents in the market for short-term bonds. The exogenous asset market participation constraint on the agents makes short-term and long-term assets imperfect substitutes. Changes in a central bank’s balance sheet make real changes in the economy. Chiu (2007) assumes that agents pay a fixed cost to participate in the asset market and choose the timing of money transfers. As a result, the degree of asset market participation is endogenous. In a micro-founded monetary framework, Williamson (2008) links asset market segmentation to the goods market segmentation and is able to generate both the liquidity effect and the Fisher effect of monetary policies.

\(^{13}\)Baumol (1952) and Tobin (1956)
Literature on unconventional asset purchase programs

In this section, I provide a summary of the literature on unconventional asset purchase programs. Currently, there is a gap between the theoretical and the empirical literature on these policies. While many empirical papers investigate different channels, there is a lack of theoretical frameworks that explain these channels. On the theory side, I discuss two papers. Curdia and Woodford (2011) add an intermediary sector to a canonical New Keynesian model. The model in Curdia and Woodford (2011) is able to analyze three separate central banks’ policies regarding quantity of reserves, interest paid on reserves, and the combination of central banks’ balance sheets. This allows them to study a rich set of central bank policies. They find that quantitative easing in the strict sense is likely to be ineffective. Williamson (2012) adds an intermediary sector to a New Monetarist monetary framework. In a version of the model with public and private assets, Williamson (2012) shows that a policy similar to the first round of quantitative easing pursued by the Federal Reserve is, at best, ineffective.

There exists a vast and growing empirical literature on unconventional monetary policies conducted in different countries. New evidence from the recent policies is adding to this literature. Here, I highlight a few of these studies that are related to the experiences of unconventional monetary policies in different countries. To summarize this literature, there is general agreement on the effects of asset purchase.

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14 Similar to the framework in Woodford (2011)
16 A model based on Lagos and Wright (2005) with heterogeneous agents similar to Rocheteau and Wright (2005).
programs on government bond yields. However, there is less agreement regarding the
channels through which asset purchase programs affect asset prices, and how these
policies spill over to the rest of the economy.

The first study of unconventional monetary policies dates back to the early 1960s.
Modigliani and Sutch (1966) and Modigliani and Sutch (1967) conduct an event
study to investigate the “operation twist” that was implemented in 1961 in the U.S.
Using the quarterly data from 1961-1965 they show that “operation twist” increased
the yield on short-term assets, but was not successful in lowering long-term yields.
Swanson, Reichlin, and Wright (2011) use higher frequency data from the same period
and show the highly statistically significant effect of the “operation twist” on longer-
term Treasury yields. They also show that the policy spilled over to the long-term
agency and corporate bond markets, but the effects were smaller.

In order to evaluate the Federal Reserve’s LSAPs, Krishnamurthy and Vissing-
Jorgensen (2011) use an event-study methodology to investigate both the daily and
the intra-daily data. They find significant signaling channel for LSAPs and significant
effects of these programs on long-term bonds. They also find that mortgage-backed
securities purchases lowered mortgage-backed security yields significantly. Hancock
and Passmore (2011) focus on purchases of mortgage-backed securities (MBS). They
develop a regression analysis that can be used to distinguish between different chan-
nels of MBS purchases. They find that the continuous purchase of these assets puts
significant downward pressure on mortgage rates.

Eser and Schwaab (2013) conduct a time series panel data regression and an event
study of the effects of ECB’s Securities Markets Programme (SMP). They find that
SMP has significant announcement effects. They also find that SMP has significant and heterogeneous persistence effects in all of the markets. Joyce, Lasaosa, Stevens, and Tong (2011) conducted an event study and a time series analysis of the Bank of England’s Asset Purchase Facility (APF) and found significant effects on asset prices through the portfolio rebalancing channel.

The Bank of Japan’s policy of QE has been widely studied and most of the studies find similar significant effects on asset prices\footnote{Marumo, Nakayama, Nishioka, and Yoshida (2003), Okina and Shiratsuka (2004), Baba, Nishioka, Oda, Shirakawa, Ueda, and Ugai (2005), and Oda and Ueda (2005) study QE in Japan and find similar results on asset prices and rates.}. Here, I focus on Bernanke, Reinhart, and Sack (2004). They use both an event study methodology and a no-arbitrage vector autoregression (VAR) model of the term structure of interest rates to study QE in Japan. Using a benchmark predicted term structure they are able to assess the impact of QE on interest rates. Although they find that the rates are significantly affected by the Bank of Japan’s QE, their model is unable to distinguish different channels of this policy.

**Organization of thesis**

The next chapter studies the liquidity effects of central banks’ asset purchase programs. Chapter 3 builds a model that focuses on the distributional effects of central banks’ asset purchase programs. Chapter 4 investigates credit rationing and heterogeneity in corporate loan contracts. Chapter 5 concludes. All proofs are relegated to Appendices A, B, and C.
Chapter 2

Liquidity Effects of Central Banks’ Asset Purchase Programs

2.1 Introduction

Central banks’ asset purchase programs involve purchasing assets and paying with assets that are different in terms of the liquidity services they provide. The liquidity characteristics of the central banks’ and households balance sheets are affected by this practice. To investigate the liquidity effects of these policies, I construct a microfounded model of monetary economy where households can trade goods with different types of assets. I use the theoretical model to show that within a specific set of parameters, open-market operations may affect the decision of households in the economy and welfare. In these cases, the central bank can affect the amount of produced goods in the economy by trading illiquid assets with money. There is an
optimum supply of bonds that maximizes welfare in the economy. In an economy with two types of government-issued assets with different liquidity characteristics, the central bank is able to use open-market operations to change the liquidity characteristics of agents’ portfolios. The central bank’s asset purchase/sale programs can improve welfare by increasing (decreasing) liquidity in periods of low (high) liquidity.

During the period 2008 – 2011 many central banks implemented a series of unconventional monetary policies in response to the financial crisis. A major part of these policies was the large-scale asset purchase programs (known as quantitative easing). The Bank of Japan implemented similar policies from 2000 – 2006. These programs are basically open-market operations that change the size or the composition of central banks’ balance sheets. Similarly, the Fed implemented two sets of policies in response to the financial crisis: 1-Quantitative easing: expanding the asset side of the central bank’s balance sheet by purchasing conventional assets\(^1\) and issuing reserves on the liability side. 2-Credit easing: changing the composition of the Fed’s balance sheet by selling conventional assets and buying unconventional assets\(^2\).

While academics discuss several channels through which these policies can affect the real economy (e.g. Krishnamurthy and Vissing-Jorgensen (2011)), policy makers (e.g. Bernanke and Reinhart (2004)) mainly highlight two: 1-Signaling lower interest rate in the long-term. 2-Increasing demand for other assets in the economy and decreasing yield on these assets\(^3\).

\(^1\)In the US this mainly takes the form of treasuries, and in Canada this mainly takes the form of bonds, term purchase, and resale agreements for the private sector.

\(^2\)Credit easing is also called an asset sterilizing program or Operation Twist.

\(^3\)Agents rebalanced their portfolios towards other assets in the economy.
The literature on open-market operations and quantitative easing falls into two categories. First, there are earlier papers that show open-market operations are irrelevant for the real economy. In these models assets are perfectly substitutable in terms of liquidity services, and open-market operations do not change the liquidity characteristics of households asset portfolios. In a model with liquid bonds, since bonds and money are perfectly substitutable, households have a similar liquidity preference toward holding bonds and money. Households cannot use bonds to affect the liquidity characteristics of their portfolios.

Second, papers show that open-market operations can affect the real economy. In a model in which interest bearing assets provide different liquidity services compared to money, open-market operations change the liquidity characteristics of households’ portfolios and have real effects on the economy. Kocherlakota (2003) uses a similar argument and shows that in a centralized market, agents use illiquid bonds to smooth consumption. Andres, Lopez-Salido, and Nelson (2004) study portfolio rebalancing channel by building a New-Keynesian model of the monetary economy. Curdia and Woodford (2011) study the effects of size of central banks’ balance sheets in a New-Keynesian model. They find out that if we do not take into account the signaling channel, pure open-market operations have no real effects. Auerbach and Obstfeld (2005) study the replacement of interest-bearing government debt with non-interest-bearing currency or reserves on central bank’s balance sheets and find out that quantitative easing has real effects. While these papers answer some of the questions concerning open-market operations, they do not discuss the liquidity channel and they use reduced form models (e.g., money in the utility function and sticky
prices). Kiyotaki and Moore (2012) study a model of monetary economy with differences in liquidity across assets. They show that open-market operations are effective when the central bank purchases the assets with partial resaleability and a substantial liquidity premium during negative liquidity shocks. The illiquid asset in their model are mainly capital and securities that are issued based on capital, and their analysis focuses on the role of open-market operations on privately provided liquidity.

I expand the existing literature on the effects of open-market operations by building a micro-founded model of a monetary economy. The basic model is a variation of Shi (2008), who uses a similar framework to study the legal restrictions on trade with nominal bonds. Agents can trade with different government-issued assets that provide different liquidity services. Contrary to Shi (2008), here the argument is not based on parameters in the utility function. Shi (2008) assumes that agents can use bonds to trade certain types of goods that yield a higher utility when consumed. Here, consumption of different goods yield the same amount of utility and my analysis hinges on the liquidity characteristics of assets. Assets are different in terms of the liquidity services they provide. Central bank’s open-market purchase of assets increases liquidity in the economy by injecting money and purchasing interest-bearing assets.

How does this model investigate the liquidity effects of central banks policies? First, I use a household structure, which helps to build a tractable model that avoids the evolving distribution of asset holding. In this structure, households do not face any intertemporal uncertainty. Therefore, there is no precautionary motive for saving. Households buy assets only for the liquidity services they provide. The yield on assets
is a pure liquidity premium. Second, I model liquidity services provided by assets. Households do not gain utility by holding assets. This allows me to investigate the effects of different central policies on the liquidity premium on assets and the overall welfare in the economy.

The model of this chapter can also be related to Ricardian equivalence. In the extension of the model with two interest bearing assets, government has two sources of financing that have different liquidity properties. There is no difference between maturities of these assets in the model, but we can think of the less liquid bond as a long term bond and the more liquid bond as a short term bond. These interest bearing assets can be used as media of exchange in some transactions. The model shows that when the households are liquidity constrained in terms of both types of bonds in the model, an asset purchase program that changes the portfolio of households asset holding would have real effect on the economy. In this situation with binding liquidity frictions, Ricardian equivalence fails. Alterations in the maturity structure of government debt would affect the liquidity characteristics of households’ portfolios of asset holding.

In the literature on monetary economics and policy liquidity traps are mostly associated with the Friedman rule\(^4\). Williamson (2012) studies a liquidity trap in cases where the economy is away from the Friedman rule and when the real interest rate is zero. He discusses the liquidity channel of open-market operations in a model with public and private liquidity in which it is costly to operate a monetary system. In this paper, we can have the properties of the liquidity trap equilibrium when the

\(^4\)Money grows at the rate that agents discount future consumption.
real interest rate is positive. In this case, marginal open-market operations do not have real effects on the economy. In an extension of the model with three assets I show that a policy of credit easing can affect welfare.

2.2 Model environment

Time is discrete and has infinite horizons. There are $H$ types of households ($H \geq 3$). Each household consumes a good that is produced by some other type of household, type $h$ household consumes good $h$ but produces good $h + 1$. There is no double coincidence of wants, and goods are perishable. Each household consists of a large number of members (measure one). These members could be sellers (measure $\sigma$), buyers (measure $N - \sigma$), or leisure seekers (measure $1 - N$). Buyers and sellers trade goods while leisure seekers are inactive. There is perfect consumption insurance between household members; members of a household share consumption and regard utility of the household as the common objective.

There are two markets in this economy, a centralized market for assets and a decentralized market for goods. Money and bonds are supplied by the central bank. The central bank implements policies by printing money at rate $\gamma$ and changing the relative composition of the stock of bonds and money in the economy. In the centralized market for assets, the government bonds are sold for money. In the decentralized market for goods, search frictions exist. Buyers and sellers of different households are randomly matched in pairs. The number of matches for each household is $\alpha N$, where $\alpha$ is a parameter of the environment and $N$ is the aggregate number of traders.
in the market. According to this matching technology, the matching rate for buyers is \( \frac{\alpha N}{N-\sigma} \) and the matching rate for the sellers is \( \frac{\alpha N}{\sigma} \). The matching process of a three household economy is shown in figure 2.1. Because of the assumed structure of the environment, a successful match is between a buyer of household “\( h \)” and a seller of household “\( h + 1 \).”

\[ \text{Figure 2.1: Matching process in a 3 household economy} \]

In the centralized market for assets, households trade government bonds for money. In the decentralized market for goods, household members trade goods for money or government bonds. Trade history is private information, agents are anonymous, and the population is large. Therefore, there is no credit. After household members are matched, a matching shock determines the type of assets they can use for trade. With probability \( 1 - l \), they can only use money (this trade is indexed by subscript “\( m \)”).

\[ ^5 \text{I assume } \alpha \text{ is low enough that matching rates are less than } 1. \]
to purchase goods, and with probability \( l \) they can use both money and bonds (this trade is indexed by subscript “\( b \)”) to purchase goods\(^6\).

### 2.2.1 Household decisions

The representative household solves the following maximization problem:

\[
v(m, b) = \max_{c_i, q_i, x_i, n, m+1, b+1} \left\{ u(c_m) - \alpha N(1 - l)\psi(Q_m) 
+ u(c_b) - \alpha N l \psi(Q_b) + h(1 - n) + \beta v(m+1, b+1) \right\}, \quad i \in \{m, b\} \tag{2.1}
\]

subject to the following constrains:

\[
x_m \leq \frac{m}{n - \sigma}, \tag{2.2}
\]

\[
x_b \leq \frac{m + b}{n - \sigma}, \tag{2.3}
\]

where lower case letters are choices of the household under consideration and capital letters are per capita variables that individual households cannot affect.

Households choose consumption \((c_i)\), terms of trade \((q_i, x_i)\), number of traders \((n)\), and asset portfolio \((m+1, b+1)\) for the next period to maximize the above value function. The utility from trade is the sum of the net utility in each type of trade. In each trade, household shares the utility from consumption of the purchased goods

\(^6\)Different types of matches can be interpreted as “monitored” and “non-monitored” matches as in Williamson (2011).
and the cost of production of the sold goods. In a money trade a representative household consumes \( c_m \) and produces \( Q_m \). The total number of money trades for the representative household is \( \alpha N(1 - l) \). Similarly, in a money and bond trade, a representative household consumes \( c_b \) and produces \( Q_b \). Since buyers have all the bargaining power, the amount sold is shown by capital letters \( Q_i \). \( u() \) is continuous and twice differentiable, and \( u'(l) > 0 \), \( u''(l) < 0 \). I assume \( \psi'(l) > 0 \), \( \psi''(l) > 0 \); \( h'(l) > 0 \) and \( h''(l) < 0 \). Each household divides its members into three groups: sellers/producers (measure \( \sigma \)), buyers (measure \( n - \sigma \)), and leisure seekers (measure \( 1 - n \)). Households choose \( n \), and \( \sigma \) is fixed\(^7\). In each type of trade, buyers are constrained by the portfolio of assets that they have. In a money trade, buyers are constrained by the amount of money they have \((2.2)\). In a money and bond trade, buyers are constrained by the total portfolio of assets they carry \((2.3)\). Goods are divisible and perishable. Consumption in each type of trade is the matching rate times the total amount of goods bought by the buyers in that trade

\[
\begin{align*}
c_b &= \frac{\alpha N}{(N - \sigma)} (n - \sigma) l q_b \\
c_m &= \frac{\alpha N (n - \sigma)(1 - l)}{(N - \sigma)} q_m.
\end{align*}
\]

\(^7\)This assumption is for simplicity. I can allow households to choose \( \sigma \) and the main results hold.
Let us define $\omega_i, \ i \in \{m, b\}$ as the marginal value of assets

$$\omega_m = \frac{\beta \partial v(m, b)}{\gamma \partial m_{+1}}$$

$$\omega_b = \frac{\beta \partial v(m, b)}{\gamma \partial b}.$$ 

$\Omega_m$ is the per capita value of money in the economy. In each trade, sellers sell goods for a portfolio of assets, which has a marginal value of $\Omega_m$. Seller’s surplus is $x_i \Omega_m - \psi(q_i), \ i \in \{m, b\}$. Since buyers have all the bargaining power, the offer sets sellers’ surplus to 0. Thus, the participation constraint is $x_i \Omega_m - \psi(q_i) = 0$, and can be written as:

$$x_i = \psi(q_i) / \Omega_m \quad i \in \{m, b\}.$$

(2.4)

The value of household’s asset portfolio in terms of money follows equation 2.5:

$$(m_{+1} + s_{+1} b_{+1} + T_{+1}) \gamma =$$

$$m + b + \alpha NLX^b + \alpha N(1 - l)X^m - \frac{\alpha N(n - \sigma)}{N - \sigma} lx^b - \frac{\alpha N(n - \sigma)}{N - \sigma} (1 - l)x^m,$$

(2.5)

where $s_{+1}$ is the price of bond in the asset market. Money balance plus the amount spent on the assets in the next period and the tax (or transfers) is equal to the portfolio of assets in the current period plus the assets that the sellers bring back minus the assets that buyers have spent on their purchases of goods.


## Timing

The timing of the events is shown in figure 4.1.

<table>
<thead>
<tr>
<th>Asset portfolio ((m, b))</th>
<th>Choices: (n, (x_i, q_i))</th>
<th>Trade in goods</th>
<th>Consumption</th>
</tr>
</thead>
</table>

At the beginning of each period, the asset market opens. Households redeem nominal bonds from the previous period for one unit of money, trade assets for money, receive transfer \(T\), and adjust their portfolios to \((m, b)\). The asset market is closed until the beginning of the next period. Households choose the amount of total traders \(n\) and give buyers instructions on how to trade in different types of trade (Goods: \(q_i\), assets: \(x_i, i \in \{m, b\}\)). Buyers and sellers search in the goods market and match according to the linear matching function. Matched sellers produce and trade and then bring goods and assets back to the household and members of the household share consumption.
CHAPTER 2. LIQUIDITY EFFECTS

2.2.2 Optimal choices

The first order condition for $q_i$ is

$$u'(c_i) = \left(\omega^m + \lambda^i\right)\frac{\psi'(q_i)}{\Omega} \quad i \in \{b, m\},$$

(2.6)

where $\lambda^b$ and $\lambda^m$ are the Lagrange multipliers on trades with bond and money, respectively. I can solve for bond prices by taking the first order conditions with respect to $b$:

$$b_{+1} : s_{+1} = \frac{\omega_b}{\omega_m}. \quad (2.7)$$

Households’ choices of the measure of traders ($n$) solves the following:

$$h'(1-n) = \frac{\alpha N}{N-\sigma} \left[ lt'(c_b)(q_b - \frac{\psi(q_b)}{\psi'(q_b)}) + (1-l)t'(c_m)(q_m - \frac{\psi(q_m)}{\psi'(q_m)}) \right]. \quad (2.8)$$

The envelope conditions for $m_{+1}, b_{+1}$ are:

$$m_{+1} : \frac{\gamma}{\beta} \omega_{+1}^m = \omega_m + \frac{\alpha N l}{N-\sigma} \lambda^b + \frac{\alpha N (1-l)}{N-\sigma} \lambda^m \quad (2.9)$$

$$b_{+1} : \frac{\gamma}{\beta} \omega_{+1}^b = \omega_m + \frac{\alpha N l}{N-\sigma} \lambda^b. \quad (2.10)$$

At the end of each period, each unit of bond is redeemed for a unit of money, therefore the value of an asset is the value of money in the following period plus the liquidity services that the asset provides, accounting for discounting and inflation. Money provides liquidity services in all types of trades, and bonds are used in certain
types of trade. Expressions 2.9 and 2.10 show that money has a liquidity premium over bonds.

2.2.3 Definition of the equilibrium

**Definition 1.** An equilibrium is households’ choices \((c_i \in \{m,b\}, q_i \in \{m,b\}, x_i \in \{m,b\}, n, m_{-1}, b_{+1})\), the value function \((v(m, b))\), shadow value of assets \((\omega^m, \omega^b)\), asset price \((s)\), and other households’ choices, such that

1. Given bond price \((s)\), and choices of others, household choices are optimal (2.1).

2. The choices and shadow prices are the same across households, i.e., \(q_i = Q^i, x_i = X^i, n = N, \omega^i = \Omega^i\).

3. Bonds market clear \((b = B)\).

4. Positive and finite values of assets \((0 < \omega < \infty)\).

5. Stationarity: quantities and prices are constant over time.

2.2.4 Welfare analysis

The envelope conditions show that the only point at which all of the constraints are non-binding is where \(\gamma = \beta\). Let us call the Lagrange multiplier on the constraint of a money trade \(\lambda^m\) and similarly the Lagrange multiplier on a constraint of a bond and money trade \(\lambda^b\).
Lemma 1. At Friedman rule ($\gamma = \beta$), $\lambda^m = \lambda^b = 0$. For $\gamma > \beta$, $\exists i \in \{m, b\}$ such that $\lambda^i > 0$.

In order to study open-market operations, let us define the ratio of stock of bonds to stock of money as:

$$z = \frac{B}{M}.$$ 

The central bank implements policies by changing the inflation rate ($\gamma$) and relative supply of assets ($z$). Changes in $z$ are the effects of open-market operations. Open-market purchase (sale) of bonds decreases (increases) $z$.

I define the welfare function as the utility function of a representative household.

$$w = u(c_b) - \alpha N l \psi(Q_b) + u(c_m) - \alpha N (1 - l) \psi(Q_m) + h(1 - n). \quad (2.11)$$

By using the above measure of welfare, I can study the welfare effects of policies. We have four types of equilibria based on the set of binding liquidity constraints. The only point at which all of the liquidity constraints are non-binding is at the Friedman rule. The Friedman rule is shown to be optimal in a wide variety of models. As the next proposition shows, the Friedman rule is optimal in this framework.

