EVALUATION OF TURBULENCE MODELS FOR UNSTEADY SEPARATION

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Abstract

Unsteady separation occurs in many physical flows due to time-varying adverse pressure gradients. This phenomenon may result in increased drag, decreased lift, and loss of efficiency or failure in flow devices. It is, therefore, important to predict and analyze unsteady separation in the design of flow devices. Turbulence models in combination with the Reynolds-Averaged Navier-Stokes equations are commonly used in the industrial design process due to their low computational cost; however, their performance in predicting steady separations is unsatisfactory, and very few studies have investigated unsteady separation to date.

This study uses high fidelity Large-Eddy Simulation (LES) of a flat plate boundary layer at $Re\delta^* = 1000$ with unsteady separation to evaluate the accuracy of the $K - \varepsilon$, $K - \omega$, and Spalart-Allmaras turbulence models in predicting flows of this type. Alternating favourable and adverse pressure gradients induce periodic acceleration and separation of the boundary layer (Case A, hereafter). Modelling errors are isolated by using an identical numerical scheme and consistent boundary conditions to the LES with a grid that has been validated in the LES study of this problem. The performance of the models is evaluated for three representative reduced frequencies $k = 10$, 1 and 0.2. An additional case is considered where separation is present throughout the entire oscillation cycle (Case B hereafter).
All three turbulence models capture the general features of this complex unsteady flow correctly, with only small discrepancies from the LES. The largest errors occur closest to the wall, particularly in predicting the downstream shedding of the separation bubble. The models are most accurate during phases of the flow where the pressure gradients are mild, however, the maximum APG-FPG phases of the flow are predicted successfully. The cyclical alternation of the pressure gradients from FPG-APG to APG-FPG with intermediate phases of ZPG are shown to contribute to the success of the turbulence models. The performance of the turbulence models is significantly better in Case A than in Case B with integrated errors (IE) of 6.8 % and 16.7 % respectively.
Co-Authorship

Candidate was the primary author, planned and performed the numerical simulations, interpreted the results, wrote the first draft and made the figures. U. Piomelli and F. Ambrogi contributed to the idea, the data interpretation and discussion. All authors commented on the manuscript and provided editorial assistance.
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Statement Of Originality

The work presented in this thesis is, to the best of my knowledge and belief, original and my own work, except as acknowledged in the text. This material has not been submitted, either in whole or in part, for a degree at this or any other university.
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Glossary of Symbols

$Re_\tau$ Reynolds number based on friction velocity.

$x_i$ Cartesian coordinates (corresponding to $x$, $y$ and $z$).

$\delta_{ij}$ Kronecker’s delta.

$\delta$ Discrete derivative operator.

$\partial$ Partial derivative operator.

$\delta_o$ Inlet displacement thickness.

$\varepsilon$ Turbulence dissipation.

$f$ Frequency of imposed unsteadiness.

$L_{pg}$ Length of pressure gradient.

$\omega$ Turbulence frequency.

$p$ Hydrodynamic pressure.

$P_k$ Turbulent energy production.
\( q \) Substep number.

\( k \) Reduced frequency.

\( Re \) Reynolds number.

\( \rho \) Density.

\( C_f \) Skin-friction coefficient.

\( S_{ij} \) Strain-rate tensor.

\( \tau_w \) Wall-shear stress.

\( K \) Turbulent kinetic energy.

\( U \) Phase-averaged streamwise velocity.

\( U_\infty \) Freestream streamwise velocity.

\( V \) Phase-averaged wall-normal velocity.

\( u_i \) Velocity component in the \( i \)-th coordinate (corresponding to \( u,v \)).

\( V_\infty \) Freestream wall-normal velocity.

\( \tilde{\nu} \) Modified eddy viscosity.

\( \nu \) Kinematic viscosity.

\( \nu_t \) Eddy viscosity.
Glossary of operators, superscripts and subscripts

⟨•⟩ Phase average.

• Time average.

○ Filtering operator.

[●] Interpolation Wake field.
Glossary of Abbreviations

**APG** Adverse Pressure Gradient.

**D** Detachment.

**DNS** Direct Numerical Simulation.

**FPG** Favourable Pressure Gradient.

**ID** Incipient Detachment.

**IE** Integrated Error.

**ITD** *Intermittent Transitory Detachment*.

**LES** Large-Eddy Simulation.

**LHS** Left Hand Side.

**MPI** Message-Passing Interface.

**NS** Navier-Stokes.

**RANS** Reynolds-Averaged Navier-Stokes.
**RHS**  Right Hand Side.

**RMS**  Root-mean square.

**SA**  Spalart Allmaras turbulence model.

**SST**  Shear Stress Transport.

**TBL**  Turbulent Boundary Layer.

**TKE**  Turbulent Kinetic Energy.

**URANS**  Unsteady Reynolds-Averaged Navier-Stokes.

**ZPG**  Zero Pressure Gradient.
Chapter 1

Introduction

1.1 Motivation

Separation is a phenomenon that frequently occurs in engineering and the natural sciences. It significantly affects the flow dynamics, such as the forces on immersed objects or the effective area (and the pressure distribution) in diverging ducts. Separation typically causes a decrease in the efficiency of devices such as turbines, diffusers and lifting bodies due to increased drag and decreased lift. The phenomenon may induce noise, unwanted vibrations, and significant stress on a turbine blade [4]. Because of its ubiquity, separation has been the subject of a considerable amount of work; [1, 4–9].