**Proposition 1.** The Friedman rule is optimal.

The proof of the above proposition is intuitive. Since buyers’ bargaining power is 1, households send too many buyers compared to the planner’s choice. Increasing $\gamma$ punishes unmatched buyers and the representative households. On the other hand, inflation decreases the amount of goods in each trade. The former effect is known
as the extensive margin of trade, and the latter is known as the intensive margin of trade. Both intensive margin \( (q) \) and extensive margin \( (n) \) decrease with inflation \( (\gamma) \). The planner chooses the lowest possible level for \( \gamma \) to maximize welfare. Therefore, the Friedman rule is optimal.

Based on the set of liquidity constraints that are binding, we can have four types of equilibria. As shown before, an equilibrium where both of the liquidity constraints are non-binding can only happen at the Friedman rule and this equilibrium is efficient. In Appendix A, I have characterized different types of equilibria and proved the following proposition:

**Proposition 2.** _Open-market operations can only have welfare effects when both of the liquidity constraints are binding. The properties of the equilibria are shown in table 2._

<table>
<thead>
<tr>
<th>Case</th>
<th>( \lambda^m )</th>
<th>( \lambda^b )</th>
<th>( s )</th>
<th>( \partial W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>+</td>
<td>0</td>
<td>( \frac{b}{\gamma} )</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>+</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>+</td>
<td>+</td>
<td>( \frac{b}{\gamma} ) &lt; ( s ) &lt; 1</td>
<td>+,-</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

According to proposition 2, marginal open market operations may have real effects when we have a type III equilibrium. The central bank’s asset purchase/sale programs may be ineffective in other cases. The literature calls these situations “liquidity traps”. In the literature on monetary economics and policy, liquidity traps are mostly
associated with the Friedman rule (type IV equilibrium where $\gamma = \beta$). Williamson (2012) studies liquidity traps in cases where the economy is away from the Friedman rule and when the real interest rate is zero. In this paper, we can have properties of the liquidity trap equilibrium even when the real interest rate is positive\(^8\). In an equilibrium where only the money constraint is binding (Case I), open-market operations have no real effects on the economy and the real interest rate is positive ($s = \frac{\beta}{\gamma}$). In this case, marginal open-market operations do not have real effects on the economy.

### 2.2.5 Numerical example

Using the following functional forms and parameters, I simulate the model. The calculations of the different types of equilibria are in the Appendix A and the results of the simulation are reported in figures 2.3 and 3.13:

\[
  u(c) = \log(c); \quad \psi(q) = \frac{q^2}{2}; \quad h(n) = 2a(n)^{1/2},
\]

where $a$ is a parameter of the model. Table 2.2 shows the properties of the equilibrium for different amounts of the liquidity parameter ($l$).

Figure 2.3 shows the range of parameters for different types of equilibria. For high enough values of $l$, the liquidity constraint for money binds and the constraint on trade with bond is slack. For low enough $l$, the constraint on bond binds and for $l < l < \overline{l}$ both of the constraints are binding.

---

\(^8\)The nominal interest rate is $\frac{1}{2}$. The real interest rate is the difference between the nominal


Table 2.2: Properties of the equilibrium for a 2-asset economy

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda^m$</th>
<th>$\lambda^b$</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>+</td>
<td>0</td>
<td>$l &gt; \bar{l} = \frac{(\gamma - 1)(N - \sigma) + \alpha N}{(2 + v) \alpha N}$</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>+</td>
<td>$l &lt; \bar{l} = \frac{\alpha N - (\gamma - 1)(N - \sigma)(1 + v)}{\alpha N (2 + v)}$</td>
</tr>
<tr>
<td>III</td>
<td>+</td>
<td>+</td>
<td>$\bar{l} \leq l \leq \bar{l}$</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Figure 2.3: Range of parameters for different types of equilibrium for a 2-asset economy
As I have proven in Proposition 2, increasing $z$ will only affect welfare when we are in type III equilibrium with both liquidity constraints binding. Figure 3.13 shows the welfare properties of the equilibrium for values of $l$ that cause both liquidity constraints binding (type III equilibrium). As shown in figure 2.4, for each inflation rate there exists an optimal level of bond supply ($z$) that maximizes welfare.

![Figure 2.4: Welfare effects of open-market operations](image-url)

interest rate an the inflation rate
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The case for legal restrictions on trading with bonds

As shown in figure 2.4, in a type II equilibrium, increasing \( z \) from zero increases the overall welfare. Similar to the argument in Shi (2008), an increase in \( z \) can be interpreted as imposing legal restrictions on trade with bonds. An economy with zero supply of bond is a pure monetary economy. An increase in \( z \) from zero represents imposing legal restrictions on trades with bonds. As figure 3.13 shows, this can improve welfare for a range of parameters.

Contrary to the argument in Shi (2008), the argument here is not based on parameters in the utility function. Shi (2008) assumes that agents can use bond to trade certain types of goods that yield a higher utility when consumed. Here, consumption of different goods yields the same amount of utility.

2.3 Model with 3 assets

In this section, I add a third asset to the model. This extension of the model allows me to study the effects of a change in the composition of the central bank’s balance sheet on the real economy. All three assets provide liquidity services, and, similar to the previous section, money is the most liquid asset in the economy. I call the least liquid asset in the economy “long-term bond” and the other asset “short-term bond.”\(^9\) The matching shock works as follows:

- Shock \( n \): With probability \( l \), agents can trade with money and short term bond

\(^9\)Here, it is assumed that a short-term bond is more liquid than a long-term bond. A short-term bond can be used in more transactions to purchase goods.
and long term bond.

- **Shock s**: With probability $k$, agents can trade with money and short term bond.

- **Shock l**: With probability $1 - l - k$, agents can only trade with money.

In each trade buyers make take-it-or-leave-it offers on the amount of goods $q_{i \in \{n, s, l\}}$ and the portfolio of assets to be traded for goods $x_{i \in \{n, s, l\}}$. Note that the portfolio of assets could be a combination of money, short-term bond, and long-term bond depending on the type of trade/shock.

Households solve the following maximization problem:

$$
v(m, b_l, b_s) = \max_{c_{i \in \{n, s, l\}}, q_{i \in \{n, s, l\}}, x_{i \in \{n, s, l\}}, n, m_{-1}, b_{l+1}, b_{s+1}} \{ u(c_l) - \alpha N(1 - l - k) \psi(Q_l) \\
+ u(c_s) - \alpha N k \psi(Q_s) + u(c_n) - \alpha N l \psi(Q_n) \\
+ h(1 - n) + \beta v(m_{-1}, b_{l+1}, b_{s+1}) \}. \tag{2.12}
$$

subject to the following constraints:

$$
x_n \leq \frac{m + b_l + b_s}{n - \sigma} \tag{2.13}
$$

$$
x_s \leq \frac{m + b_s}{n - \sigma} \tag{2.14}
$$

$$
x_l \leq \frac{m}{n - \sigma}. \tag{2.15}
$$
According to constraints 2.13, 2.14 and 2.15, in each type of trade (money and short-term bond trade, money and long-term bond trade, and money trade), buyers are constrained by the portfolio of assets that they have. Consumption in each type of trade is characterized by the following:

\[
\begin{align*}
    c_n &= \frac{\alpha N(n - \sigma)l}{(N - \sigma)} q_n \\
    c_s &= \frac{\alpha N(n - \sigma)k}{(N - \sigma)} q_s \\
    c_l &= \frac{\alpha N(n - \sigma)(1 - k - l)}{(N - \sigma)} q_l.
\end{align*}
\]

where \( \frac{\alpha N}{(N - \sigma)} \) is the matching rate and \((n - \sigma)l\), \((n - \sigma)k\) and \((n - \sigma)(1 - k - l)\) are the number of buyers in each type of trade.

Let’s define \( \omega_i \quad i \in \{m, b_s, b_l\} \) as the marginal value of assets

\[
\begin{align*}
    \omega_m &= \frac{\beta}{\gamma} \frac{\partial}{\partial m_{+1}} v(m, b_l, b_s) \\
    \omega_{b_s} &= \frac{\beta}{\gamma} \frac{\partial}{\partial b_{s1}} v(m, b_l, b_s) \\
    \omega_{b_l} &= \frac{\beta}{\gamma} \frac{\partial}{\partial b_{l1}} v(m, b_l, b_l).
\end{align*}
\]

Since buyers have all the bargaining power, the offer sets sellers’ surplus to 0. Thus, the participation constraint is:

\[
x_i = \psi(q_i) / \Omega_i \quad i \in \{l, s, n\}. \tag{2.16}
\]
The value of household’s asset portfolio in terms of money follows equation 2.17.

\[(m_{-1} + s_{+1} b^l_{+1} + s_{+1} b^s_{+1} + T_{-1})\gamma = \]

\[m + b_l + b_s + \alpha NLX^n + \alpha NkX^s + \alpha N(1 - k - l)X^l \]

\[-\frac{\alpha N(n - \sigma)}{N - \sigma}l_{x^n} - \frac{\alpha N(n - \sigma)}{N - \sigma}k_{x^s} - \frac{\alpha N(n - \sigma)}{N - \sigma}(1 - l - k)x^l. \tag{2.17}\]

where \(s^l_{+1}\) and \(s^s_{+1}\) are the prices of long-term bonds and short-term bonds respectively. \(\alpha NLX^n + \alpha NkX^s + \alpha N(1 - k - l)X^l\) is the amount of assets that households’ sellers spend and \(\frac{\alpha N(n - \sigma)}{N - \sigma}l_{x^n} - \frac{\alpha N(n - \sigma)}{N - \sigma}k_{x^s} - \frac{\alpha N(n - \sigma)}{N - \sigma}(1 - l - k)x^l\) is the amount of assets that households’ buyers buy.

### 2.3.1 Optimal choices

The first order condition for \(q_i\) is:

\[u'(c_i) = (\omega^m + \lambda^i)\frac{\psi'(q_i)}{\Omega} \quad i \in l, s, n. \tag{2.18}\]

I can solve for bond prices by taking the first order conditions with respect to \(b^l_+, b^s_+\).

\[b^l_{+1} : s^l_{+1} = \frac{\omega_{b_l}}{\omega_m}. \tag{2.19}\]

\[b^s_{+1} : s^s_{+1} = \frac{\omega_{b_s}}{\omega_m}. \tag{2.20}\]
The envelope conditions for \( m_{+1}, b_{+1}^s, b_{+1}^l \) are:

\[
m_{+1} : \frac{\gamma}{\beta} \omega^{-1}_m = \omega_m + \frac{\alpha Nl}{N - \sigma} \lambda^n + \frac{\alpha Nk}{N - \sigma} \lambda^s + \frac{\alpha N(1 - l - k)}{N - \sigma} \lambda^l
\]  
(2.21)

\[
b_{+1}^s : \frac{\gamma}{\beta} \omega^{-1}_{bs} = \omega_m + \frac{\alpha Nl}{N - \sigma} \lambda^n + \frac{\alpha Nk}{N - \sigma} \lambda^s
\]  
(2.22)

\[
b_{+1}^l : \frac{\gamma}{\beta} \omega^{-1}_{bl} = \omega_m + \frac{\alpha Nl}{N - \sigma} \lambda^n.
\]  
(2.23)

At the end of each period, each asset is redeemed for a unit of money, therefore the value of an asset is the value of money in the next period plus the transaction services of each asset accounting for discounting and inflation. Money provides transaction service in all types of trades, but bonds are used as medium of exchange in certain types of trade. Similar to the 2-asset economy, the Friedman rule \( (\gamma = \beta) \) is optimal. Here, the central bank has 3 policy variables: money growth rate \( (\gamma) \), long-term bond supply \( (z_l = B_l/M) \), and short-term bond supply \( (z_s = B_s/M) \).

In order to study the different equilibria and welfare effects of policy, I will focus on the log-utility and quadratic cost functions:

\[
u(c) = \log(c)
\]

\[
\psi(q) = q^2/2.
\]

**Lemma 2.** With \( u(c) = \log(c) \) and \( \psi(q) = q^2/2 \), \( N = n \) is the same for different
cases of equilibrium. An equilibrium exists if \( h'(1 - N) = \frac{3}{2(N - \sigma)} \) has a real solution for \( N \).

With log-utility and quadratic cost functions, the first order condition for \( n \) becomes \( h'(1 - N) = \frac{3}{2(N - \sigma)} \) in all of the cases. These functional forms shut down variations in the extensive margin of trade.

In the next proposition, I characterize these different types of equilibrium.

**Proposition 3.** Eight types of equilibrium exist, all of which have different sets of binding liquidity constraints, as defined in table 2.3.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda^r )</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>( \lambda^s )</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>( \lambda^l )</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

The properties of these equilibria are summarized in table 2.4.

As the proposition shows, in equilibriums with at least two binding liquidity constraints, there exists a set of parameters that indicate that open-market operations affect welfare. In these cases, replacing less liquid bonds in household portfolios with liquid money would increase the intensive margin of trade and welfare.

Table 2.4 also shows that a policy of changing the relative supply of bonds while keeping the size of the central bank’s balance sheet growing with the rate of inflation can affect the overall welfare when we have a type II or VII equilibrium. Credit easing
CHAPTER 2. LIQUIDITY EFFECTS

Table 2.4: Properties of the equilibrium for the 3-asset economy

<table>
<thead>
<tr>
<th>Case</th>
<th>Prices</th>
<th>$\frac{\partial q}{\partial z_l}$</th>
<th>$\frac{\partial q}{\partial z_s}$</th>
<th>$\frac{\partial q}{\partial z_l}$</th>
<th>$\frac{\partial W}{\partial z_l}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\beta/\gamma &lt; s_l = s_s &lt; 1$</td>
<td>$\frac{\partial q}{\partial z_l} = 0$</td>
<td>$\frac{\partial q}{\partial z_s} &gt; 0$</td>
<td>$\frac{\partial q}{\partial z_l} &lt; 0$</td>
<td>$\frac{\partial W}{\partial z_l} &gt;= 0$</td>
</tr>
<tr>
<td>II</td>
<td>$s_l = \beta/\gamma &lt; s_s &lt; 1$</td>
<td>$\frac{\partial q}{\partial z_l} = 0$</td>
<td>$\frac{\partial q}{\partial z_s} &lt; 0$</td>
<td>$\frac{\partial q}{\partial z_l} &gt;= 0$</td>
<td>$\frac{\partial W}{\partial z_l} = 0$</td>
</tr>
<tr>
<td>III</td>
<td>$\beta/\gamma &lt; s_l &lt; s_s = 1$</td>
<td>$\frac{\partial q}{\partial z_l} = 0$</td>
<td>$\frac{\partial q}{\partial z_s} &gt; 0$</td>
<td>$\frac{\partial q}{\partial z_l} &lt; 0$</td>
<td>$\frac{\partial W}{\partial z_l} &gt;= 0$</td>
</tr>
<tr>
<td>IV</td>
<td>$s_l = s_s = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>$s_l = s_s = \beta/\gamma &lt; 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VI</td>
<td>$s_l = \beta/\gamma &lt; s_s = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VII</td>
<td>$\beta/\gamma &lt; s_l &lt; s_s &lt; 1$</td>
<td>$\frac{\partial q}{\partial z_l} &gt;= 0$</td>
<td>$\frac{\partial q}{\partial z_s} &lt; 0$</td>
<td>$\frac{\partial q}{\partial z_l} &lt; 0$</td>
<td>$\frac{\partial W}{\partial z_l} &lt;= 0$</td>
</tr>
<tr>
<td>IIX</td>
<td>$s_l = s_s = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

can be implemented by changing the relative supply of bonds while the following relationship holds

$$s_l dz_l + s_s dz_s = 0$$

The above relationship allows the central bank to keep the size of its balance sheet growing at rate $\gamma^{10}$.

Figures 2.5, 2.6, and 2.7 show different types of equilibria for different bond supply $(z_s, z_l)$ and inflation rates $(\gamma)$.

---

10 An example of a policy of credit easing in a type II equilibrium is

$$\frac{\beta}{\gamma} dz_l + s_l dz_s = 0$$

This policy has real effects on the economy and the central bank keeps the size of its balance sheet growing at a constant rate.
Figure 2.5: Range of parameters for different types of equilibrium for the 3-asset economy \( z_s = z_l = 0.7 \)
Figure 2.6: Range of parameters for different types of equilibrium for the 3-asset economy ($z_s = 2.7, z_l = 0.7$)
Figure 2.7: Range of parameters for different types of equilibrium for the 3-asset economy ($z_s = 0.7, z_l = 2.7$)
Chapter 3

Central Banks’ Asset Purchase Programs,
Asset Distributions,
and Endogenous Market Segmentation

3.1 Introduction

What are the long-run effects of open-market operations on the distribution of assets and prices in the economy? Why do some people participate in the market for interest-bearing assets, while others do not participate in the asset market and only
hold money? In order to answer these questions, I construct a model of a monetary economy with heterogeneous agents, in which the central bank implements policies by changing the supply of nominal bonds and money. I show that a segmented asset market arises under a specific parameters set; households with high income participate in the bond market and hold positive portfolios of bond and money, while low income households do not participate in the bond market and only keep money for transaction purposes. When deciding whether to participate in the asset market, households compare liquidity services provided by money with returns on bond. In an equilibrium with a segmented asset market, open-market operations affect the participation decisions of the households and, therefore, have real effects on the distribution of assets and prices in the economy.

The distribution of asset holding and segmentation in the asset market is well documented. Some agents participate in the market for interest bearing assets and hold positive portfolios of different assets, while others do not participate in the asset market.\(^1\) Explaining these facts requires a heterogeneous agent model in which households choose to hold different portfolios of asset holdings. In this paper, households have different preferences towards labor supply, and they experience different matching shocks. Heterogeneity in preferences towards labor supply and idiosyncratic matching shocks allows me to generate an equilibrium distribution of asset holding.

\(^1\)In 2009, 7.7% of the surveyed U.S. households did not have access to banking products and services, and at least 71% of these unbanked U.S. households earned less than $30,000 in a year. Also, 26.5% of U.S. households did not have any savings in a bank account; moreover, they did not hold any financial assets similar to a bank account. Source: FDIC (2009) and The Panel Study of Income Dynamics (PSID). According to the Survey of Consumer Finances (2010), 92.5% of households had access to transaction accounts and 12% held saving bonds.
among different households and a distribution of prices among different markets.

A branch of literature uses asset market segmentation to explain persistence responses to monetary shocks observed in the data\textsuperscript{2}. In these models with segmented asset markets, only the fraction of agents who are active in the asset markets immediately receive the monetary shocks. As a result, it would take time for the monetary shocks to affect other agents in the economy. This literature explains the real effects of money injection and open-market operations using the generated segmentation in the asset market. These models use two ways to generate the segmented asset market: limited participation models that assume only certain agents attend the asset market and models that assume agents must pay a fixed cost to enter the asset market or to transfer assets between the asset market and the goods market. In a cash-in-advance framework, Grossman and Weiss (1983) assume that only a fixed fraction of the population can withdraw funds from banks each period.\textsuperscript{3} In Alvarez, Atkeson, and Kehoe (2000), agents must pay a fixed cost to transfer money between the asset market and the goods market. In a similar fashion, Khan and Thomas (2010) assume agents pay idiosyncratic fixed costs to transfer wealth between interest-bearing assets and money. Chiu (2007) assumes that agents pay a fixed cost to attend the asset market, and they choose the timing of money transfers. In a micro-founded monetary framework, Williamson (2008) links the the asset market segmentation to the goods market segmentation.

In this paper, I generate segmentation in the asset market without assuming any

\textsuperscript{2}For an overview of this literature see Edmond and Weill (2008)

\textsuperscript{3}Similarly Alvarez, Lucas, and Weber (2001), assumes only a fixed fraction of agents attend the asset market.
rigidities and frictions in the asset market. All of the agents can participate in the asset market every period, and there is no transaction cost or any other frictions that prohibit agents from trading in the asset market. Segmentation in the asset market is generated endogenously. When deciding whether to participate in the asset market, households compare liquidity services provided by money with return on bond. Agents hold different amounts of assets, and some agents choose to hold no bond in their asset portfolio. In a segmented asset market, the return on bond is not high enough to attract all of the households to the asset market. Here, the real and welfare effects of open-market operations and money injections are not caused by the assumed segmentation in the asset market. However, open-market operations have real effects on the participation decisions of agents in the asset market, and as a result, on the distributions of assets and prices when the markets are segmented. In a segmented asset market, agents at the participation margin of trading assets may change their decision with a marginal change in the bond supply. Numerical exercises show that the central bank can improve welfare by purchasing bonds and supplying money. The policy of open-market purchase of bond is most effective when the asset market is segmented. This policy would increase the participation rate in the asset market and help households smooth consumption. By participating in the asset market, households are able to better insure themselves against idiosyncratic income shocks. Moreover, in a segmented asset market, open-market purchase of bond decreases both the intensive and extensive margins of trade in the decentralized market. The results are robust to exogenous segmentation in the asset market.

Shi (2008) and Williamson (2012) assume that assets other than money provide
partial liquidity services. In these models, open-market operations change the overall liquidity in the economy. Because of partial liquidity, government bonds are not perfect substitutes for money and a Modigliani-Miller argument does not hold. The same logic is applied in this paper. Agents can only trade with money, and government bond is completely illiquid in the market for goods. Government bond is an imperfect substitute for money, thus open-market operations can have real effects on the economy. Here, households with good shocks use nominal bond to smooth their consumption over time, and the illiquidity of bond is important for this purpose. In a model with liquid bonds, households are indifferent between holding bond and money, since bond and money are perfectly substitutable. They cannot use bonds to smooth consumption. Kocherlakota (2003) uses a similar argument and shows that in a centralized market, agents use illiquid bonds to smooth consumption intertemporally.

This paper is related to the literature on the distribution of money and assets in the economy. In a search model of monetary economy with bargaining, after each round of trading there would be agents that have been matched and have succeeded in trade and agents that have not traded. This would generate an evolving distribution of asset holdings among agents, which is a state variable and makes the model intractable. Camera and Corbae (1999) generate distribution of asset holdings among agents and price dispersion in equilibrium in a framework based on Kiyotaki and Wright (1989).\textsuperscript{4} The evolving distribution of asset holdings makes their model highly intractable for policy analysis. A large section of the monetary literature avoids the distribution

\textsuperscript{4}Zhu (2003) and Green and Zhou (1998) use a similar approach and have distribution of asset holdings and price dispersion as an equilibrium object.
of asset holdings by simplifying assumptions. Lucas (1990) and Shi (1995) assume a large household structure, and with this insurance mechanism, agents within a household share consumption and asset holdings after each round of trade. The sharing mechanism collapses the distribution of asset holdings to a single point. Lagos and Wright (2005) assume a quasi-linear preference structure for the agents along with one round of centralized trading. These assumptions make the distribution of money holdings degenerate and the model highly tractable. By using competitive search in the decentralized market for goods, Menzio, Shi, and Sun (2011) are able to make the distribution of money holdings non-degenerate. Sun (2012) puts Menzio, Shi, and Sun (2011) in a Lagos-Wright framework and, by using a household structure, the model becomes more tractable for studying the effects of different fiscal and monetary policies. Models in Sun (2012) and Menzio, Shi, and Sun (2011) are block recursive; the household’s problem can be solved without involving the endogenous distribution of asset holdings.

My paper closely follows Sun (2012) in using competitive search in the decentralized market together with a centralized market to adjust asset balances. Households and firms trade goods in markets with and without frictions. The frictional markets are characterized by competitive search, where households face a trade-off between higher matching probability and better terms of trade. Competitive search in the
goods market makes the model highly tractable.\textsuperscript{5} Households with different idiosyn-
cratic labor cost shocks choose to hold different amounts of assets. Search is directed
in the sense that households with different portfolios of asset holdings have differ-
ent preferences towards matching probability and terms of trade and choose different
submarkets. Agents with a high income shock choose a submarket with high price
and higher matching probability. Despite a nontrivial distribution of money and
bond across agents, the competitive search in the frictional markets makes this model
highly tractable.

Unlike models with bargaining and due to the competitive nature of the frictional
goods market, the distribution of households across asset holdings does not directly
affect the firms’ cost/benefit of opening a shop in a submarket. Households’ decisions
do not affect matching probabilities and terms of trade in the frictional goods mar-
ket. Households take the specification of the submarkets as given and choose which
submarket to participate in. Households only need to know the prices in the econ-
omy, and these prices contain all of the information about the distributions in the
economy. Hence, the equilibrium is partially block recursive.\textsuperscript{6} Households’ decisions
do not directly depend on the distribution of asset holding in the economy.

\textsuperscript{5}Aside from tractability, comparing to random search, competitive search is closer to the real
world. As Howitt (2005) states: “In contrast to what happens in search models, exchanges in actual
market economies are organized by specialist traders, who mitigate search costs by providing facilities
that are easy to locate. Thus, when people wish to buy shoes they go to a shoe store; when hungry
they go to a grocer; when desiring to sell their labor services they go to firms known to offer
employment. Few people would think of planning their economic lives on the basis of random
encounters with nonspecialists...”.

\textsuperscript{6}Unlike a block recursive equilibrium (Shi (2009), Menzio and Shi (2010) in labor, and Menzio,
Shi, and Sun (2011) and Sun (2012) in monetary economics), here distributions affect households’
decision through prices.
The rest of the paper is organized as follows. In Section 2, I develop the model environment and characterize value and policy functions. Section 3 defines and characterizes the stationary equilibrium. Section 4 presents the computational algorithm and the results of a numerical example. In Section 5, I introduce exogenously segmented asset markets to the model. Section 6 concludes the paper.

3.2 Model environment

Time is discrete and has an infinite horizon. Each period consists of four subperiods: labor market, asset market, frictionless goods market and frictional goods market. They operate sequentially, one in each subperiod. The economy is populated by measure one of \textit{ex-ante} homogeneous households. Each household consists of a worker and a buyer. There is a general good that can be produced and consumed by all of the households. There are also at least three types of special goods. Each household is specialized in the production and consumption of one of the special goods, and there is no double coincidence of wants. Because of the specialized structure of households and the no-double-coincidence-of-wants assumption, a medium of exchange is necessary in this environment. The utility function of the household is

\[ U_h(y, q, l) = U(y) + u(q) - \theta l \]

where \( y \) is the consumption of the general good, \( q \) is the consumption of the special goods, and \( l \) is the labor supply in a period of time. The parameter \( \theta \in [\underline{\theta}, \bar{\theta}] \) is
the random disutility of labor. It is iid across households and time, and it is drawn from the probability distribution $F(\theta)$ at the beginning of each period. $\theta$ captures the heterogeneity of households. $U()$ and $u()$ are continuous and twice differentiable. $u' > 0$, $U'' > 0$; $u'' < 0$, $U'' < 0$; $u(0) = U(0) = u'(\infty) = U'(\infty) = 0$; and $u'(0)$ and $U'(0)$ are large and finite. Goods are divisible and perishable. There are two fiat objects in the economy: money and nominal bonds. Both are supplied by the central bank. Nominal bonds are supplied in a centralized market after the utility shocks have been realized. Agents redeem each unit of bonds from the last period for one unit of money at the beginning of each period. Bonds can be costlessly counterfeited by all households, and they cannot differentiate between the original bond that is printed by the central bank and fake bonds that are printed by other households\(^7\). As a result, nobody accepts bonds as a medium of exchange and, households cannot trade with bonds.

Agents can trade the general good in a perfectly competitive market. There are search frictions in the market for special goods. There is a measure one of competitive firms, who hire workers from the households at the beginning of a period in a competitive labor market. Firms pay hired workers by issuing IOUs. These IOUs can be used to trade goods for money and are redeemed at the end of the period\(^8\). Households own equal shares in these firms. Firms need labor for production of the general good and one type of special goods. These firms are destroyed at the end of

---

\(^7\)Bonds liquidity services have been discussed in the literature, e.g., Shi (2008), Mahmoudi (2011), and Kocharlakota (2003).

\(^8\)Because of the large structure of firms, they do not face unpredictable matching shocks and there is no commitment problem in redeeming IOUs.
each period, and new firms are formed in the second subperiod of each period\(^9\).

Following Sun (2012), I assume a competitive search environment where agents choose to search in submarkets indexed by terms of trade and matching probability. Agents are randomly matched, and only matched agents can trade goods. Firms choose the measure of shops to operate in each submarket. There is free entry in these submarkets. The fixed cost of operating a shop in a submarket is \(k > 0\) units of labor. In producing \(q\) units of special goods, firms incur \(\psi(q)\) units of labor in production costs where, \(\psi()\) is twice continuously differentiable and \(\psi' > 0, \psi'' > 0\) and \(\psi(0) = 0\).

Each submarket is a particular set of terms of trade \((q: \text{amount of special goods and } x: \text{money to be paid})\) and matching probabilities \((b: \text{matching probability for buyers and } e: \text{matching probability for sellers/shops})\). Firms and households take terms of trade and matching frictions as given and decide which submarket to participate in. In each submarket, buyers and shops randomly match according to the respective matching probability. Households and firms decide which submarket to enter, therefore matching probabilities are a function of terms of trade \((x, q)\). Each submarket can be indexed by the respective terms of trade. Matching probability is characterized by a constant return to scale matching function \((e = \mu(b))\), which has the standard characteristics of a matching function\(^{10}\).

In the asset market, central bank prints money at rate \(\gamma\), redeems last period nominal bonds \((A_{-1})\) for one unit of money, issues and sells bonds \((A)\) for the current

\(^9\)With this assumption, there is no need to keep track of firms’ asset holdings.
\(^{10}\)For a survey of literature on the properties of matching functions, see Petrongolo and Pissarides (2001)
period at nominal price $s$, and balances budget by a lump sum tax/transfers ($T$).
The asset market is a competitive market, and households take bonds price ($s$) as
given.

I study the steady state equilibrium, and I will use labor as the numeraire of the
model. Figure 4.1 shows the timing of events.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig31}
\caption{Timing}
\end{figure}

\subsection{Firms’ decision}

Firms have access to a linear production technology. For each unit of labor input,
they produce one unit of output. Firms decide how much to produce in the frictionless
market ($Y$) and determine the measure of shops in each submarket ($dN(x,q)$). They
sell the produced general good at the given market price $P$. In each submarket the
matching probability for each shop is $e(x,q)$. Shops sell the produced special goods
to matched buyers at price $x$. In the production process, firms incur $k$ units of labor
in fixed cost and $\psi(q)$ units of labor in variable costs. Firms maximize the following
profit function:

\[
\pi = \max_Y \{ PY - Y \} + \\
\max_{dN(x,q)} \int \{ e(x, q)x - [k + e(x, q)\psi(q)] \} dN(x, q).
\]  \hspace{1cm} (3.1)

The term \(e(x, q)x - [k + e(x, q)\psi(q)]\) is the expected profit of a shop. If the expected profit in a submarket is strictly positive, firms will choose \(dN(x, q) = \infty\). If the expected profit is strictly negative, firms will choose \(dN(x, q) = 0\). Therefore, the optimal \(dN(x, q)\) satisfies the following inequalities with complementary slackness.

\[
e(x, q)[x - \psi(q)] \leq k, \quad dN(x, q) \geq 0.
\]  \hspace{1cm} (3.2)

As is standard in the competitive search literature, I assume that the profit maximizing condition holds for the submarkets that are not visited by any buyers and firms. For all submarkets where \(k < x - \psi(q)\), we have:

\[
e(x, q)[x - \psi(q)] = k
\]

\[
dN(x, q) = 0.
\]

For the submarkets where \(k \geq x - \psi(q)\) we have \(dN(x, q) = 0\), and I assume \(e = 1\).
and $b = 0$. I can write these two cases as:

$$e(x, q) = \begin{cases} \frac{k}{x - \psi(q)} & k \leq x - \psi(q) \\ 1 & k > x - \psi(q) \end{cases} \quad (3.3)$$

Note that the matching probabilities do not depend on the distributions in the economy. This property of the frictional market simplifies the households’ problems, and we can write households’ matching probabilities as a function of terms of trade $(x, q)$.

### 3.2.2 Households’ decision

#### Decision in the frictionless goods market

In the asset market, households redeem each unit of their nominal bonds from the previous period for one unit of money. Government prints and injects money at rate $\gamma$. Government supplies one period nominal bonds in a centralized market at the competitive price $s$. Then the asset market closes until the next period.

Let $W(m, a_{-1}, \theta)$ be the value function of a household at the beginning of a period. The household holds a portfolio composed of $m$ units of money and $a_{-1}$ units of nominal bonds in units of labor at the beginning of the period. Let $w$ be the normalized wage rate, which is the nominal wage rate divided by the money stock ($M$). The nominal wage rate associated with real balance $m$ is $w M m$.

Given the prices $(p, s)$ and transfers $(T)$, the household decides how much to consume in the frictionless market $(y \geq 0)$, labor supply $(l \geq 0)$, money balances
for transaction purposes \((z)\), money balances for precautionary saving \((h)\), and bond holdings at the beginning of the following period \((a)\). Let \(V(z, h, a)\) be the value function of the household at the start of the next subperiod (frictional market). The household chooses an asset portfolio consisting of the money needed for transaction purposes \((z)\), precautionary savings \((h)\), and bond holdings \((a)\) for the frictional market. In order to purchase nominal bonds, one has to pay the nominal price \(s\) this period to receive the nominal return of one in the following period. In order to have a real return \(a\) in the following period, one needs to pay \(s\gamma a\) in terms of labor units. The households solve the following optimization problem subject to a standard budget constraint:

\[
W(m, a_{-1}, \theta) = \max_{y, l, z, h, a} U(y) - \theta l + V(z, h, a)
\]

\[
\text{st. } py + z + h + s\gamma a \leq m + a_{-1} + l + T.
\]

Let us assume that \(V(z, h, a)\) is differentiable\(^{11}\) and the choice of \(l\) is an interior solution. As \(U()\) is positively sloped the budget constraint is binding. I use the binding budget constraint to eliminate \(l\) from the optimization problem. Using the equilibrium condition \(p = 1\), the value function of the representative household can be written as:

\[
W(m, a_{-1}, \theta) = \theta(m + T + a_{-1}) + \max_{y \geq 0} \{U(y) - \theta y\} + \max_{z, a, h} \{-\theta(z + s\gamma a + h) + V(z, h, a)\}.
\]

\(^{11}\)I will prove this later
The above expression is linear in the households’ portfolio of asset holding at the beginning of the period \((m, a_{-1})\). As I will show later, this linearity will simplify the problem of the household in the decentralized market for goods. Furthermore, the households’ choice of asset holding for the following subperiod \((z, h, a)\) is independent of the asset holding of the current subperiods \((m, a_{-1})\).\(^{12}\)

The optimal choices of \(y\) must satisfy:

\[ U'(y) = \theta. \] (3.4)

In the above equation, I have used the equilibrium condition \(p = 1\). Similarly \(z\), \(h\) and \(a\) satisfy:

\[
V_z(z, h, a) \begin{cases} 
\leq \theta & z \geq 0 \\
\geq \theta & z \leq m - s\gamma a - h 
\end{cases} \tag{3.5}
\]

\[
V_h(z, h, a) \begin{cases} 
\leq \theta & h \geq 0 \\
\geq \theta & h \leq m - s\gamma a - z 
\end{cases} \tag{3.6}
\]

\[
V_a(z, h, a) \begin{cases} 
\leq \theta s\gamma & a \geq 0 \\
\geq \theta s\gamma & sa \gamma \leq m - z - h 
\end{cases} \tag{3.7}
\]

where the inequalities hold with complimentary slackness. \(m\) is the maximum amount of money that households can hold in terms of labor units. Clearly, households’ money balance \((m)\) and bond holdings \((a_{-1})\) does not affect the choices of \(y\), \(z\), \(h\), and \(a\). This is an important property of households’ policy function. Households’ decisions

---

\(^{12}\)The quasi-linear preference structure allows me to remove wealth effects.
are independent of their current portfolio of asset holdings. As a result, I can write policy functions as functions of household type \((\theta)\). Using the optimization problem of the household, I can write the value function as a linear function of \(m\) and \(a_{-1}\):

\[
W(m, a_{-1}, \theta) = W(0, 0, \theta) + \theta m + \theta a_{-1},
\]

where

\[
W(0, 0, \theta) = U(y(\theta)) - \theta y(\theta) + V(z(\theta), h(\theta), a(\theta)) - \theta(z(\theta) + h(\theta) + s\gamma a(\theta)).
\]

It is clear that the value function is continuous and differentiable. The following lemma summarizes these findings.

**Lemma 3.** The value function \(W(m, a_{-1}, \theta)\) is continuous and differentiable in \((m, a_{-1}, \theta)\). It is also affine in \(m\) and \(a_{-1}\).

Lemma 3 shows the standard linearity property that is shared by frameworks based on Lagos and Wright (2005). I can use this property to simplify households’ decision in the frictional market.

**Decision in the frictional market**

A household’s decision in the frictional market is similar to Sun (2012). The household chooses which submarket to participate in. As I can index the submarkets by the respective terms of trade, the household chooses the terms of trade \((x\) and \(q)\) to maximize the expected value of attending the respective submarket. In choosing
which submarket to participate in, households are constrained by their amount of
money holding \((x \leq z)\).\(^{13}\) In a submarket, the household matches with probability
\(b(x, q)\) and trades according to the stated terms of trade. The matching probability
comes from the firms’ decision problems. In each match, the representative household
spends \(x\) amount of money and consumes \(q\) amount of special good. With probability
\(1 - b(x, q)\) there is no match and the representative household exits the frictional
market with the starting portfolio of assets. As is standard in the search and matching
literature, I assume \(b(x, q)\) is nonincreasing. The representative household solves the
following optimization problem:

\[
v(z, h, a) = \max_{x \leq z, q} b(x, q) \left[ u(q) + \beta E \left[ W \left( \frac{z - x + h}{\gamma}, a, \theta \right) \right] \right] + \left[1 - b(x, q)\right] \beta E \left[ W \left( \frac{z + h}{\gamma}, a, \theta \right) \right]. \tag{3.10}
\]

Using the linearity of \(W(.)\) (3.8) and firms’ optimization problem (3.3), I can
eliminate \(q\) from the above expression. The household’s problem becomes:

\[
v(z, h, a) = \max_{x \leq z, b} \left\{ b \left[ u(\psi^{-1}(x - \frac{k}{\mu(b)})) - \beta E(\theta) \frac{x}{\gamma} \right] + \beta E \left[ W \left( \frac{z + h}{\gamma}, a, \theta \right) \right] \right\} \tag{3.11}
\]

\(^{13}\)Because households are committed to posted terms of trade they cannot choose a submarket in which they cannot afford to trade.
The optimal choices of $x$ and $b$ satisfy the following first-order conditions:

$$
\frac{u'(\psi^{-1}(x - \frac{k}{\mu(b)})}{\psi'(\psi^{-1}(x - \frac{k}{\mu(b)}) - \frac{\beta E(\theta)}{\gamma}} \geq 0, \quad x \leq z
$$

(3.12)

$$
u \left( \psi^{-1} \left( x - \frac{k}{\mu(b)} \right) \right) - \frac{\beta E(\theta)x}{\gamma} + \frac{u' \left( \psi^{-1} \left( x - \frac{k}{\mu(b)} \right) \right) k b \mu'(b)}{\psi' \left( \psi^{-1} \left( x - \frac{k}{\mu(b)} \right) \right) [\mu(b)]^2} \leq 0, \quad b \geq 0.

(3.13)

where the two sets of inequalities hold with complementary slackness. Note that $b = 1$ cannot be an equilibrium outcome\(^{14}\).

For $b(z) = 0$, I assume $x(z) = z$. Define $\Phi(q) = u'(q)/\psi'(q)$. As is shown in Sun (2012), without loss of generality, I can focus on the case $x(z) = z$. Similar to Sun (2012), households do not need to hold more money than they want to spend. If the following condition holds: \(^{15}\)

$\frac{u(q^*) - \beta E(\theta)}{\gamma} \left[ \psi(q^*) + \frac{k}{\mu(b^*)} \right] + \left[ \frac{u'(q^*)}{\psi'(q^*)} \right] k b^* \mu'(b^*) \frac{k b^*}{[\mu(b^*)]^2} = 0.$

Given $q^*$, 3.13 can be written as:

$u(q^*) - \beta E(\theta) \gamma [\psi(q^*) + k] + \frac{u'(q^*)}{\psi'(q^*)} k b^* \mu'(b^*) \frac{k b^*}{[\mu(b^*)]^2} = 0.$

The left-hand side of the above equation is strictly increasing in $b^*$, and $b^*$ exists and is unique if $\frac{E(\theta)}{\gamma}$ satisfies:

$u(q^*) - \beta E(\theta) \gamma [\psi(q^*) + k] > 0.$

For all $z < x^* = \psi(q^*) + \frac{k}{\mu(b^*)}$, $x(z) = z$. For $z \geq x^*$, $x(z) = x^*$.
\[ u\left( \phi^{-1} \left[ \frac{\beta E(\theta)}{\gamma} \right] \right) - \frac{\beta E(\theta)}{\gamma} \left( \psi \left( \phi^{-1} \left[ \frac{\beta E(\theta)}{\gamma} \right] \right) + k \right) > 0. \quad (3.14) \]

the household’s problem becomes:

\[ B(z) + \beta E \left[ W \left( \frac{z + h}{\gamma}, a, \theta \right) \right], \quad (3.15) \]

where

\[ B(z) = \max_{b \in [0, 1]} b \left[ u(\psi^{-1}(z - \frac{k}{\mu(b)})) - \frac{\beta z}{\gamma} E(\theta) \right]. \quad (3.16) \]

The value function \( B(z) \) may not be concave in \( z \). Furthermore, equation 3.16 is the product of the choice variable \( b \), and a function of \( b \) and this product may not be concave. Following Menzio, Shi, and Sun (2011) and Sun (2012), I introduce lotteries to make the households’ value function concave\(^\text{16}\). A lottery is a choice of probabilities \((\pi_1, \pi_2)\) and respective payments \((L_1, L_2)\) that solves the following problem:

\[ \tilde{V}(z) = \max_{L_1, L_2, \pi_1, \pi_2} [\pi_1 B(L_1) + \pi_2 B(L_2)], \quad (3.17) \]

subject to:

\[ \pi_1 L_1 + \pi_2 L_2 = z; \quad L_2 \geq L_1 \geq 0 \]

\[ \pi_1 + \pi_2 = 1; \quad \pi_i \in [0, 1]. \]

Note that the agent’s policy functions for the lottery choices are: \( L_{i \in \{1, 2\}}(z) \) and

\(^\text{16}\)The numerical exercise in Section 4 shows that households play lotteries only when they have very low real balances and this does not happen in equilibrium.
π_{i \in \{1,2\}}(z)$. $\tilde{V}(z)$ is the household’s value function after playing the lottery. After playing this lottery, the value function of the household becomes concave.

### 3.2.3 Government

Government imposes policies by either changing the inflation rate ($\gamma$) or changing the relative supply of bond ($\lambda$). I assume that the government runs a balanced budget at each period. Let us define

$$\lambda = \frac{A_{-1}}{M}$$

as the ratio of stock of bond to stock of money in the economy. $\lambda$ shows the composition of the central bank’s balance sheet. A temporary jump in $\lambda$ indicates that the central bank has issued more short-term debt and the composition of its balance sheet has shifted to short-term financing of the government transfers. The total real transfer that a household receives ($T$) is the sum of transfers from printing money (seigniorage) and the transfers received from the bond market.\(^{17}\)

$$T = \frac{\gamma - 1}{w\gamma} + \frac{sA - A_{-1}}{wM'}.$$  \hspace{1cm} (3.18)

### 3.2.4 Properties of value and policy functions

As shown in the previous section, the choice of bonds holdings and bonds prices do not directly affect households’ decisions in the frictional market. The solution

\(^{17}\)Note that because of the quasi-linear structure of households’ utility function, government transfers can be interpreted as a public good.
to firms’ problem (3.3) shows that the matching probabilities do not depend on the distributions in the economy. Sun (2012) discusses fiscal policy in a framework similar to the one used here. In that environment, fiscal policy variables do not directly affect households’ decisions in the frictional market. As a result, the properties of value functions and the households’ choice of which submarket to search \((x, q)\) are the same as in Sun (2012). Let us define \(\widehat{z}\) as the maximum value of real balance \((z)\) that equation 3.14 holds. The following lemma shows the properties of the value functions and policy functions:

**Lemma 4.** The following statements about the value functions and policy functions are true

1. The value function \(B(z)\) is continuous and increasing in \(z \in [0, \widehat{z}]\).

2. The value function \(\widetilde{V}(z)\) is continuous, differentiable, increasing, and concave in \(z \in [0, \widehat{z}]\).

3. For \(z\) such that \(b(z) = 0\), the value function \(B(z) = 0\) and the choice of \(q\) is irrelevant.

4. If and only if there exists a \(q > 0\) that satisfies

\[
\frac{\beta E(\theta)}{\gamma} [\psi(q) + k] > 0
\]

there exists a \(z > 0\) such that \(b(z) > 0\).
5. For $z$ such that $b(z) > 0$, the value function $B(z)$ is differentiable, $B(z) > 0$, and $B'(z) > 0$.

6. $b(z)$ and $q(z)$ are unique and

$$b'(z) > 0$$

$$q'(z) > 0$$

7. $b(z)$ solves

$$\max_{b \in [0,1]} \left\{ u(q(z)) - \frac{\beta E(\theta)z}{\gamma} + \frac{u'(q(z)) k b \mu'(b)}{\psi'(q(z)) [\mu(b)]^2} \right\} \quad (3.19)$$

where

$$q(z) = \psi^{-1} \left( z - \frac{k}{\mu(b(z))} \right) \quad (3.20)$$

8. $b(z)$ strictly decreases with $E(\theta)$, and $q(z)$ strictly increases in $E(\theta)$.

9. There exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z \in (z_1, \hat{z}]$.

10. There exists $z_0 > z_1$ such that a household with $z < z_0$ will play the lottery with the prize $z_0$.

Since the choices of bond holdings and bond prices do not directly affect households' decisions in the frictional market, the proof of lemma 4 is exactly similar to lemma 2 in Sun (2012). Lemma 4 summarizes the characteristics of the value functions. According to part 6, households with higher money balances choose to trade in submarkets with higher matching probabilities and higher terms of trade. They sort
themselves in to different submarkets according to their money holdings. A household with a higher money balance has lower marginal value for money. This household wants to get rid of a high amount of money in a short period of time and therefore chooses a submarket with high price and high matching probability.

Equations 3.8, 3.10, 3.16, and 3.17 give:

\[
V(z, h, a) = \bar{V}(z) + \beta E \left[ W \left( \frac{z + h}{\gamma}, a, \theta \right) \right] \\
= \bar{V}(z) + \beta E [W(0, 0, \theta)] + \frac{\beta E(\theta)z}{\gamma} + \frac{\beta E(\theta)h}{\gamma} + \beta E(\theta)a. \quad (3.21)
\]

Equation 3.21 shows that \( V(z, h, a) \) is linear in \( a \) and \( h \), and the slopes are

\[
V_a(z, h, a) = \beta E(\theta) \quad \quad \quad (3.22)
\]
\[
V_h(z, h, a) = \frac{\beta E(\theta)}{\gamma} \quad \quad \quad (3.23)
\]

Using lemma 4, equations 3.22, and 3.23 and policy functions 3.26 and 3.25, I can conclude the following lemma.

**Lemma 5.** The value function \( V \) is continuous and differentiable in \( (z, h, a) \). \( V(z, h, a) \) is increasing and concave in \( z \in [0, \bar{z}] \). \( V(z, h, a) \geq \beta E[W(0, 0, \theta)] > 0 \) for all \( z \).

Continuity and differentiability of \( V \) with respect to precautionary savings \( (h) \) and bond holdings \( (a) \) is trivially concluded from the linearity condition \( (3.21, 3.22, 3.23) \). \( V \) is increasing and concave in \( z \in [0, \bar{z}] \), because equation 3.21 can be differentiated
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as:

\[
\frac{\partial V(z, h, a)}{\partial z} = \frac{d\tilde{V}(z)}{dz} + \frac{\beta E(\theta)}{\gamma}
\]  

(3.24)

and lemma 4 shows that \( \tilde{V}(z) \) is increasing and concave in \( z \).

Using conditions 3.6, 3.7, 3.22, and 3.23, I can write the household’s choice of bond holdings and precautionary savings in money as follows:

\[
\begin{align*}
    h(\theta) &\geq 0 & \theta &\geq \frac{\beta E(\theta)}{\gamma} \\
    h(\theta) &\leq \bar{m} - z(\theta) - s\gamma a(\theta) & \theta &\leq \frac{\beta E(\theta)}{\gamma}
\end{align*}
\]

(3.25)

\[
\begin{align*}
    a(\theta) &\geq 0 & \theta &\geq \frac{\beta E(\theta)}{s\gamma} \\
    s\gamma a(\theta) &\leq \bar{m} - z(\theta) - h(\theta) & \theta &\leq \frac{\beta E(\theta)}{s\gamma}
\end{align*}
\]

(3.26)

where the inequalities hold with complementary slackness. Using equations 3.21, 3.26, 3.25, and 3.24, I can characterize the policy functions of the households with respect to asset holdings and labor supply in Lemma 6.

**Lemma 6.** \( a(\theta), h(\theta), z(\theta), \) and \( l(m, a_{-1}, \theta) \) follow the following rules:
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I: Negative nominal interest rate ($s \geq 1$)

\[
\begin{align*}
\theta < \frac{\beta \bar{E}(\theta)}{\gamma} & \quad \begin{cases} 
  h(\theta) = \overline{m} - z(\theta) \\
  a(\theta) = 0 \\
  l(m, a_{-1}, \theta) = py(\theta) + \overline{m} - a_{-1} - T \\
  V_z = \widetilde{V}_z(z)
\end{cases} \\
\theta \geq \frac{\beta \bar{E}(\theta)}{\gamma} & \quad \begin{cases} 
  h(\theta) = 0 \\
  a(\theta) = 0 \\
  l(m, a_{-1}, \theta) = py(\theta) - m - a_{-1} - T \\
  V_z = \widetilde{V}_z(z) + \frac{\beta E(\theta)}{\gamma}
\end{cases}
\end{align*}
\]

(3.27)

II: Positive nominal interest rate ($s < 1$)

\[
\begin{align*}
\theta < \frac{\beta E(\theta)}{s\gamma} & \quad \begin{cases} 
  h(\theta) = 0 \\
  a(\theta) = \frac{\overline{m} - z(\theta)}{s\gamma} \\
  l(m, a_{-1}, \theta) = py(\theta) + z(\theta)(1 - s\gamma) + s\gamma \overline{m} - a_{-1} - T \\
  V_z = \widetilde{V}_z(z) + \beta E(\theta)(\frac{1}{\gamma} - 1)
\end{cases} \\
\theta \geq \frac{\beta E(\theta)}{s\gamma} & \quad \begin{cases} 
  h(\theta) = 0 \\
  a(\theta) = 0 \\
  l(m, a_{-1}, \theta) = py(\theta) + z(\theta) - m - a_{-1} - T \\
  V_z = \widetilde{V}_z(z) + \frac{\beta E(\theta)}{\gamma}
\end{cases}
\end{align*}
\]

(3.28)

Lemma 6 and equation 3.5 show the characteristics of the policy functions in two
cases: when the nominal interest rate is negative (3.27) and when it is positive (3.28).

In an equilibrium with a negative nominal interest rate, households choose to hold all of their portfolio in terms of money. Money has a greater return compared to bonds in this case and households decide to hold all of their precautionary savings in terms of money. Higher amount of portfolio from the previous period \((m, a_{-1})\) reduces the labor supply \((l(m, a_{-1}, \theta))\), and households choose to work less because of the higher value of their asset portfolio.

Lemma 6 shows that the equilibrium is partially block recursive. Households do not need to know the distribution of the asset holding for their decision problems, and prices \((p, s, w)\) contain all the information they need about the distributions in the aggregate economy. In the next section, I show that we cannot have a negative nominal interest rate in the stationary equilibrium and households’ policy functions can be described by 3.28.

### 3.3 Stationary Equilibrium

Here, I characterize the stationary equilibrium.

**Definition 2.** A stationary equilibrium is the set of household value functions \((W, B, V, \tilde{V})\); household choices \((y, l, z, a, h, q, b, L_1, L_2, \pi_1, \pi_2)\); firm choices \((Y, dN(q, b))\); and prices \((p, s, w)\) that satisfy the following conditions:

1. Given the prices \((p, s, w)\), realization of shocks \((\theta)\), asset balances, and terms of trade in all submarkets \((q, x)\), household choices solve households’ optimality conditions (3.28 and 3.27).
2. Given prices and the terms of trade in all submarkets, firms maximize profit (3.1).

3. Free entry condition (3.3)

4. Stationarity: quantities, distributions and prices are constant over time.

5. Symmetry: Households with the same shock values and the same asset portfolios make the same decisions.

6. Bond market clears (3.29), labor market clears (3.30), and general goods market clears \((p = 1)\).

In the bonds market, the total amount of bonds supplied equals the sum of demanded bonds by households of different type. The nominal amount of supplied bonds by the central bank is \(A_s\), and therefore the real supply of bonds is \(\frac{A_s}{wM}\). The market clearing for bonds gives:

\[
\frac{A_s}{wM} = \int \int_\theta \int_\theta a \, dF(\theta)dG(m)dH(a_1) = \int_\theta a(\theta)dF(\theta), \tag{3.29}
\]

where in the second equality, I use the fact that households’ decisions on their asset holdings is only based on their labor supply shock.

**Lemma 7.** No positive bond supply \((\lambda > 0)\) can support an equilibrium with negative nominal interest rate \((s \geq 1)\). Households choose to hold bonds as precautionary saving, and they only choose money for transaction purposes:

- \(h(\theta) = 0\).
• $z(\theta) \geq 0$.

From condition 3.27 and bond market clearing condition 3.29, it is straightforward to show that positive amounts of bond supply would not clear the market when $s \geq 1$. With a positive real interest rate, households never use money for precautionary motives.

There are two cases for the equilibrium: First, when $\theta < \frac{\beta E(\theta)}{s \gamma} < \overline{\theta}$, households with low enough $\theta$ choose to hold a positive amount of bond (traders in the asset market), while households with high $\theta$ only hold money for transaction purposes (non-traders in the asset market). Figure 3.2 shows that the threshold $\frac{\beta E(\theta)}{s \gamma}$ determines who participates in the asset market.

Second, in the case where $\overline{\theta} < \frac{\beta E(\theta)}{s \gamma}$, all of the households hold a portfolio of bonds and money\(^{18}\). In figure 3.3, $\frac{\beta E(\theta)}{s \gamma}$ is very high and everyone participates in the market for bond. In deciding whether to participate in the asset market, households compare two scenarios: 1-working this period and buying bonds and redeeming purchased bonds for money next period, and 2-not working this period and working next period.

\(^{18}\)Note that with positive bonds supply, we cannot have the case in which $\frac{\beta E(\theta)}{s \gamma} < \theta$. 
From lemma 7 and equations 3.28, 3.17, and 3.5, I can show that changes in bond supply ($\lambda$) would change the threshold ($\beta_E(\theta)$). Figure 3.2 shows that an equilibrium with a segmented asset market arises when $\theta < \frac{\beta_E(\theta)}{s^\gamma} < \overline{\theta}$. In an equilibrium with a segmented asset market, open-market operations affect the decision of the households regarding the composition of their real portfolio of assets for households at the participation margin and therefore have real effects on the distributions of asset portfolios in the economy.

Lemma 6 shows that when we have an equilibrium with no segmentation in the asset market, money holding from the previous period does not affect agents’ labor supply. In the same type of equilibrium, households’ bond holding has a negative effect on their labor supply. This property of the equilibrium is due to the fact that households in an asset market with no segmentation and households with good shocks in a segmented asset market ($\theta < \frac{\beta_E(\theta)}{s^\gamma}$) hold money balances only for transaction purposes. Note that this is different from the pure precautionary motive ($h(\theta) > 0$), and bonds always dominate money because of positive real interest rate ($s < 1$). In a segmented asset market, households with bad labor supply shocks ($\theta \geq \frac{\beta_E(\theta)}{s^\gamma}$) consider unmatched buyers’ expected money balances as a precautionary motive for
saving.

As shown in appendix B, the labor market clearing condition is

$$\frac{w}{\gamma} [\gamma - 1 + \lambda(s - 1)] =$$

$$(2 - s\gamma) \int_{\theta}^{\bar{\theta}} a(\theta) dF(\theta) + (1 - \frac{1}{\gamma}) \int_{\theta}^{\bar{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$

$$+ (1 - \frac{1}{\gamma}) \int_{\theta}^{\bar{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \frac{1}{\gamma} \int_{\theta}^{\bar{\theta}} h(\theta)dF(\theta). \quad (3.30)$$

Using lemmas 6 and 7, market clearing condition for the asset market and the fact that households do not save money for precautionary motives, I can summarize the market clearing conditions to a single equation that could be solved for bond price (s).

$$\left[ \frac{1}{\gamma^2 s} + (1 - \gamma - \lambda(s - 1)(2 - s\gamma)) \right] \int_{\theta}^{\bar{\theta}} \frac{\partial E(\theta)}{\partial \gamma} a(\theta) dF(\theta) =$$

$$\gamma(1 - \gamma) \int_{\theta}^{\bar{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$

$$+ \gamma(1 - \gamma) \int_{\theta}^{\bar{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta). \quad (3.31)$$

For $\gamma < 2$, the left-hand side of the above expression does not increase with bond price (s). Note that equation 3.31 cannot solely be used for numerical computations, and we need to compute wage (3.30) and check for positive wages. From equations
3.28, 3.29, and 3.31, I can characterize the set of prices in equilibrium. Let us define \( \bar{s} \) and \( s \) as:

\[
\bar{s} = \frac{\beta E(\theta)}{\theta \gamma} \\

s = \frac{\beta E(\theta)}{\theta \gamma}.
\]

Let bond price be in the range: \( \bar{s} \leq s \). The left-hand side of 3.31 is 0, while the right-hand side is a positive number. In this case, there is no equilibrium. Previously, I have shown that \( s < 1 \) in equilibrium. Therefore, the market clears at a price in the range \( s < \min\{1, \bar{s}\} \).

Let us define \( \zeta(\gamma, \lambda) \) as:

\[
\zeta(\gamma, \lambda) = \sum_{i=1,2} \int_{\theta} \pi_i(z(\theta))(1 - b(L_i(z(\theta))))L_i(z(\theta))dF(\theta).
\]

(3.32)

For the case where \( s < \bar{s} \), figure 3.3 shows the policy functions. In this case, \( \zeta(\gamma, \lambda) \) is independent of \( \lambda \), and I show it by \( \zeta_1(\gamma) \). The only policy variable on the right-hand side of 3.31 is \( \gamma \). For a constant positive rate of inflation (\( \gamma > 1 \)), the left-hand side shows a positive relationship between bond price and bond supply\(^{19}\).

From figures 3.2 and 3.3 and the positive relationship between bond price and bond supply, I can summarize the demand for bond in figure 3.4.

\(^{19}\)We can rewrite the expression on the left hand side of 3.31 as

\[
\left[ \frac{1}{\gamma^2 s} + (2 - s \gamma)(1 - \gamma) + \lambda(s - 1)(s \gamma - 2) \right] \int_{\theta} \frac{dE(\theta)}{s \gamma} a(\theta)dF(\theta)
\]

The above expression shows a positive relationship between bond price \( s \) and bond supply \( \lambda \) for positive real interest rates \( s < 1 \) and positive inflation rate \( \gamma > 1 \).
For low bond price, the return on the bond is high enough to attract all of the households to the asset market. They hold a positive portfolio of money and bond according to figure 3.2. With higher bond price (lower interest rate), we have a segmented asset market, and higher price in this type of equilibrium leads to low asset market participation.

From 3.31 and 3.32 proposition 4 follows:

**Proposition 4.** Let us define $\bar{s} = \min(1, \bar{s})$. Then for a positive rate of inflation ($\gamma > 1$), there exists the following thresholds: $s_{\min}$ and $\lambda_u$, which solves the following equations:

$$
\left[ \frac{1}{\gamma \bar{s}} + (1 - \gamma - \lambda(\bar{s} - 1)(2 - \bar{s}\gamma)) \right] \int_{\theta} \frac{\beta E(\theta)}{\bar{s}^2} a(\theta) dF(\theta) = \gamma(1 - \gamma)\zeta(\gamma, \lambda_u).
$$

(3.33)
\[
\left[\frac{1}{\gamma^2 s_{\min}} + 1 - \gamma\right] \int_\beta E(\theta) \frac{\beta E(\theta)}{s_{\min}^\gamma} a(\theta) dF(\theta) = \gamma (1 - \gamma) \zeta(\gamma, 0), \tag{3.34}
\]

where

1. Bond price is in the range: \( s_{\min} < s < \tilde{s} \).

2. Bond supply is in the range: \( 0 < \lambda < \lambda_u \).

In the above proposition, I have used the positive relationship between bond price \( s \) and bond supply \( \lambda \) from equation 3.31 to derive the equilibrium limits for bond price and bond supply.

### 3.3.1 Welfare analysis

I have shown in the appendix B that the steady state welfare can be calculated using the following expression:

\[
\varpi = \int [U(y(\theta)) - \theta y(\theta) + u(q(z(\theta))) - \theta z(\theta) - \theta h(\theta) - s \gamma \theta a(\theta)] dF(\theta)
+ \left[ \int_{\beta}^{\tilde{\theta}} \pi_1(z(\theta)) [1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \right] \int \theta dF(\theta)
+ \left[ \int_{\beta}^{\tilde{\theta}} \pi_2(z(\theta)) [1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) \right] \int \theta dF(\theta)
+ \left( 1 + \frac{1}{\gamma} \right) \left[ \int a_{-1} dH_{a_{-1}} \right] \int \theta dF(\theta) + \frac{1}{\gamma} \left[ \int h_{-1} dJ_{h_{-1}} \right] \int \theta dF(\theta)
+ \frac{1}{w \gamma} \left[ \gamma - 1 - \lambda + s \lambda \right] \int \theta dF(\theta).
\]
Using lemma 7 and equations 3.30 and 3.32 the measure of welfare can be simplified as:

\[
\varpi = \int [U(y(\theta)) - \theta y(\theta)]dF(\theta) + \int [u(q(z(\theta))) - \theta z(\theta) - s\gamma a(\theta)]dF(\theta)
+ \left[\zeta(\gamma, \lambda) + (2 - s + 1 + \frac{1}{\gamma}) \int a(\theta)dF(\theta)\right] \int \theta dF(\theta).
\]

(3.35)

3.4 Numerical Example

In order to simulate the economy, I use the partial block recursivity of the equilibrium. I use the following algorithm:

1. For given supply of bonds (\(\lambda\)) and inflation (\(\gamma\)), and an arbitrary bond price (\(s\)) calculate policy functions (\(a(\theta), h(\theta), z(\theta), y(\theta), l(\theta), b(z(\theta)), q(z(\theta))\)) (6) and lottery choices (\(\pi_1(z(\theta)), \pi_2(z(\theta)), L_1(z(\theta)), L_2(z(\theta))\)) (3.17)

2. Calculate the value functions (\(B(z(\theta)), \tilde{V}(z(\theta))\))

3. Calculate wage (\(w\)) using labor market clearing condition 3.30

4. If \(w < 0\) change \(s\) and start from 1.

5. Check bonds market clearing condition 3.29, adjust bond price and start from
   1. until bond market clears.

I simulate the economy using the following functional forms:
\[ u(c) = u_0 \frac{(c + a)^{1-\sigma} - a^{1-\sigma}}{1 - \sigma} \]  
\[ U(c) = U_0 \frac{(c + a)^{1-\sigma_u} - a^{1-\sigma_u}}{1 - \sigma_u} \]

\[ \psi(q) = \psi_0 q^\psi; \mu(b) = 1 - b; F(\theta) \text{ is continuous uniform on } [\theta, \bar{\theta}]. \]

I use the following parameter values:

\[ \beta = 0.96 \quad u_0 = 1 \quad U_0 = 100 \quad a = 0.01 \]
\[ \sigma = 2 \quad \sigma_u = 2 \quad \phi = 2 \quad \psi_0 = 1 \]
\[ k = 1 \quad \bar{m} = 20 \quad \theta \in [0.25, 1.75] \quad F(\theta) \text{ uniform} \]

Figure 3.5 shows the policy functions regarding households’ portfolio and the effects of open-market operations. In a segmented asset market, high income households choose a positive portfolio of bond and money. The threshold that determines who participates in the asset market is affected by open-market operations. As it can be seen in figure 3.5 an open-market purchase of bond would increase the real interest rate and shift the threshold to the right. More households decide to participate in the bond market as a result of an open-market purchase of assets.

Figure 3.6 shows the asset portfolio choice of households in an equilibrium with no segmentation. Comparing to figure 3.5, here bond supply is so low that high real interest rates attract all of the households to the asset market and they hold a positive portfolio composing of money and bond. A marginal policy of pure open-market operation would not affect the decision of households regarding their real asset holding, and would not have real effects on the distributions in the economy.

Figure 3.7 shows labor supply in an equilibrium with no segmentation in the asset market. Households with bad shocks (high \( \theta \)) and high asset balance work less.
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Figure 3.5: Choice of asset holding in a segmented asset market

Figure 3.6: Choice of asset holding in an equilibrium with no segmentation
Households with good shocks (low $\theta$) work more and hold more money for transaction purposes. A pure policy of open-market operations (marginal change in $\lambda$) would shift the labor supply. Higher real interest rate would change labor supply of households but households’ real asset holding would not change (Figure 3.6).

Figure 3.7: Labor supply in an equilibrium with no segmentation ($\gamma = 1.01, \lambda = 0.003$)

Figure 3.8 shows labor supply in an equilibrium with segmented asset market. Households with bad shocks ($\theta \geq \frac{\beta E(\theta)}{s\gamma}$) supply labor only to fund their money holding for the next subperiod ($z(\theta)$). Households who received better shocks than the threshold for asset market participation ($\theta < \frac{\beta E(\theta)}{s\gamma}$) provide high labor supply to buy bonds ($a(\theta)$) as a precautionary saving for the next period. A pure policy of open-market operations (marginal changes in $\lambda$) has two effects: First, it has a level effect
on the labor supply. This is similar to the case with no segmentation. Higher real interest rates requires higher labor supply for the same real asset holding. Second, open-market operations changes the threshold \( \left( \frac{\beta E(\theta)}{\sigma_\gamma} \right) \), and therefore affects the participation decision of households in the market for bonds. Higher real interest rate, attracts some of households who were not participating in the asset market, and these households supply more labor.

Figure 3.8: Labor supply in an equilibrium with segmented asset market \( (\gamma = 1.01, \lambda = 0.0085) \)

Figure 3.9 shows the characteristics of submarkets in the decentralized market. Agents with higher money holdings search in submarkets with higher price, output and matching probabilities. This property of the equilibrium is shared with many
competitive search models\textsuperscript{20}. Agents sort themselves according to their money holdings. Households with higher money balances have low marginal value for money. As shown in lemma 4, they decide to get rid of a high amount of money as soon as they can and choose submarket with higher price and higher matching probability compared to households with low money balances. Unlike models of bargaining, buyers and sellers know the marginal value of money holdings of all of the households in the economy and they commit to posted terms of trade.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3_9.png}
\caption{Policy function in decentralized market}
\end{figure}

Figures 3.10 shows the output choice of households in the decentralized market. Generally, households with better shocks participate in submarkets with higher output. As shown in figures 3.5 and 3.6, conditional on participation (/not participation) in the asset market, households with better shocks choose higher amounts of money balances. Figure 3.9 shows that households with higher money balances choose higher

\textsuperscript{20}e.g. equilibrium in Menzio, Shi, and Sun (2011) and Sun (2012) shows similar properties.
output. Therefore, we can see that conditional on participating (not participating) in the asset market households with better shocks choose submarkets with higher output and figure 3.10 confirms this. In the case with a segmented asset market (figure on the right) a marginal open-market purchase of bond would increase asset market participation and reduce real output choice of households on the participation margin to lower values.

Figure 3.10: Output choice of households in decentralized market

Figure 3.11 shows the matching probability choice of households in the decentralized market. As shown in figures 3.5 and 3.6, conditional on participation (not participation) in the asset market, households with better shocks choose higher amounts of money balances. Figure 3.9 shows that households with higher money balances choose submarkets with higher matching probabilities. As a result, conditional on participating (not participating) in the asset market households with better shocks choose submarkets with higher matching probability and figure 3.11 confirms this. In
the case with a segmented asset market (figure on the right) a marginal open-market purchase of bond would increase asset market participation and reduce real output choice of households on the participation margin to lower values.

Figure 3.11: Matching probability choice of households in decentralized market

I can discuss the effects of open-market operations on the extensive and intensive margins using figures 3.10 and 3.11. A marginal open-market purchase of bond would decrease $\lambda$ and bond price ($s$). This policy will have no effects on the intensive margin (right graph in 3.10) and extensive margin (right graph in 3.11) when we have an asset market with no segmentation. As shown in figures 3.10 and 3.11 In a segmented asset market, open market purchase of bonds would shift the threshold for asset market participation to the right. This will decrease both the intensive and the extensive margins of trade for households in the participation margin. Higher real interest rate attracts a subset of households to the bond market. In the decentralized market these households choose to apply to submarkets with lower matching probability and lower
output and this will decrease both the extensive margin and the intensive margin of trade.

Figure 3.12 shows that conditional on participating (/not participating) in the asset market, household with better income shock pay lower price per unit of output in the decentralized market.

![Figure 3.12: Price per unit choice of households in decentralized market](image)

Figures 3.13 shows welfare for different values of bond supply and inflation rate. The central bank can generally affect overall welfare by purchasing bonds and supplying money. The policy of open-market purchase of bond is most effective when the asset market is segmented. This policy would increase the participation rate in the asset market and help households smooth consumption. By participating in the asset market, households are able to better insure themselves against idiosyncratic income shocks. When the asset market is not segmented, marginal open-market purchase/sale of bond would only change the real interest rate.
Figure 3.13: Welfare

Figure 3.14 shows equilibrium bond price ($s$) for different amounts of bond supply ($\lambda$) and different inflation rates ($\gamma$). At each level of inflation bond price increases with higher supply of bonds.\footnote{Note that the price of bond is the inverse of nominal return on bonds}

Figure 3.15 shows equilibrium wage ($w$) for different amounts of bond supply ($\lambda$). For a fixed rate of inflation, wage increases with bond supply.

**Exogenously segmented asset market**

In this section I introduce another source of heterogeneity to the model. Following Alvarez, Lucas, and Weber (2001), I assume only a fixed fraction of households attend the asset markets (traders), and the remaining never has access to the asset market.
Figure 3.14: Bond prices

Figure 3.15: Wage
(non-traders). This extension allows me to compare the results of this paper to the literature that assumes the asset markets are exogenously segmented\(^{22}\). Proposition 5 shows that the same logic from the case with endogenous asset market segmentation applies and asset market traders and non-traders solve optimization problems similar to the problem in the previous sections. The households’ decisions are only linked through the market clearing conditions and prices. Households do not take in to account the distribution of asset holdings among traders and non-traders. The following proposition shows that the main results in the previous sections are robust to adding exogenously segmented asset market.

**Proposition 5.** With exogenously segmented asset markets, value functions, policy functions and labor choices of traders in the asset market have the same properties as the case without exogenously segmented asset market.

The formal proof is in the appendix B.

\(^{22}\text{e.g. Alvarez, Lucas, and Weber (2001)}\)
Chapter 4

Credit Rationing and Heterogeneity in Corporate Loan Contracts

4.1 Introduction

In a friction-less credit market, firms that find it optimal to have bank debt in their capital structure should be able to issue bank debt at a correct market price. This correct price for debt (cost of debt) would depend on the incoming cash flow as well as the risk of the firm’s future projects. In the real world, however, there is an excess demand for loanable funds; i.e., certain firms apply for loans but fail to receive them. Stiglitz and Weiss (1981) argue that potential borrowers who are denied loans would not be able to borrow even if they indicated a willingness to pay more than
the market interest rate. Faulkender and Petersen (2006) and Colla, Ippolito, and Li (2013) also show that bank credit is indeed rationed and certain types of borrowers end up with certain types of debt securities. Williamson (1987) uses a model with costly monitoring technology to explain credit rationing.

In the corporate finance literature, it is commonly assumed that information asymmetries between borrowers and lenders may be reflected by premiums in interest rates. This per se does not explain why some potential borrowers may fail to find a lender. Furthermore, there are other models suggesting that information frictions may be reflected in liquidity of funds. The literature, however, lacks models in which both liquidity distortions and price dispersions are allowed to operate simultaneously.\(^1\) This paper attempts to bridge this gap by proposing a model of credit market using the tools provided in the competitive search literature (i.e., Guerrieri, Shimer, and Wright (2010)). We develop a model of a market with adverse selection and search frictions, and show that there exists a unique separating equilibrium in which each type of firm in need of bank credit applies to a different type of contract. In this model, banks post the terms and conditions of credit agreements and potential borrowers choose where to direct their search. Banks have imperfect information about borrowers, and through the posted terms of trade they can attract certain types of

\(^1\)Stiglitz and Weiss (1981) provide the first theoretical framework that justifies credit rationing in equilibrium. They show that banks with imperfect information will formulate the terms of the contract, specifically the interest rate, to attract low-risk borrowers. Willingness to lend at a higher rate will attract riskier borrowers and the overall effect is value-destroying. In a similar environment with information frictions, Bester (1985) shows that when we allow banks to post different contracts to screen borrowers, credit rationing disappears. In an environment with only information frictions, one cannot explain both credit rationing and price dispersion. We add search friction to a standard environment with information frictions and show that a separating equilibrium with credit rationing always exists.
borrowers and screen out others. Contract terms and market tightness (i.e., ratio of borrowers to lenders in each submarket) is public information. In this framework, borrowers potentially face a trade-off between the terms of loans and market tightness. In a benchmark setup of the model with no information frictions, we show the properties of the offered loan contracts. In the second best version of the model with information frictions on the borrowers’ side, we show how private information distorts the terms of loan contracts. Competitive search frictions allow us to investigate the effects of private information on two margins: 1- extensive margin of loan contracts: the number of matched agents and offered loans is identified by the tightness in each submarket. 2- intensive margin of the loan contracts: the amount of loans offered may be distorted downward when we add information problem to the model.

In the rest of the paper, we provide empirical evidence on the findings in the theoretical part of this paper. Using a novel dataset that records successful and unsuccessful applications for bank credit as well as the conditions of each credit agreement and firm characteristics, we verify the co-existence of unmatched credit seekers and distribution in types of contracts. This finding has two immediate implications. First, there is a difference between a desired capital structure and a realized one. There are two types of firms whose capital structure does not have bank debt: firms that prefer to not have a bank loan and firms that prefer to have a bank loan but are unsuccessful in acquiring it. Even if a firm is successful in issuing bank debt, due to restrictions on the supply side, the amount of debt issued might be different from what is desired. Due to the lack of data on unsuccessful bank loan applications, empirical banking studies may ignore this difference and explain debt structure solely
as a function of firm characteristics. Using data on a large cross-section of firms, we investigate the magnitude of this issue. We provide empirical evidence that there are potential borrowers who are unsuccessful in acquiring bank loans even if they are willing to pay higher interest rates, a point ignored or at best acknowledged based on anecdotal evidence in prior work.

Our paper also relates to the recent literature on corporate debt structure and debt heterogeneity. Rauh and Sufi (2010) and Colla, Ippolito, and Li (2013) show that different borrowers specialize in certain types of debt. Furthermore, the degree and type of specialization varies widely across different firm types measured by size, degree of information asymmetry, maturity, profitability, etc., a finding that is also confirmed in our study. In analyzing debt specialization, Colla, Ippolito, and Li (2013) extend the work of Rauh and Sufi (2010)\footnote{Who use a sample of 305 randomly selected rated public U.S firms for the period 1996 to 2006.} by studying a sample of 3,296 U.S. public firms from 2002 to 2009. This number includes all the firms with available financial data on both Standard and Poor’s Capital IQ database and the Compustat database. Our study complements these two papers by taking advantage of the European Commission’s EFIGE (European Firms in a Global Economy) database. This data is based on a survey of 15,000 manufacturing firms in seven key European countries that is validated with each firm’s balance sheet information, from Bureau van Dijck Amadeus database. This data helps us deepen our understanding of capital structure beyond prior findings in this area, especially since it includes rich data on the borrowing behaviour of private firms. Studying private firm-bank relationships is important because private firms have limited access to public funds, and as a result,
acquiring bank loans is crucial for them. Furthermore, since the level of information asymmetry between lenders and private borrowers is higher, the role of search frictions in forming capital structure is more clearly demonstrated. Also, as mentioned earlier, this dataset provides additional insight by including data on unsuccessful bank loan applications.

The rest of the article proceeds as follows. The next section provides the theoretical model. In the next section, we develop the general environment, define equilibrium, and solve constrained optimization problems with and without borrower-specific information frictions. Section 4.3 describes the data and empirical evidence. Proofs for the solutions in theoretical models are relegated to the appendix C.

### 4.2 Theoretical Model

There is a measure one of borrowers. Measure $0 < \pi_1 < 1$ of these borrowers are type 1, and the remaining $(\pi_2 = 1 - \pi_1)$ are type 2 agents. There are three time periods: matching, loan, and repayment. Borrowers have access to a measure one of Lucas trees with stochastic return. Lucas trees produce fruits on the last subperiod (repayment). A type $i = 1, 2$ borrower receives $R$ units of fruit in the repayment subperiod with probability $p_i$, where $p_1 < p_2$, and with probability $1 - p_i$ this borrower receives 0 fruit. There is a large measure of ex ante homogeneous lenders who may decide to enter the market. If they decide to enter the market, they incur $k > 0$ in the fixed cost of entering. Lenders can produce in the loan subperiod and they incur $c(q)$ cost when they produce $q$ units. Borrowers cannot produce in the loan subperiod and
their utility of consuming \( q \) is \( u(q) \). We assume \( u'(q) > 0, u''(q) < 0, c'(q) > 0, \) and \( c''(q) > 0 \).

Lenders post contracts in the frictional loan market and each borrower directs his search to a single submarket. Contracts are observable by all participants in the market. Each contract is a loan amount \( (q) \) and a repayment level \( (x) \). The fraction of lenders to borrowers in each submarket is called tightness and is represented by \( \theta \). In each submarket, borrowers and lenders match according to a matching function. Each borrower is matched with a lender with probability \( \mu(\theta) \), where \( \mu'(\theta) > 0 \) and \( \mu''(\theta) < 0 \). Each lender matches with a borrower with probability \( \eta(\theta) = \frac{\mu(\theta)}{\theta} \), where \( \eta(\theta) \) is non-increasing. Timing of the events is shown in Figure 4.1

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3"The concavity assumption on the matching function is a standard assumption in many models with search frictions. For a survey on the properties of matching functions, see Petrongolo and Pissarides (2001).
4.2.1 First best

Assume borrowers’ types are common knowledge. Instead of solving the competitive search equilibrium directly, we solve a set of optimization problems for each type. Maximization problems in 4.1 show how the optimal contract is chosen.

\[
\mathcal{U}_{i=1,2} = \max_{\theta, q, x} \mu(\theta)[u(q) + p_i(R - x)] \\
st. \quad \eta(\theta)(-c(q) + p_i x) \geq k
\]

(4.1)

In the optimization problem for each type, market tightness \((\theta)\), loan amount \((q)\), and repayment level \((x)\) are chosen to maximize the expected utility of each type subject to the bank making nonnegative profits when only type \(i\) borrowers apply. A borrower’s expected profit is the probability that he matches \((\mu(\theta))\) times his expected return given that he finds a match. If he finds a match, he will enjoy \(u(q)\) in utility of consuming the loan. At the repayment period, a matched borrower pays back \(x\) of his fruits, if his tree yields any.

A lender’s expected profit conditional on entering is her matching probability \((\eta(\theta))\) times her expected profit. Her expected profit is her expected fruit collection in the repayment period, and she suffers \((c(q))\) when giving out loans. Here we assume that \(R\) is large enough such that the constraint \(R > x\) is not binding. Later we will show the specific minimum value for \(R\) that guarantees a nonbinding constraint.
The constraint in problem 4.1 is binding:

\[ x_i = \frac{k}{p_i \eta(\theta)} + \frac{c(q)}{p_i} \quad i = 1, 2 \]  

(4.2)

Substitute in the objective function, and the problem becomes:

\[ \overline{U}_{i=1,2} = \max_{\theta, q} \mu(\theta)u(q) + \mu(\theta)p_iR - \theta k - \mu(\theta)c(q) \]  

(4.3)

The first order condition for \( q_i \) is:

\[ u'(q^*_i) = c'(q^*_i) \]  

(4.4)

\[ q^*_1 = q^*_2 = q^* \]

Note that \( q^* \) is the first best amount of lending in an environment with no frictions. This result shows that without information frictions, search frictions do not distort the optimal amount of loans. As seen in equation 4.2, search frictions may distort the repayment levels. The amount of loan is also independent of borrowers’ types. The first order condition for \( \theta_i \) gives the first-best market tightness for each submarket:

\[ \mu'(\theta^*_i) = \frac{k}{u(q^*) - c(q^*) + p_iR} \]  

(4.5)

Using the concavity property of the matching function, we can see that \( \theta^*_2 < \theta^*_1 \). Additionally, equation 4.2 shows that \( x^*_2 < x^*_1 \). The market for high-type borrowers is tight comparing to the low-type borrowers, but high-type borrowers pay a lower
interest rate. Without information frictions, search frictions only affect the number of successful matches through market tightness in different submarkets. There is no credit rationing along the intensive margin in the loan market: The amounts of offered loans are the same for high-type and low-type borrowers.

The lowest value for $R$ such that the constraint $R > x$ is nonbinding is $R_{\text{min}}^* = \max\{x_1^*, x_2^*\}$, where $x_i^*$ is the first best value for repayment level.

$$x_i^* = \frac{k}{p_i \eta(\theta^*)} + \frac{c(q^*)}{p_i} \quad i = 1, 2$$  \hspace{1cm} (4.6)

Therefore, the constraint in problem 4.1 is $R > R_{\text{min}}^*$.

### 4.2.2 Private information

Assume borrowers’ types are private information. Agents with the lowest type (type 1) do not face incentive compatibility constraints. Therefore, their problem is similar to the first best with complete information. We can find the market tightness that they face, their loan amount, and their repayment level by solving the following first-best problem:

$$\bar{U}_1 = \max_{\theta, q, x} \mu(\theta)[u(q) + p_1(R - x)]$$

$$\text{st.} \quad \eta(\theta)(-c(q) + p_1 x) \geq k$$  \hspace{1cm} (4.7)

In a type 2 optimization problem borrowers face an incentive compatibility constraint: compared to type 2 contracts, type 1 contracts should be more attractive for
In 4.8 we use the binding participation constraint to eliminate $x$ and the problem becomes:

$$
\bar{U}_2 = \max_{\theta,q} \mu(\theta)[u(q) + p_2(R - x)]
$$

$$
st. \quad \eta(\theta)(-c(q) + p_2x) \geq k
$$

$$
\mu(\theta)(u(q) + p_1(R - x)) \leq \bar{U}_1 \quad (4.8)
$$

The algorithm to solve the type-2 problem is to first solve the first best problem without the incentive compatibility constraint. Then, we can check whether the solution for a type-2 borrower $(q^*, \theta^*_2, x^*_2)$ satisfies the incentive compatibility constraint. If the solution satisfies the incentive compatibility constraint then information problems do not distort the type 2 problem and we get the first best. The allocations are efficient.

If the solution to the first best problem for a type-2 borrower $(q^*, \theta^*_2, x^*_2)$ does not satisfy the incentive compatibility constraint in problem 4.9, we have to solve the above constrained optimization problem (4.8).

**Proposition 6.** The amount of loan received by the good type (type 2) under binding
private information is less than the amount they receive in the first best case

\[ q_2 < q^*_2 = q^* \]

The proof of the above proposition is in Appendix C. The above proposition shows that the information problem intensifies credit rationing. Search friction generates credit rationing in the extensive margin of loans contracts: the number of matched agents and offered loans is identified by the market tightness in each submarket, and market tightness may be distorted upward or downward. Information problems affect both the extensive margin and the intensive margin of the loan contracts: the amount of loans offered may be distorted downward when we add the information problem to the model.

### 4.2.3 Existence and uniqueness

The existence and uniqueness of the equilibrium follow from the assumed preference and payoff structure. Guerrieri, Shimer, and Wright (2010) show that under very mild assumptions on preferences and payoffs, this problem has a unique equilibrium. These assumptions are met here. Assumptions A1 and A2 in Guerrieri, Shimer, and Wright (2010) hold because the assumed preferences are monotone, and also the

---

In problem 4.8, we can use the binding free entry condition to eliminate \( x \). The first order condition for \( \theta \) gives

\[
\mu'(\theta_2) = \frac{k}{\left| \frac{1 - \kappa}{p_2} \right| u(q_2) - c(q_2) + p_2 R}
\]

where \( \kappa \) is the multiplier on the incentive compatibility constraint. Then it is straightforward to show that compared to \( \theta^* \), \( \theta_2 \) may be distorted upward or downward.
contract allows transfers. We have the single crossing property here, therefore the sorting assumption A3 is met too. As a result a unique equilibrium always exists. Note that the nonexistence in the adverse selection problems is resolved here\footnote{Comparing to Rothschild and Stiglitz (1976) equilibrium always exists here. As Guerrieri, Shimer, and Wright (2010) state: “...a key difference in our paper is that matching is bilateral and that each principal can serve at most one agent. This can create distortions along the extensive margin and implies that principals must form expectations about which agents are most attracted to a contract.”}.

### 4.2.4 A model with multiple types

In previous sections, we developed a simple model with two types of borrowers to prove the main properties in the cases with and without private information. Here we generate the above model to include multiple types of borrowers. This version of the model will generate multiple submarkets, but the basic intuition of the model and the results are the same as before. Let us assume we have a measure one of agents and a fraction $\pi_i > 0$ of them are type $i$, where $0 \leq i \leq I$. The setup of the model is the same as in the previous section with $p_i > p_j$ for $i > j$. In the case without private information the setup of the model and the results are exactly the same as before. A more detailed solution to this problem is presented in Appendix C.

In the case with private information the problem of the type 1 agent is the same
as in 4.7. The problem of a type $i > 1$ agent is:

$$
\bar{U}_i = \max_{\theta,q,x} \mu(\theta)[u(q) + p_i(R - x)]
$$

subject to:

$$
\eta(\theta)(-c(q) + p_i x) \geq k
$$

$$
\mu(\theta)(u(q) + p_j(R - x)) \leq \bar{U}_j \quad i > j
$$

The solution algorithm for the above optimization problems is similar to that presented in the previous sections. First we solve the first best problem for the type 1 agents. Then we use the maximized value for a type 1 agent ($\bar{U}_1$) to solve the problem of a type 2 agent and continue the process until we solve the problem of all of the agents. Again, information problems do not distort the choices of the lowest types (type 1), however choices of higher type agents ($i > 1$) may be distorted in cases with private information.

### 4.3 Data and Evidence

We start with 14,759 European firms with available data on the EFIGE database\(^\text{6}\). EFIGE was primarily created to “examine the pattern of internationalization of European firms” by the European Commission. The database provides firm-level quantitative and qualitative information on about 150 items ranging from R&D and innovation, labour organisation, financing and organisational activities, and pricing.

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\(^6\)EFIGE stands for “European Firms in Global Economy: internal policies for external competitiveness”; it is supported by the Directorate General Research of the European Commission through its 7th Framework Programme and coordinated by Bruegel, a European think tank.
behaviour. It is designed to be a representative sample of manufacturing firms in seven European economies (Germany, France, Italy, Spain, United Kingdom, Austria, Hungary). Data was collected in 2010 through survey questionnaires, covering the years from 2007 to 2009. Data gathered through surveys were validated by assessing the comparability of the survey data with official statistics. We focus on the part of this database that is dedicated to firms’ financing activities. Table 4.1 and Table 4.2 report descriptive statistics on the distribution of surveyed firms by country, industry, and size classes (extracted from Altomonte and Aquilante (2012)).

<table>
<thead>
<tr>
<th>Class Size</th>
<th>AUT</th>
<th>FRA</th>
<th>GER</th>
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<th>ITA</th>
<th>SPA</th>
<th>UK</th>
<th>Total</th>
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<tbody>
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<td>1,001</td>
<td>701</td>
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<td>1,036</td>
<td>635</td>
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<td>1,135</td>
<td>176</td>
<td>1,407</td>
<td>1,244</td>
<td>805</td>
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<td>Employees (50-249)</td>
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<td>429</td>
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<td>519</td>
<td>2,970</td>
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<tr>
<td>Employees (over 250)</td>
<td>46</td>
<td>214</td>
<td>306</td>
<td>45</td>
<td>145</td>
<td>146</td>
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<tr>
<td>Total</td>
<td>443</td>
<td>2,973</td>
<td>2,935</td>
<td>488</td>
<td>3,021</td>
<td>2,832</td>
<td>2,067</td>
<td>14,759</td>
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</table>

The first relevant survey question for our research is “Did your firm recur to external financing in the period 2008-2009? By external financing we mean funds not generated internally (i.e., not through self-financing).” Possible responses are “Yes,” “No,” and “DK/DA” (Do not know/Did not answer). A follow-up question is: “Have firms actually increased the total amount of external financing over that period?”. In Table 4.3, we show that 43.0% of 14,759 firms (6,344 firms) sought for external financing during 2008-2009. Out of this number, 2,692 firms (42.4%) indeed...
Table 4.2: Distribution of firms by country and NACE2 industries

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<tr>
<td>29</td>
<td>48</td>
<td>249</td>
<td>503</td>
<td>68</td>
<td>381</td>
<td>305</td>
<td>208</td>
<td>1,762</td>
</tr>
<tr>
<td>31</td>
<td>20</td>
<td>121</td>
<td>134</td>
<td>19</td>
<td>152</td>
<td>66</td>
<td>124</td>
<td>636</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
<td>94</td>
<td>56</td>
<td>9</td>
<td>49</td>
<td>25</td>
<td>101</td>
<td>339</td>
</tr>
<tr>
<td>33</td>
<td>15</td>
<td>58</td>
<td>192</td>
<td>6</td>
<td>71</td>
<td>25</td>
<td>80</td>
<td>447</td>
</tr>
<tr>
<td>34</td>
<td>6</td>
<td>73</td>
<td>41</td>
<td>11</td>
<td>47</td>
<td>64</td>
<td>33</td>
<td>275</td>
</tr>
<tr>
<td>35</td>
<td>2</td>
<td>16</td>
<td>20</td>
<td>3</td>
<td>33</td>
<td>42</td>
<td>21</td>
<td>137</td>
</tr>
<tr>
<td>36</td>
<td>5</td>
<td>16</td>
<td>172</td>
<td>18</td>
<td>211</td>
<td>258</td>
<td>258</td>
<td>938</td>
</tr>
<tr>
<td>Total</td>
<td>339</td>
<td>2,756</td>
<td>2,904</td>
<td>482</td>
<td>2,997</td>
<td>2,810</td>
<td>2,057</td>
<td>14,345</td>
</tr>
</tbody>
</table>
increased the total amount of external financing and 3,636 firms (57.3%) were not successful in raising external financing.

Table 4.3: Distribution of firms in need of external financing

<table>
<thead>
<tr>
<th>Asked for External Financing</th>
<th>Raised External Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
</tr>
<tr>
<td>Yes</td>
<td>6,344</td>
</tr>
<tr>
<td>No</td>
<td>3,636</td>
</tr>
<tr>
<td>DN/DK</td>
<td>16</td>
</tr>
<tr>
<td>Total</td>
<td>14,759</td>
</tr>
</tbody>
</table>

Table 4.4 shows the breakdown of the type of financial instrument used as a means of external financing. Of the 2,692 firms who succeeded in raising external financing, 16% (430 firms) used equity; only 3% (81 firms) relied on venture capital (VC) and private equity financing (PE); whereas 45.3% (1,220 firms) and 72.5% (1,952 firms) relied on short-term and medium- or long-term bank credit, respectively. This result confirms a heavy reliance of the European economy on bank financing as opposed to VC/PE financing. However, this result should be interpreted with caution as firms with less than 10 employees (which is more likely to include entrepreneurial firms and start-ups) are excluded from the data. Table 4.4 also shows that firms use other types of external financing, such as financial securities, public funds, tax incentives, leasing or factoring, and other (3.2%, 8.4%, 4.5%, 29.5% and 10.4% of all the firms in the sample, respectively). In the rest of this section, we focus on the firm-bank relationship and bank credit rationing.
Table 4.4: Type of financial instrument used for external financing

<table>
<thead>
<tr>
<th>Type of Instrument</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>430</td>
<td>16.0%</td>
</tr>
<tr>
<td>Venture capital and private equity</td>
<td>81</td>
<td>3.0%</td>
</tr>
<tr>
<td>Short-term bank credit</td>
<td>1,220</td>
<td>45.3%</td>
</tr>
<tr>
<td>Medium or long term bank credit</td>
<td>1,952</td>
<td>72.5%</td>
</tr>
<tr>
<td>Securities</td>
<td>86</td>
<td>3.2%</td>
</tr>
<tr>
<td>Public funds</td>
<td>226</td>
<td>8.4%</td>
</tr>
<tr>
<td>Tax incentives</td>
<td>120</td>
<td>4.5%</td>
</tr>
<tr>
<td>Leasing or factoring</td>
<td>794</td>
<td>29.5%</td>
</tr>
<tr>
<td>Other financing methods</td>
<td>279</td>
<td>10.4%</td>
</tr>
</tbody>
</table>

The most direct way of detecting credit rationing is through knowing who the unsuccessful credit seekers are and identifying the price they were willing to pay to obtain credit. Regulatory authorities around the world normally do not require firms to report unsuccessful bank loan applications. Also, there is no comprehensive data collected from the supply side that provide the details of denied corporate loans. We believe the EFIGE database can be used to provide new insights into the mechanism of credit rationing in the corporate loan market.

Surveyed firms were asked “During the last year, did the firm apply for more credit?” In Table 4.5, we show that out of the 2,710 firms who responded to this question, 1,997 (73.7%) indicated they applied for more credit, although only 1,407 firms (70.4%) were successful in obtaining credit. Table 4.5 also shows that out of the 29.6% of unsuccessful applicants, almost everyone (98.5%) was willing to borrow at the interest rate that they currently pay or the rate they previously paid and 60.7% (358 applicants) indicated they were prepared to borrow at a higher rate of interest if needed.
| Did the firm apply for more credit? | Was the firm willing to increase borrowing at the same rate of interest? | Number (%)
|---|---|---|
| Yes | Yes | 1,997 (73.7\%)
| No | No | 713 (26.3\%)
| Total | Total | 2,710 (100.0\%)

Was the firm willing to increase borrowing at a higher rate of interest?

| Number (%)
|---|
| Yes | 358 (60.7\%)
| No | 230 (39.0\%)
| DK/DY | 2 (0.3\%)
| Total | 590 (100.0\%)

Was the firm successful?

| Number (%)
|---|
| Yes | 581 (98.5\%)
| No | 9 (1.5\%)
| DK/DY | 2 (0.3\%)
| Total | 590 (100.0\%)

Total Number of Respondents

| Number (%)
|---|
| Yes | 1,997 (73.7\%)
| No | 713 (26.3\%)
| Total | 2,710 (100.0\%)
We next focus on the screening process of credit applications. In the theoretical part of this paper, we show that banks screen borrowers by posting contracts with certain terms and conditions and by requiring certain types of information from potential borrowers. Their goal is to attract specific groups of borrowers by making it difficult for others to apply. Table 4.6 provides descriptive statistics on the distribution of the type of information and guarantees required in credit applications. We divide firms that apply for bank credit into three groups: 1. firms with successful applications; 2. firms with unsuccessful applications that were not willing to pay a higher interest rate; and 3. firms with unsuccessful applications that were willing to pay a higher interest rate. We exclude the two unsuccessful firms (Table 4.5) for which their willingness to pay a higher rate is not known. We also divide terms and conditions into two main groups: First, the information that banks required from potential borrowers in order to process their application, and second, the type of guarantee or collateral that borrowers were asked to provide. Information required includes these seven categories: collateral, balance sheet information, interviews with management on firm’s policy and prospects, business plan and firms’ targets, historical records of payments and debt service, brand recognition, and other. The types of guarantee/collateral required include five categories: personal guarantees from the person who manages or owns the firm, guarantees on assets belonging to the firm, guarantees on assets of the group the firm belongs to, third party collateral (by a consortium), and other collaterals. The number and percentage of loan contracts in each firm category that are subject to each term and condition are presented. For instance, Table 4.6 shows that under “Information Required,” 61.5%, 77% and 79.3%
of loans applied for by successful applicants, unsuccessful applicants not willing to pay a higher rate, and unsuccessful applicants willing to pay a higher rate, respectively, were required to provide information on some sort of collateral. Also, as an example under “Type of Guarantee/Collateral Required”, it is shown that personal guarantees from the person who manages or owns the firm were required for 39.7%, 53.5% and 54.7% of loans applied for by successful applicants, unsuccessful applicants not willing to pay a higher rate and unsuccessful applicants willing to pay a higher rate, respectively. In addition to providing statistics on the conditions of acquiring loans, Table 4.6 also shows that denied loans were more difficult to acquire originally, as applicants have to provide more guarantees and collaterals at the time of loan applications.

To sum up, we provide evidence on two issues: First, the existence of credit rationing in the corporate loan market, that is, the existence of excess demand for loanable funds. Our results demonstrate that there are indeed potential borrowers in need of bank credit who are willing to pay a higher interest rate but their applications are denied. This implies that firms are rationed by the lenders in the corporate loan market. Second, we provide evidence on the heterogeneity that exists in the corporate loan market in the terms and conditions of loans and also in the information required in the screening process of loan applications.
Table 4.6: Distribution of conditions of granted and denied credit agreements

<table>
<thead>
<tr>
<th>Information Required:</th>
<th>Successful credit application (N=1,407)</th>
<th>Unsuccessful credit application, not willing to pay higher rates (N=230)</th>
<th>Unsuccessful credit application, willing to pay higher rates (N=358)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Percentage</td>
<td>Number</td>
</tr>
<tr>
<td>Collateral</td>
<td>865</td>
<td>61.5%</td>
<td>177</td>
</tr>
<tr>
<td>Balance sheet information</td>
<td>1,253</td>
<td>89.1%</td>
<td>212</td>
</tr>
<tr>
<td>Interviews with management on firm’s policy and prospects</td>
<td>835</td>
<td>59.3%</td>
<td>105</td>
</tr>
<tr>
<td>Business plan and firms’ targets</td>
<td>746</td>
<td>53.0%</td>
<td>117</td>
</tr>
<tr>
<td>Historical records of payments and debt service</td>
<td>613</td>
<td>43.6%</td>
<td>104</td>
</tr>
<tr>
<td>Brand recognition</td>
<td>224</td>
<td>15.9%</td>
<td>28</td>
</tr>
<tr>
<td>Other</td>
<td>147</td>
<td>10.4%</td>
<td>21</td>
</tr>
<tr>
<td>Type of Guarantee/Collateral Required:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Personal guarantees from the person who manages or owns the firm</td>
<td>558</td>
<td>39.7%</td>
<td>123</td>
</tr>
<tr>
<td>Guarantees on assets belonging to the firm</td>
<td>572</td>
<td>40.7%</td>
<td>107</td>
</tr>
<tr>
<td>Guarantees on assets of the group the firm belongs to</td>
<td>114</td>
<td>8.1%</td>
<td>22</td>
</tr>
<tr>
<td>Third party collateral (i.e., by a consortium, etc.)</td>
<td>63</td>
<td>4.5%</td>
<td>22</td>
</tr>
<tr>
<td>Other collaterals</td>
<td>68</td>
<td>4.8%</td>
<td>19</td>
</tr>
</tbody>
</table>
Chapter 5

Conclusion

This dissertation investigates the effects of central banks’ asset purchase programs on the economy and the role of frictions in the corporate loan markets. In chapter 2, I construct a model of the monetary economy in which different assets provide liquidity services. Adding illiquid nominal bonds to a microfounded model of monetary economy allows me to study the welfare effects of central banks asset purchase programs. I show that the central bank can change the overall liquidity and welfare in the economy by changing the relative supply of assets with different liquidity characteristics. My model also enables me to study the welfare effects of a restriction on trade with government bonds. I show that in a non-empty set of parameters restricting trade with government bonds can affect welfare. A liquidity trap can exist away from the Friedman rule and with a positive real interest rate. One possible extension of the model in chapter 2 is to add privately issued assets to the model and investigate the liquidity effects of a richer set of central banks’ asset purchase (or sale) programs.
In chapter 3, I construct a model of the monetary economy with heterogeneous agents, in which the central bank implements policies by changing the supply of nominal bond and money. Chapter 3 studies the central bank’s open-market operations in a model with heterogeneous agents. Using competitive search in the frictional market for goods allows me to study the distribution of asset holding in a tractable model. The central bank can implement monetary policies by supplying money and trading bonds in the asset market. There are two types of equilibria. In an equilibrium with low bond supply, the asset market is not segmented. All of the agents participate in the asset market and hold positive portfolios of bonds and money. In an equilibrium with high bond supply, segmentation is generated endogenously. Households with high labor supply shocks participate in the asset market and hold positive portfolios of bonds and money. Households with low labor supply shock only hold money in their portfolios. In an equilibrium with no segmentation, open-market operations have no real effects on the distribution of real asset holding. In an equilibrium with segmented asset market, open-market operations change the decision of a subset of households and have real effects on the distribution of asset holdings. The main results are robust to exogenously segmented asset market.

One possible extension of the model is to relax the quasi-linear preference of the households to a more general preference structure. With a more general preference structure, one can study the wealth effects in this framework. In this setup, it would be difficult to analytically show some of the properties of the equilibrium and computational exercises would be more critical.

In chapter Chapter 4, we studied corporate loan contracts. If the price mechanism
works then we should not observe credit rationing in corporate loan markets. Using a novel dataset on a large sample of European firms seeking external financing, we show that credit rationing indeed exists. Out of 1,997 firms that apply for more bank credit, 590 firms (29.6%) cannot obtain any. The application by 60.7% of these firms (358 firms) was denied in spite of them being prepared to borrow at a higher rate of interest. We also show that banks provide credit agreements with different terms and conditions. In addition, they screen potential borrowers by requiring various information and guarantees during the process of loan application.

The corporate finance literature clearly lacks models in which both credit rationing and distribution in the terms and conditions of loans are allowed to operate simultaneously. To fill this gap, chapter 4 presents a simple model of the credit market with adverse selection and search frictions. In this model, banks post the terms and conditions of credit agreements and potential borrowers with private information choose where to direct their search. In equilibrium, banks post separating contracts. The terms and conditions of each contract serve as a device by which lenders induce borrowers to self-select across credit sub-markets. This leads to a number of findings. First, in equilibrium, there exists certain firms in need of bank credit that remain unmatched. Second, the credit market becomes segmented into different “sub-markets,” and each sub-market attracts certain types of borrowers.

Chapter 4 suggests several directions for further research. First, our finding that a significant fraction of firms fail to achieve their desired capital structure suggests that studies that link realized capital structure to firms’ characteristics and even firms’ unobserved heterogeneity are not complete unless they account for frictions
in the credit market. In the empirical part of this paper, we focus on the cross-sectional heterogeneity in bank-firm relationships and variations in capital structure. It will be useful to study the impact of various credit supply shocks and regimes over time on the evolution of capital structure and heterogeneity in debt contracts. Another interesting research idea is how the deviation of realized capital structure from the desired capital structure of firms changes over time as a result of shocks in the supply of credit. Understanding how search frictions work in credit markets requires collecting a time-series of successful and unsuccessful bank loan applications as well as the terms and conditions of the granted and denied loan agreements. With such a data set, one can study the market tightness in different submarkets and compare the results with the predictions of the theoretical model.

Second, our treatment of credit agreement focuses on a simple contract that only includes interest rate. Another possible venue of future research is to extend the theoretical part of this paper and examine the joint determination of the amounts, rates, and guarantees of credit agreements in each sub-market.
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Appendix A

Omitted Proofs from Chapter 2

A.1 The 2-asset economy

I characterize 3 cases of the equilibria based on the set of liquidity constraints that are binding. In all of these cases at least one of the constraints are binding. The case where none of them are binding only happens at the Friedman rule, and it is shown to be efficient.

Case I: $\lambda^m > 0$ and $\lambda^b = 0$

The first order conditions are

$$u'(c_b) = \psi'(q_b)$$

$$u'(c_m) = \psi'(q_m) + \frac{\psi'(q_m)}{\Omega^m} \lambda^m$$
I can rewrite the above equation as

\[ \lambda^m = \left( \frac{u'(c_m)}{\psi(q_b)} - 1 \right)\Omega^m \]

Envelope condition gives

\[ \frac{\gamma}{\beta} \omega^m = \omega^m + \frac{\alpha N(1-l)}{N - \sigma} \left[ \left( \frac{u'(c_m)}{\psi'(q_m)} - 1 \right)\Omega^m \right] \]

By applying stationarity, I can write the envelope as

\[ \frac{\gamma}{\beta} - 1 = \frac{\alpha N(1-l)}{N - \sigma} \right[ \left( \frac{u'(c_m)}{\psi'(q_m)} - 1 \right] \]

The price for nominal bond is

\[ s = \frac{\omega^b}{\omega^m} = \frac{\beta}{\gamma} \]

**Case II**: \( \lambda^m = 0 \) and \( \lambda^b > 0 \)

The first order conditions are

\[ u'(c_m) = \psi'(q_m) \]

\[ \lambda^b = \left( \frac{u'(c_b)}{\psi(q_b)} - 1 \right)\Omega^b \]

Envelope condition gives
The price for the nominal bond is 1. By applying stationarity, I can write the envelope as

\[ \frac{\gamma}{\beta^{m-1}} = \omega^m + \frac{\alpha N l}{N - \sigma} \left[ \left( \frac{u'(c_b)}{\psi'(q_b)} - 1 \right) \Omega^m \right] \]

It is straightforward to see that changing \( z \) would not affect households’ decision and welfare when at least one of the liquidity constraints is not binding.

**Case III:** \( \lambda^m > 0 \) and \( \lambda^b > 0 \)

The first order conditions are

\[ \lambda^m = \left( \frac{u'(c_m)}{\psi'(q_m)} - 1 \right) \Omega^m \]

\[ \lambda^b = \left( \frac{u'(c_b)}{\psi'(q_b)} - 1 \right) \Omega^m \]

Envelope conditions give

\[ \frac{\gamma}{\beta^{m-1}} = \omega^m + \frac{\alpha N l}{N - \sigma} \left[ \left( \frac{u'(c_b)}{\psi'(q_b)} - 1 \right) \Omega^m \right] + \frac{\alpha N (1 - l)}{N - \sigma} \left[ \left( \frac{u'(c_m)}{\psi'(q_m)} - 1 \right) \Omega^m \right] \]

\[ \frac{\gamma}{\beta^{m-1}} = \omega^m + \frac{\alpha N l}{N - \sigma} \left[ \left( \frac{u'(c_b)}{\psi'(q_b)} - 1 \right) \Omega^m \right] \]
By applying stationarity, I can write the envelope conditions as

\[
\frac{\gamma}{\beta} s - 1 = \frac{\alpha N l}{N - \sigma} \left[ \frac{u'(c_b)}{\psi'(q_b)} - 1 \right]
\]

\[
\frac{\gamma}{\beta} - 1 = \frac{\alpha N l}{N - \sigma} \left[ \frac{u'(c_b)}{\psi'(q_b)} - 1 \right] + \frac{\alpha N (1 - l)}{N - \sigma} \left[ \frac{u'(c_m)}{\psi'(q_m)} - 1 \right]
\]

It is straightforward to see that changing \( z \) affects the decisions of households and has real effects on the economy.
A.2 Numerical example for the 2-asset economy

I solve the model for the following functional forms

\[ u(c) = \log(c) \]
\[ \psi(q) = \frac{q^2}{2} \]
\[ h(n) = 2an^{1/2} \]

Now I solve the model for 3 different cases of liquidity constrains

Case I:

\[ q_b = \frac{1}{(\alpha N l)^{1/2}} \]
\[ q_m = \frac{1}{\left(\left(\frac{\gamma}{\beta} - 1\right)(N - \sigma) + \alpha N (1 - l)\right)^{1/2}} \]
\[ \frac{a}{(1 - N)^{1/2}} = N - \sigma \]

By using the constraints it is straightforward to show that this equilibrium happens for high enough \( l \)

\[ l > \bar{l} = \frac{(\frac{\gamma}{\beta} - 1)(N - \sigma) + \alpha N}{(2 + z)\alpha N} \]

Case II:

\[ q_m = \frac{1}{(\alpha N (1 - l))^{1/2}} \]
APPENDIX A. OMITTED PROOFS FROM CHAPTER 2

\[ q_b = \frac{1}{\left( (\frac{\gamma}{\beta} - 1)(N - \sigma) + \alpha N l \right)^{1/2}} \]

\[ \frac{a}{(1 - N)^{1/2}} = N - \sigma \]

By using the constraints it is straightforward to show that this equilibrium happens for low enough \( l \)

\[ l < l = \frac{\alpha N - (\frac{\gamma}{\beta} - 1)(N - \sigma)(1 + z)}{\alpha N(2 + z)} \]

Case III:

\[ q_m = \frac{1}{\left( (\frac{\gamma}{\beta} - 1)(N - \sigma) + \alpha N (1 - l) \right)^{1/2}} \]

\[ q_b = \frac{1}{\left( (\frac{\gamma}{\beta} s - 1)(N - \sigma) + \alpha N l \right)^{1/2}} \]

\[ s = \frac{\frac{\gamma}{\beta}(N - \sigma) + \alpha N (1 - l) + (1 + z)(N - \sigma - \alpha N l)}{\frac{\gamma}{\beta}(2 + z)(N - \sigma)} \]

\[ \frac{a}{(1 - N)^{1/2}} = N - \sigma \]


A.3 The 3-asset economy

Define

\[ \zeta(q_i) = \psi'(q_i)q_i - \psi(q_i) \quad i \in \{l, s, n\} \]

In what follows I solve the problem in different cases of equilibrium.

Cases:

1: \( \lambda^s = 0 < \lambda^n, \lambda^l \)

From the envelope conditions it follows

\[ s_s = s_l < 1 \]

The first order conditions are

\[ u'(c_s) = \psi'(q_s) \]

\[ \frac{\gamma}{\beta} s_s - 1 = \frac{\alpha N l}{N - \sigma} \frac{u'(c_n)}{\psi'(q_n) - 1} \]

\[ \frac{\gamma}{\beta} (1 - s_s) = \frac{\alpha N (1 - l - k)}{N - \sigma} \frac{u'(c_l)}{\psi'(q_l) - 1} \]
\[ h'(1 - N) = \left( \frac{\gamma}{\beta} s_s - 1 + \frac{\alpha NL}{N - \sigma} \right) \zeta(q_n) + \frac{\alpha N k}{N - \sigma} \zeta(q_s) + \left( \frac{\gamma}{\beta} (1 - s_s) + \frac{\alpha N (1 - k - l)}{N - \sigma} \right) \zeta(q_l) \]

Solution for \( u(c) = \log(c) \) and \( \psi(q) = q^2/2 \)

\[ \frac{1}{q_s^2} = \alpha N k \]
\[ \frac{1}{q_n^2} = \left( \frac{\gamma}{\beta} s_s - 1 \right)(N - \sigma) + \alpha N l \]
\[ \frac{1}{q_l^2} = \frac{\gamma}{\beta} (1 - s_s)(N - \sigma) + \alpha N (1 - l - k) \]
\[ h'(1 - N) = \frac{3}{2(N - \sigma)} \]

The solution to the above equations is

\[ s_s = \frac{1 + \frac{\beta \alpha N (1 - l - k)}{\gamma (N - \sigma)} + \beta / \gamma (1 + z_s + z_l)(1 - \frac{\alpha NL}{N - \sigma})}{2 + z_l + z_s} \]
\[ \frac{1}{q_n^2} = \frac{\alpha N (1 - l - k) + \gamma / \beta (N - \sigma) + \alpha N l - \left( 1 + z_l + z_s \right)(N - \sigma)}{2 + z_l + z_s} \]
\[ \frac{1}{q_l^2} = \frac{1 + z_l + z_s}{2 + z_l + z_s} \left( \alpha N (1 - k) + (N - \sigma)(\gamma / \beta - 1) \right) \]

Using some algebra I can solve for the criteria for this equilibrium

\[ \frac{1 + z_l + z_s}{(1 + z_s)(2 + z_l + z_s) + (1 + z_l + z_s)} \left( 1 + \frac{N - \sigma}{\alpha N} (\gamma / \beta - 1) \right) \leq k \]
\[
\frac{(1 + z_l + z_s)(1 + \frac{N - \sigma}{\alpha N}(\gamma/\beta - (1 + z_s + z_l)))}{(1 + z_s)(2 + z_l + z_s) + (1 + z_l + z_s)} \leq k
\]

The left hand side of the second equation is greater than the first.

**II:** \(\lambda^n = 0 < \lambda^s, \lambda^l\)

\[
s_l = \frac{\beta}{\gamma} < s_s < 1
\]

\[
u'(c_n) = \psi'(q_n)
\]

\[
\frac{\gamma}{\beta} s_s - 1 = \frac{\alpha N k}{N - \sigma} \left[\frac{u'(c_s)}{\psi'(q_s)} - 1\right]
\]

\[
\frac{\gamma}{\beta} (1 - s_s) = \frac{\alpha N (1 - l - k)}{N - \sigma} \left[\frac{u'(c_l)}{\psi'(q_l)} - 1\right]
\]

\[
h'(1 - N) = \frac{\alpha N l}{N - \sigma} \zeta(q_n) + (\frac{\gamma}{\beta} s_s - 1 + \frac{\alpha N k}{N - \sigma}) \zeta(q_s) +
\]

\[
(\frac{\gamma}{\beta} (1 - s_s) + \frac{\alpha N (1 - k - l)}{N - \sigma}) \zeta(q_l)
\]

Solution for \(u(c) = \log(c)\) and \(\psi(q) = q^2/2\)

\[
1/q_s^2 = (\frac{\gamma}{\beta} s_s - 1)(N - \sigma) + \alpha N k
\]
\[ 1/q_n^2 = \alpha N l \]

\[ 1/q_i^2 = \frac{\gamma}{\beta} (1 - s_s)(N - \sigma) + \alpha N (1 - l - k) \]

\[ h'(1 - N) = \frac{3}{2(N - \sigma)} \]

The solution to the above equations is

\[ s_s = \frac{1 + \alpha N (1 - l - k) \beta/\gamma - (\alpha N k - \beta/\gamma)(1 + z_s)}{2 + z_s} \]

\[ 1/q_s^2 = \frac{\alpha N (1 - l) + (1 + z_s)(\alpha N K (1 - \gamma/\beta(N - \sigma)) - (N - \sigma)) - (N - \sigma)(\gamma/\beta - 1)}{2 + z_s} \]

\[ 1/q_i^2 = \frac{1 + z_s(\gamma/\beta(N - \sigma)(1 - \alpha N k) + \alpha N (1 - l - k) - (N - \sigma))}{2 + z_s} \]

Using some algebra I can solve for the criteria for this equilibrium

\[ k \leq \frac{((1 + z_s + z_l)(2 + z_s) + 1 + z_s)l - (1 + z_s)(1 - \frac{N - \sigma}{\alpha N}(\gamma/\beta - 1)) + \frac{(1 + z_s)^2(N - \sigma)}{\alpha N}}{(1 + z_s)^2(1 - \gamma/\beta(N - \sigma))} \]

\[ \frac{((1 + z_s)(1 + z_l + z_s) + (1 + z_s)l)}{(1 + \gamma/\beta(N - \sigma))(1 + z_s)} \leq k \]

\( \text{III: } \lambda^l = 0 < \lambda^n, \lambda^s \]

\[ s_l < s_s = 1 \]
APPENDIX A. OMITTED PROOFS FROM CHAPTER 2

\[ u'(c_l) = \psi'(q_l) \]

\[ \frac{\gamma}{\beta} s_l - 1 = \frac{\alpha N l}{N - \sigma} \left[ \frac{u'(c_n)}{\psi'(q_n)} - 1 \right] \]

\[ \frac{\gamma}{\beta} (1 - s_l) = \frac{\alpha N K}{N - \sigma} \left[ \frac{u'(c_s)}{\psi'(q_s)} - 1 \right] \]

\[ h'(1 - N) = \frac{\alpha N(1 - k - l)}{N - \sigma} \zeta(q_i) + \left( \frac{\gamma}{\beta} s_l - 1 + \frac{\alpha N l}{N - \sigma} \right) \zeta(q_n) + \left( \frac{\gamma}{\beta} (1 - s_l) + \frac{\alpha N k}{N - \sigma} \right) \zeta(q_s) \]

Solution for \( u(c) = \log(c) \) and \( \psi(q) = q^2/2 \)

\[ 1/q_s^2 = \frac{\gamma}{\beta} (1 - s_s)(N - \sigma) + \alpha N k \]

\[ 1/q_n^2 = \left( \frac{\gamma}{\beta} s_s - 1 \right)(N - \sigma) + \alpha N l \]

\[ 1/q_l^2 = \alpha N (1 - l - k) \]

\[ h'(1 - N) = \frac{3}{2(N - \sigma)} \]

And by some algebra
\[ s_l = \frac{(1 + z_s)(1 + \frac{\alpha N k}{N - \sigma}) - \beta / \gamma(\frac{\alpha N k}{N - \sigma} - 1)(1 + z_s + z_l)}{2 + 2 z_s + z_l} \]

\[ 1/q_l^2 = \gamma / \beta (N - \sigma) + \alpha N k - (1 + z_s) (\gamma / \beta (N - \sigma) + \alpha N k) - (\alpha N l - (N - \sigma))(1 + z_l + z_s) \]

\[ 1/q_n^2 = \frac{\alpha N (1 + z_s)(k + l) - (N - \sigma)(1 + z_l + z_s)(\gamma / \beta - 1)}{2 + 2 z_s + z_l} \]

Using some algebra I can solve for the criteria for this equilibrium

\[ k + l \leq \frac{(2 + 2 z_s + z_l) + \frac{N - \sigma}{\alpha N} (1 + z_s + z_l)^2 (\gamma / \beta - 1)}{(1 + z_s)(1 + z_s + z_l) + (2 + 2 z_s + z_l)} \]

\[ ((1 + z_s)(1 + z_s + z_l) + 2 + 2 z_s + z_l)k + \frac{N - \sigma}{\alpha N} (1 + z_s)(1 + z_s + z_l) + 2 + 2 z_s + z_l)l \leq \]

\[ 2 + 2 z_s + z_l - (1 + z_s + z_l)(1 + z_s)(\frac{N - \sigma}{\alpha N} (\gamma / \beta + 1)) \]

IV: \( \lambda^* = \lambda^l = 0 < \lambda^n \)

\[ s_l = s_s = 1 \]

\[ u'(c_l) = \psi'(q_l) \]

\[ u'(c_s) = \psi'(q_s) \]
\[
\frac{\gamma}{\beta} - 1 = \frac{\alpha N l}{N - \sigma} u'(c_n) - 1
\]

\[
h'(1 - N) = \frac{\alpha N k}{N - \sigma} \zeta(q_s) + \left(\frac{\gamma}{\beta} - 1 + \frac{\alpha N l}{N - \sigma}\right) \zeta(q_n) + \frac{\alpha N (1 - k - l)}{N - \sigma} \zeta(q_l)
\]

As the above equations show, marginal open-market operations (small changes in
\(z_s\) and \(z_l\)) do not change the real decisions of the households and welfare. Solution
for \(u(c) = \log(c)\) and \(\psi(q) = q^2/2\)

\[
1/q_s^2 = \alpha N k
\]

\[
1/q_n^2 = \left(\frac{\gamma}{\beta} - 1\right)(N - \sigma) + \alpha N l
\]

\[
1/q_l^2 = \alpha N (1 - l - k)
\]

\[
h'(1 - N) = \frac{3}{2(N - \sigma)}
\]

Using some algebra I can solve for the criteria for this equilibrium

\[
(\gamma/\beta - 1) \left(\frac{N - \sigma}{\alpha N}\right) (1 + z_s + z_l) \leq (1 + z_s) k - (1 + z_s + z_l) l
\]

\[
(2 + z_s + z_l) l + k \leq 1 - (1 + z_s + z_l)(\gamma/\beta - 1) \frac{N - \sigma}{\alpha N}
\]

\(V: \lambda^s = \lambda^n = 0 < \lambda^l\)
\[ s_t = s_s = \beta / \gamma < 1 \]

\[ u'(c_n) = \psi'(q_n) \]

\[ u'(c_s) = \psi'(q_s) \]

\[ \frac{\gamma}{\beta} - 1 = \frac{\alpha N (1 - k - l)}{N - \sigma} \left[ \frac{u'(c_n)}{\psi'(q_n)} - 1 \right] \]

\[ h'(1 - N) = \frac{\alpha N l}{N - \sigma} \zeta(q_n) + \left( \frac{\gamma}{\beta} - 1 + \frac{\alpha N (1 - k - l)}{N - \sigma} \right) \zeta(q_l) + \frac{\alpha N k}{N - \sigma} \zeta(q_s) \]

As the above equations show, marginal open-market operations (small changes in \( z_s \) and \( z_t \)) do not change the real decisions of the households and welfare.

Solution for \( u(c) = \log(c) \) and \( \psi(q) = q^2 / 2 \)

\[ 1/q_s^2 = \alpha N k \]

\[ 1/q_n^2 = \alpha N l \]

\[ 1/q_l^2 = \left( \frac{\gamma}{\beta} - 1 \right) (N - \sigma) + \alpha N (1 - l - k) \]
\[ h'(1 - N) = \frac{3}{2(N - \sigma)} \]

Using some algebra I can solve for the criteria for this equilibrium:

\[ 1 + (\gamma/\beta - 1)\frac{N - \sigma}{\alpha N} \leq (2 + z_s + z_l)l + k \]

\[ 1 + (\gamma/\beta - 1)(\frac{N - \sigma}{\alpha N}) \leq (2 + z_s)k + l \]

**VI:** \( \lambda^n = \lambda^l = 0 < \lambda^s \)

\[ s_l = \beta/\gamma < s_s = 1 \]

\[ u'(c_n) = \psi'(q_n) \]

\[ u'(c_l) = \psi'(q_l) \]

\[ \frac{\gamma}{\beta} - 1 = \frac{\alpha Nk}{N - \sigma}[u'(c_s)/\psi'(q_s) - 1] \]

\[ h'(1 - N) = \frac{\alpha Nl}{N - \sigma}\zeta(q_n) + (\frac{\gamma}{\beta} - 1 + \frac{\alpha Nk}{N - \sigma})\zeta(q_s) + \frac{\alpha N(1 - k - l)}{N - \sigma}\zeta(q_l) \]

As the above equations show, marginal open-market operations (small changes in \( z_s \)
and $z_l$ do not change the real decisions of the households and welfare.

Solution for $u(c) = \log(c)$ and $\psi(q) = q^2/2$:

$$1/q_s^2 = (\frac{\gamma}{\beta} - 1)(N - \sigma) + \alpha N k$$

$$1/q_n^2 = \alpha N l$$

$$1/q_l^2 = \alpha N(1 - l - k)$$

$$h'(1 - N) = \frac{3}{2(N - \sigma)}$$

Using some algebra I can solve for the criteria for this equilibrium:

$$(2 + z_s)k + l \leq 1 - (\gamma/\beta - 1)\frac{N - \sigma}{\alpha N}(1 + z_s)$$

$$(\gamma/\beta - 1)(\frac{N - \sigma}{\alpha N})(1 + z_s) \leq (1 + z_s + z_l)l - (1 + z_s)k$$

**VII:** $0 < \lambda^s, \lambda^l, \lambda^n$

$$s_l < s_s < 1$$

$$\frac{\gamma}{\beta} s_l - 1 = \frac{\alpha N l}{N - \sigma} [u'(c_n) - 1]$$

$$\frac{\gamma}{\beta} (s_s - s_l) = \frac{\alpha N k}{N - \sigma}[u'(c_s) - 1]$$
\[
\frac{\gamma}{\beta}(1 - s_s) = \frac{\alpha N(1 - l - k)}{N - \sigma} \left[ \frac{u'(c_l)}{\psi'(q_l)} - 1 \right]
\]

\[
h'(1 - N) = (\frac{\gamma}{\beta} s_l - 1 + \frac{\alpha N l}{N - \sigma}) \zeta(q_n) + \left(\frac{\gamma}{\beta}(s_s - s_l) + \frac{\alpha N k}{N - \sigma}\right) \zeta(q_s) + \left(\frac{\gamma}{\beta}(1 - s_s) + \frac{\alpha N (1 - l - k)}{N - \sigma}\right) \zeta(q_l)
\]

Solution for \(u(c) = \log(c)\) and \(\psi(q) = q^2/2\):

\[
1/q_s^2 = \frac{\gamma}{\beta}(s_s - s_l)(N - \sigma) + \alpha N k
\]

\[
1/q_n^2 = (\frac{\gamma}{\beta} s_l - 1)(N - \sigma) + \alpha N l
\]

\[
1/q_l^2 = \frac{\gamma}{\beta}(1 - s_s)(N - \sigma) + \alpha N (1 - l - k)
\]

\[
h'(1 - N) = \frac{3}{2(N - \sigma)}
\]
Appendix B

Omitted Proofs from Chapter 3

B.1 Market clearing conditions

I can find the cumulative distribution of money before lotteries by:

$$G(m) = \int \int_{z^{-1}(m)} dF(\theta)dH$$  \hspace{1cm} (B.1)

and similarly the distribution of bond before lotteries follows:

$$H(a_{-1}) = \int \int_{a^{-1}(a_{-1})} dF(\theta)dG$$  \hspace{1cm} (B.2)

I assume a balanced budget for government at each period of time. The total real transfer that a household receives is the sum of transfers from printing money and the transfers received from bond market:
\[ T = \frac{\gamma - 1}{w^{\gamma}} + \frac{sA - A_{-1}}{wM'} \]  

(B.3)

In the bond market the total amount of bonds supplied equals the sum of demanded bonds by households of different type. Thus, the market clearing for bonds gives:

\[ \frac{A_{s}}{w_{M}} = \int \int \int_{\tilde{\theta}} a(\theta) dF(\theta) dG(m) dH(a_{-1}) \]  

(B.4)

In the general-good market, the market clearing condition is:

\[ Y = \int_{\tilde{\theta}} y(\theta) dF(\theta) \]  

(B.5)

LD is the same as Sun (2012):

\[ LD = Y + \int_{\tilde{\theta}} \frac{\pi_1(z(\theta)) b(L_1(z(\theta)))}{\mu(b(L_1(z(\theta))))} [k + \psi(q(L_1(z(\theta)))) \mu(b(L_1(z(\theta))))] dF(\theta) \]

\[ + \int_{\tilde{\theta}} \frac{\pi_2(z(\theta)) b(L_2(z(\theta)))}{\mu(b(L_2(z(\theta))))} [k + \psi(q(L_2(z(\theta)))) \mu(b(L_2(z(\theta))))] dF(\theta) \]  

(B.6)

The firms zero-profit condition gives:

\[ k + \psi(q(L_i(z(\theta)))) \mu(b(L_i(z(\theta)))) = L_i(z(\theta)) \]
Then LD becomes:

\[
LD = \int_{\bar{\theta}}^{\bar{\theta}} y(\theta)dF(\theta) + \int_{\bar{\theta}}^{\bar{\theta}} \pi_1(z(\theta))b(L_1(z(\theta)))L_1(z(\theta))dF(\theta) + \int_{\bar{\theta}}^{\bar{\theta}} \pi_2(z(\theta))b(L_2(z(\theta)))L_2(z(\theta))dF(\theta) \quad (B.7)
\]

Aggregate labor supply is the sum of households labor supply:

\[
LS = \int_{\bar{\theta}}^{\bar{\theta}} \int l(m, a, \theta)dF(\theta)dG_a(m)dH(a_{-1})
\]

in which \( dG_m \) is the distribution of money holdings at the beginning of the period. Substituting for \( l \) in the above equation we get

\[
LS = \int_{\bar{\theta}}^{\bar{\theta}} \int [py(\theta) + z(\theta) + s\gamma a(\theta) - m - a_{-1} - T]dF(\theta)dG_a(m)dH(a_{-1}) \quad (B.8)
\]

Substituting for \( T \):

\[
LS = \int_{\bar{\theta}}^{\bar{\theta}} \int [py(\theta) + z(\theta) + s\gamma a(\theta) - m - a_{-1} - \frac{\gamma - 1}{w^\gamma} - \frac{sA}{wM^\gamma} + \frac{A_{-1}}{w^\gamma M^\gamma}]dF(\theta)dG_a(m)dH(a_{-1}) \quad (B.9)
\]
LS becomes:

$$LS = \frac{A_{-1}}{w\gamma M} - \frac{sA}{wM'} - \frac{\gamma - 1}{w\gamma} - \int mdG_a(m) - \int a_{-1}dH(a_{-1})$$

$$+ \int_{\theta}^{\bar{\theta}} [y(\theta) + z(\theta) + s\gamma a(\theta)]dF(\theta) \quad (B.10)$$

Labor market clearing condition gives:

$$\int_{\theta}^{\bar{\theta}} s\gamma a(\theta)dF(\theta) + \int_{\theta}^{\bar{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$

$$+ \int_{\theta}^{\bar{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta)$$

$$= \frac{sA}{wM'} + \frac{\gamma - 1}{w\gamma} - \frac{A_{-1}}{wM} + \int mdG_a(m) + \int a_{-1}dH(a_{-1})$$

$m$ is the distribution of money at the beginning of the period. Therefore, it consists of balances that are not spent plus the payments on nominal bonds:

$$\int mdG_a(m) =$$

$$\int_{\theta}^{\bar{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta)$$

$$+ \int_{\theta}^{\bar{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) + \int \frac{a_{-1}}{\gamma}dH(a_{-1}) + \int \frac{h_{-1}}{\gamma}dJ_{h_{-1}}$$

Plug in the labor market clearing condition:
\[ \frac{s A}{w M} + \frac{\gamma - 1}{w \gamma} - \frac{A_{-1}}{w \gamma M} = \]
\[ \int_{\theta}^{\bar{\theta}} s \gamma a(\theta) dF(\theta) + (1 - \frac{1}{\gamma}) \int_{\theta}^{\bar{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta) \]
\[ + (1 - \frac{1}{\gamma}) \int_{\theta}^{\bar{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \int_{\theta}^{\bar{\theta}} a_{-1}(1 + \frac{1}{\gamma})dH(a_{-1}) - \int_{\theta}^{\bar{\theta}} \frac{h_{-1}}{\gamma}dJ_{h_{-1}} \]

The labor-market-clearing can be written as:

\[ \frac{1}{w \gamma} [\gamma - 1 - \lambda + s \lambda] = \]
\[ (2 - s \gamma) \int_{\theta}^{\bar{\theta}} a(\theta) dF(\theta) + (1 - \frac{1}{\gamma}) \int_{\theta}^{\bar{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta) \]
\[ + (1 - \frac{1}{\gamma}) \int_{\theta}^{\bar{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \int_{\theta}^{\bar{\theta}} \frac{h(\theta)}{\gamma}dF(\theta) \quad (B.11) \]
B.2 Welfare analysis

I use the household’s utility function to calculate welfare:

\[ \varpi = \int \int \int \{ U(y) + u(q) - \theta l \} dF(\theta) dG(m) dH(a_{-1}) \]

\[ = \int U(y(\theta)) dF(\theta) + \int u(q(z(\theta))) dF(\theta) - \int \int \int \{ \theta l \} dF(\theta) dG(m) dH(a_{-1}) \]

I can write the last integral as:

\[ \int \int \int (\theta l) dF(\theta) dG(m) dH(a_{-1}) = \]

\[ \int \left[ \theta (y(\theta) + z(\theta) + h(\theta) + s \gamma a(\theta)) \right] dF(\theta) \]

\[ - \int \theta \left( \int m dG_a \right) dF(\theta) - \int \theta \left( \int a_{-1} dH(a_{-1}) \right) dF(\theta) - T \int \theta dF(\theta) \]

I have shown in the market clearing appendix the distribution of money before the lotteries is:

\[ \int m dG_a(m) = \]

\[ \int_\theta \pi_1(z(\theta)) [1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \]

\[ + \int_\theta \pi_2(z(\theta)) [1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) + \int a_{-1} \gamma dH(a_{-1}) + \int \frac{h_{-1}}{\gamma} dJ_{h_{-1}} \]

I can substitute for the distribution of money \( m dG_a \) and labor supply \( l \) from
the above equations, and for government transfers \((T)\) from the market clearing appendix to simplify the equation for welfare:

\[
\varpi = \int [U(y(\theta)) - \theta y(\theta) + u(q(z(\theta))) - \theta z(\theta) - \theta h(\theta) - s_\gamma \theta a(\theta)] dF(\theta) \\
+ \int \frac{\pi_1(z(\theta))}{\gamma} \left[1 - b(L_1(z(\theta))) \right] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \\
+ \int \frac{\pi_2(z(\theta))}{\gamma} \left[1 - b(L_2(z(\theta))) \right] \frac{L_2(z(\theta))}{\gamma} dF(\theta) \\
+ \int \frac{1}{\gamma} \left[ \int a_{-1} dH_{a-1} \right] \int \theta dF(\theta) + \int \frac{1}{\gamma} \left[ \int h_{-1} dJ_{h-1} \right] \int \theta dF(\theta) \\
+ \frac{1}{w_\gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF(\theta)
\]
B.3 Proof of proposition 5

Let’s assume there are two types of agents in the economy, traders in the asset market (denoted by subscript T) and non-traders (denoted by subscript N).

Value function of a trader:

\[ W_T(m_T, a_{-1}, \theta) = \max_{y_T, l_T, z_T, a_T} \left\{ U(y_T) - \theta l_T + V_T(z_T, h_T, a_T) \right\} \]

\[ \text{st. } py_T + z_T + s\gamma a \leq m_T + a_{-1} + l_T + T \]

Value function of a non-trader:

\[ W_N(m_N, \theta) = \max_{y_N, l_N, z_N} \left\{ U(y_N) - \theta l_N + V_T(z_N, h_N) \right\} \]

\[ \text{st. } py_N + z_N \leq m_N + l_N + T \]

Using the budget constraint to eliminate \( l_{i=N,T} \):

\[ W_T(m_T, a_{-1}, \theta) = \theta(m_T + T + a_{-1}) + \max_{y_T \geq 0} \left\{ U(y_T) - \theta py_T \right\} + \max_{z_T, a, h_T} \left\{ -\theta(z_T + s\gamma a + h_T) + V_T(z_T, h_T, a) \right\} \]

\[ W_N(m_N, \theta) = \theta(m_N + T) + \max_{y_N \geq 0} \left\{ U(y_N) - \theta py_N \right\} + \max_{z_N, h_N} \left\{ -\theta(z_N + h_N) + V_N(z_N, h_N) \right\} \]

The optimal choices of \( y_{i \in \{T,N\}}, \ z_{i \in \{T,N\}} \) and \( a \) must satisfy:
\[ U'(y_T) = U'(y_N) = \theta \] (B.12)

The above expression shows that a trader and a non-trader choose the same amount of consumption in the frictionless market:

\[ y_T(\theta) = y_N(\theta) = y(\theta) \]

\[
\frac{\partial V_T(z_T, h_T, a)}{\partial z_T} \begin{cases} 
\leq \theta & z_T \geq 0 \\
\geq \theta & z_T \leq \bar{m} - s\gamma a - h_T
\end{cases} \quad (B.13)
\]

\[
\frac{\partial V_T(z_T, h_T, a)}{\partial h_T} \begin{cases} 
\leq \theta & h_T \geq 0 \\
\geq \theta & h_T \leq \bar{m} - s\gamma a - z_T
\end{cases} \quad (B.14)
\]

\[
\frac{\partial V_T(z_T, h_T, a)}{\partial a} \begin{cases} 
\leq \theta s\gamma & a \geq 0 \\
\geq \theta s\gamma & sa \leq \bar{m} - z_T - h_T
\end{cases} \quad (B.15)
\]

\[
\frac{\partial V_N(z_N, h_N)}{\partial z_N} \begin{cases} 
\leq \theta & z_N \geq 0 \\
\geq \theta & z_N \leq \bar{m} - h_N
\end{cases} \quad (B.16)
\]

\[
\frac{\partial V_N(z_N, h_N)}{\partial h_N} \begin{cases} 
\leq \theta & h_N \geq 0 \\
\geq \theta & h_N \leq \bar{m} - z_N
\end{cases} \quad (B.17)
\]

The value functions can be written as:
\[ W_T(m_T, a_{-1}, \theta) = W_T(0, 0, \theta) + \theta m_T + \theta a_{-1} \]  
(B.18)

Where:

\[ W_T(0, 0, \theta) = U(y(\theta)) - \theta y(\theta) + V_T(z_T(\theta), h_T(\theta), a(\theta)) - \theta(z_T(\theta) + h_T(\theta) + s\gamma a(\theta)) \]  
(B.19)

\[ W_N(m_T, \theta) = W_T(0, \theta) + \theta m_N \]  
(B.20)

Where:

\[ W_N(0, \theta) = U(y(\theta)) - \theta y(\theta) + V_N(z_N(\theta), h_N(\theta)) - \theta(z_N(\theta) + h_N(\theta)) \]  
(B.21)

We can see that the value function \( W() \) is linear in household’s asset holdings for both traders and non-traders. Agents problem in the frictional market for traders and non-traders are similar. The difference comes from their value function which has 3 state variables for traders and 2 state variables for non-traders. After simplification and applying the lotteries as the previous section I can write agents value function as:

\[
\begin{align*}
V_T(z_T, h_T, a) &= \widehat{V}_T(z) + \beta E \left[ W_T\left( \frac{z_T + h_T}{\gamma}, a, \theta \right) \right] \\
&= \widehat{V}_T(z) + \beta E [W_T(0, 0, \theta)] + \frac{\beta E(\theta)z_T}{\gamma} + \frac{\beta E(\theta)h_T}{\gamma} + \beta E(\theta) \quad \text{(B.22)}
\end{align*}
\]
\[ V_N(z_N, h_N) = \tilde{V}_N(z) + \beta E \left[ W_N \left( \frac{z_N + h_N}{\gamma}, \theta \right) \right] = \tilde{V}_N(z_N) + \beta E \left[ W_N(0, \theta) \right] + \frac{\beta E(\theta) z_N}{\gamma} + \frac{\beta E(\theta) h_N}{\gamma} \] (B.23)

Trader’s and non-trader’s choice of bond holding follows the following condition with complementary slackness:

\[
\begin{align*}
\begin{cases}
    a(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\
    a(\theta) \leq \bar{m}_T - z_T(\theta) - h_T(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma}
\end{cases} 
\end{align*}
\] (B.24)

\[
\begin{align*}
\begin{cases}
    h_T(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\
    h_T(\theta) \leq \bar{m}_T - z_T(\theta) - a'(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma}
\end{cases} 
\end{align*}
\] (B.25)

\[
\begin{align*}
\begin{cases}
    h_N(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\
    h_N(\theta) \leq \bar{m}_N - z_N(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma}
\end{cases} 
\end{align*}
\] (B.26)

The labor choices of traders are the same as the labor choices in equations 3.28 and 3.27. Labor choices of non-traders are as B.27:

\[
l_N(m, \theta) = \begin{cases}
p y(\theta) + z_N(\theta) - m_N - T_N & \theta > \frac{\beta E(\theta)}{\gamma} \\
p y(\theta) + z_N(\theta) - m_N - T_N & \theta = \frac{\beta E(\theta)}{\gamma} \\
p y(\theta) + \bar{m}_N - m_N - T_N & \theta < \frac{\beta E(\theta)}{\gamma}
\end{cases} \] (B.27)
B.3.1 Market clearing condition and welfare measure

Similar to the case where all of the agents trade in the asset market the real transfer is:

$$T = \frac{\gamma - 1}{w\gamma} + \frac{sA - A_{-1}}{wM'}$$  \hspace{1cm} (B.28)

The market clearing condition for the bond market and the general good market is the same as 3.29 and B.5.

Similar to the case with only one type of agent, the labor demand can be written as:

$$LD = \int_{\underline{\theta}}^{\overline{\theta}} y(\theta)dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta))b(L_1(z_T(\theta)))L_1(z_T(\theta))dF_T(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta))b(L_1(z_N(\theta)))L_1(z_N(\theta))dF_N(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta))b(L_2(z_T(\theta)))L_2(z_T(\theta))dF_T(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta))b(L_2(z_N(\theta)))L_2(z_N(\theta))dF_N(\theta)$$  \hspace{1cm} (B.29)

Labor supply is the sum of households labor supply:

$$LS = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} l_T(m_T, a, \theta)dF_T(\theta)dG_a(m)dH(a_{-1}) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} l_N(m_N, \theta)dF_N(\theta)dG_a(m)$$

Substituting for labor choices and transfers:
\[ \text{LS} = \int_{\overline{\theta}} \int_{\theta} \left[ p(y(\theta)) + z_T(\theta) + s\gamma a(\theta) - m_T - a_{-1} \right] dF_T(\theta) dG_a(m_T) dH(a_{-1}) \]

\[ + \int_{\overline{\theta}} \int_{\theta} \left[ p(y(\theta)) + z_N(\theta) - m_N \right] dF_N(\theta) dG_a(m_N) \]

\[ - \frac{\gamma - 1}{w\gamma} - \frac{sA}{wM'} + \frac{A_{-1}}{w\gamma M} \]  

\[ (B.30) \]

\[ \text{LS} = \frac{A_{-1}}{w\gamma M} - \frac{sA}{wM'} - \frac{\gamma - 1}{w\gamma} - \int m_T dG_a(m_T) - \int m_N dG_a(m_N) - \int a_{-1} dH(a_{-1}) \]

\[ + \int_{\overline{\theta}} [y(\theta) + z_T(\theta) + s\gamma a(\theta)] dF_T(\theta) + \int_{\theta} [y(\theta) + z_N(\theta)] dF_N(\theta) \]  

\[ (B.31) \]

Labor market clearing condition gives:

\[ \frac{A_{-1}}{w\gamma M} - \frac{sA}{wM'} - \frac{\gamma - 1}{w\gamma} - \int m_T dG_a(m_T) - \int m_N dG_a(m_N) \]

\[ - \int a_{-1} dH(a_{-1}) + \int_{\overline{\theta}} [z_T(\theta) + sa(\theta)] dF_T(\theta) + \int_{\theta} [z_N(\theta)] dF_N(\theta) \]

\[ = \int_{\overline{\theta}} \pi_1(z_T(\theta)) b(L_1(z_T(\theta))) L_1(z_T(\theta)) dF_T(\theta) + \int_{\overline{\theta}} \pi_1(z_N(\theta)) b(L_1(z_N(\theta))) L_1(z_N(\theta)) dF_N(\theta) \]

\[ + \int_{\overline{\theta}} \pi_2(z_T(\theta)) b(L_2(z_T(\theta))) L_2(z_T(\theta)) dF_T(\theta) + \int_{\overline{\theta}} \pi_2(z_N(\theta)) b(L_2(z_N(\theta))) L_2(z_N(\theta)) dF_N(\theta) \]

Similar to the appendix A:
\[
\int m_T dG_a(m_T) = \\
\int_\Theta \pi_1(z_T(\theta))[1 - b(L_1(z_T(\theta)))] \frac{L_1(z_T(\theta))}{\gamma} dF_T(\theta) \\
+ \int_\Theta \pi_2(z_T(\theta))[1 - b(L_2(z_T(\theta)))] \frac{L_2(z_T(\theta))}{\gamma} dF_T(\theta) + \int \frac{a-1}{\gamma} dH(a-1) + \int \frac{h-1_T}{\gamma} dJ_{h-1T}
\]

where \(dG_a(m_T)\) is the traders’ distribution of money holdings at the beginning of the period. Similarly, I can state the same for distribution of money holding among non-traders.

\[
\int m_N dG_a(m_N) = \\
\int_\Theta \pi_1(z_N(\theta))[1 - b(L_1(z_N(\theta)))] \frac{L_1(z_N(\theta))}{\gamma} dF_N(\theta) \\
+ \int_\Theta \pi_2(z_N(\theta))[1 - b(L_2(z_N(\theta)))] \frac{L_2(z_N(\theta))}{\gamma} dF_N(\theta) + \int \frac{h-1_N}{\gamma} dJ_{h-1N}
\]

Plug in the labor market clearing condition:
\[
\frac{1}{w_\gamma} [\gamma - 1 - \lambda + s\lambda] = \\
(2 - s\gamma) \int_\theta^{\theta} a(\theta) dF_T(\theta) + (1 - \frac{1}{\gamma}) \int_\theta^{\theta} \pi_1(z_T(\theta))(1 - b(L_1(z_T(\theta))))L_1(z_T(\theta))dF_T(\theta) \\
+ (1 - \frac{1}{\gamma}) \int_\theta^{\theta} \pi_2(z_T(\theta))(1 - b(L_2(z_T(\theta))))L_2(z_T(\theta))dF_T(\theta) - \int \frac{h_T(\theta)}{\gamma} dF_T(\theta) \\
+ (1 - \frac{1}{\gamma}) \int_\theta^{\theta} \pi_1(z_N(\theta))(1 - b(L_1(z_N(\theta))))L_1(z_N(\theta))dF_N(\theta) - \int \frac{h_N(\theta)}{\gamma} dF_N(\theta) \\
+ (1 - \frac{1}{\gamma}) \int_\theta^{\theta} \pi_2(z_N(\theta))(1 - b(L_2(z_N(\theta))))L_2(z_N(\theta))dF_N(\theta)
\]

It can be shown as in appendix for the benchmark model that the measure of welfare is:
\[ \varpi = \int [U(y(\theta)) - \theta y(\theta) - u(q(z_T(\theta))) - \theta z_T(\theta) - s\gamma a(\theta)]dF_T(\theta) \\
+ \left[ \int_{\theta}^{\theta_T} \pi_1(z_T(\theta))[1 - b(L_1(z_T(\theta)))]L_1(z_T(\theta))dF_T(\theta) \right] \int \theta dF_T(\theta) \\
+ \left[ \int_{\theta}^{\theta_T} \pi_2(z_T(\theta))[1 - b(L_2(z_T(\theta)))]L_2(z_T(\theta))dF_T(\theta) \right] \int \theta dF_T(\theta) \\
+ (1 + \frac{1}{\gamma}) \left[ \int a_{-1}dH_{a_{-1}} \right] \int \theta dF_T(\theta) + \frac{1}{\gamma} \left[ \int h_{-1T}dJ_{h_{-1T}} \right] \int \theta dF_T(\theta) \\
\quad - \frac{1}{w\gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF_T(\theta) \\
+ \int [U(y(\theta)) - \theta y(\theta) + u(q(z_N(\theta))) - \theta z_N(\theta)]dF_N(\theta) \\
+ \left[ \int_{\theta}^{\theta_N} \pi_1(z_N(\theta))[1 - b(L_1(z_N(\theta)))]L_1(z_N(\theta))dF_N(\theta) \right] \int \theta dF_N(\theta) \\
+ \left[ \int_{\theta}^{\theta_N} \pi_2(z_N(\theta))[1 - b(L_2(z_N(\theta)))]L_2(z_N(\theta))dF_N(\theta) \right] \int \theta dF_N(\theta) \\
+ \frac{1}{\gamma} \left[ \int h_{-1N}dJ_{h_{-1N}} \right] \int \theta dF_N(\theta) + \frac{1}{w\gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF_N(\theta) \\
\]

As I have shown above, the problem of traders and non-traders in the bond market are very similar to the case with no exogenous segmentation in the asset market. The same logic from the case with endogenous asset market segmentation applies and asset market traders and non-traders solve optimization problems similar to the problem in the previous sections. The households’ decisions are only linked through the market clearing conditions and prices. We have a partial block recursive equilibrium in which the distributions in the economy affects households’ decision through
prices. Households do not take into account the distribution of asset holdings among traders and non-traders. The main results in the previous sections are robust to adding exogenously segmented asset market.
Appendix C

Omitted Proofs from Chapter 4

C.1 Proof for proposition 6

The constraint problem is the following

\[
\mathcal{U}_2 = \max_{\theta, q, x} \mu(\theta)[u(q) + p_2(R - x)] \\
st. \quad \eta(\theta)(-c(q) + p_2 x) \geq k \\
\mu(\theta)(u(q) + p_1(R - x)) \leq \mathcal{U}_1
\]

(C.1)

Let us call the Lagrangian multipliers of the above constraints \( \lambda \) and \( \nu \). The solution for these multipliers are

\[
\lambda = \theta \frac{p_2}{p_1} \frac{1}{p_2} - \frac{c'(q_2)}{u'(q_2)}
\]

(C.2)
\[
\nu = \mu(\theta) \frac{p_2(1 - \frac{c'(q_2)}{u'(q_2)})}{p_1(\frac{p_2}{p_1} - \frac{c'(q_2)}{u'(q_2)})}
\]  
(C.3)

We are interested in cases where multipliers are positive. Therefore, from C.2

\[
\frac{p_2}{p_1} > \frac{c'(q_2)}{u'(q_2)}
\]

Moreover, from C.3

\[
\frac{c'(q_2)}{u'(q_2)} < 1
\]

and using curvature properties of \(u()\) and \(c()\) we can see that

\[
q_2 < q_2^*
\]

### C.2 The model with multiple types

In the first-best model the participation constraint in the problem 4.10 binds:

\[
x_i = \frac{k}{p_i \eta(\theta)} + \frac{c(q)}{p_i}, \quad i = 1, 2, ..., n
\]  
(C.4)

Substitute in the objective function, and the problem becomes:

\[
U_{i=1,2,...,n} = \max_{\theta, q} \mu(\theta)u(q) + \mu(\theta)p_1R - \theta k - \mu(\theta)c(q)
\]  
(C.5)
Similar to the case with two assets, the first order condition is: The first order condition for \( q_i \) is:

\[
    u'(q_i^*) = c'(q_i^*) \tag{C.6}
\]

\[
    q_i^* = q_j^* = q^*, \quad \forall i, j
\]

Again this result shows that without information frictions, search frictions do not distort the optimal amount of loans. Similar to the case for two assets search frictions may distort the repayment levels. The amount of loan is also independent of borrowers’ types. The first order condition for \( \theta_i \) gives the first-best market tightness for each submarket:

\[
    \mu'(\theta_i^*) = \frac{k}{u(q^*) - c(q^*) + p_iR} \tag{C.7}
\]

Using the concavity property of the matching function, we can see that \( \theta_n^* < ... < \theta_2^* < \theta_1^* \). Additionally, equation C.4 shows that \( x_n^* < ... < x_2^* < x_1^* \). These results are similar to the case with two assets.

In the case with private information, agents with the lowest type (type 1) do not face incentive compatibility constraints. Therefore, their problem is similar to the first best with complete information. We can find the market tightness that they face, their loan amount, and their repayment level by solving the following first-best problem:

\[
    \bar{U}_1 = \max_{\theta,q,x} \mu(\theta)[u(q) + p_1(R - x)]
\]

\[
    st. \quad \eta(\theta)(-c(q) + p_1x) \geq k \tag{C.8}
\]
In a type $i > 1$ optimization problem borrowers face an incentive compatibility constraint:

$$\mathcal{U}_i = \max_{\theta, q, x} \mu(\theta)[u(q) + p_i(R - x)]$$

subject to:

$$\eta(\theta)(-c(q) + p_i x) \geq k$$

$$\mu(\theta)(u(q) + p_j(R - x)) \leq \mathcal{U}_j, \quad j < i$$

(S.9)

Solving problem C.9 in its general form is not analytically feasible for all $n$ type agents. One can use specific functional forms and computational methods to solve the above problems.