Most of the studies in the past have concentrated on steady separation. In nature, however, separation is often unsteady, for instance, in helicopter and turbine blades, pitching airfoils and in the motion of aquatic animals. An important engineering problem in which unsteady separation plays a role is dynamic stall; dynamic stall occurs over a pitching airfoil, where alternating adverse and favourable pressure gradients are induced due to the airfoils changing the angle of attack. The stall
is characterised by vortex shedding from the leading edge, which causes a lift change, a dramatic drag increase, and a downward pitching moment. Dynamic stall may occur in turbomachinery, rotorcraft and wind turbines. For example, in the case of a wind turbine, dynamic stall occurs when the blades’ steady-state stall angle of attack is exceeded [10] due to in-flow turbulence, misalignment or external gusts. Dynamic stall negatively influences the turbine’s performance and decreases the device’s life.

Another example of unsteady separation can be observed in the locomotion of aquatic animals. As the fish propel themselves, the rear part of their body oscillates from side to side (or, in some cases, up and down). The side of the body with convex curvature experiences an adverse pressure gradient (APG), while the concave side experiences a favourable one (FPG). As the tail moves in the other direction, the FPG becomes APG and vice-versa. Given the metabolic benefits of a lowered cost of locomotion for a given swimming speed, it seems reasonable to assume that active flow sensing by fish may contribute to improvements in the efficiency of locomotion. It has been found that flow sensing is employed by rainbow trout (*Oncorhynchus mykiss*) to modify their rhythmic motor pattern profile. This hydrodynamic mechanism enables rainbow trout to reduce their axial muscle activity, which implies an adaptive response to reduce the cost of locomotion [11].

While most of the early work on separation consisted of experimental studies, high-fidelity numerical simulations have become more widespread over recent years. Direct Numerical Simulations (DNS) and Large-Eddy Simulations (LES) have been applied to steady and unsteady separated flows. DNS (which resolves all temporal and spatial scales) and LES (in which only the integral scales of turbulence are resolved) have been used. However, for both methods, very fine grids and significant computational
resources are needed and unsuitable for some industrial applications requiring rapid turnaround.

In the industrial environment, the solution of the Reynolds-Averaged Navier-Stokes (RANS) equations are prevalent. In this case, turbulence models are used to parameterize the effect of all the turbulent scales of motion. Turbulence models, however, are not always accurate in separated flows. One of the standard test-cases for turbulence models is the backward-facing step, in which the sharp corner causes separation; most models have difficulties predicting the reattachment length (see Wilcox [12]). However, a consistent evaluation of the model’s accuracy is difficult because comparison with experimental data does not permit the isolation of modelling errors from numerical and experimental ones.

The present study considers the unsteady separation on a flat-plate boundary layer, generated by an unsteady pressure gradient imposed through blowing and suction at the upper boundary of the domain. We aim to use a high-fidelity numerical database [13] to evaluate the accuracy of turbulence models for the Unsteady Reynolds-Averaged Navier-Stokes (URANS) equations consistently.

The rest of the chapter will review previous steady and unsteady separation studies. Then, we will discuss the results of recent numerical simulations, focusing on the performance of URANS for unsteady separations. The objectives of this study and the plan for the remainder of this thesis will conclude the Chapter.
1.2 Literature Review

1.2.1 Separation dynamics

Separation is a complex phenomenon which refers to the departure or breakdown of boundary-layer flow. In this process, an abrupt thickening of the rotational flow in the near-wall region occurs. The wall-normal velocity component increases and the near-wall fluid interacts with the free stream flow [1]. Separation can be caused by sharp geometrical changes or by strong APGs. This work focuses on pressure-induced separation.

Separation can be divided into three distinct regions: (1) intermittent detachment, where instantaneous backflow is present 1% of the time; (2) transitory detachment, where instantaneous backflow is present 50% of the time, and (3) detachment, which occurs on a smooth surface when the time-averaged wall shear stress is zero. This behaviour is sketched in Figure 1.1.

As a turbulent boundary layer (TBL) is subjected to an adverse pressure gradient, the near-wall flow decelerates until some backflow occurs at the incipient detachment point Fig. 1.1. Here, the motion of small flow packets is reversed; this fluid moves upstream for a distance, then is carried back downstream. These reversed flows occur where kinetic energy is low, and are caused by forces arising from large-scale structures and APG. Large eddies carry momentum from the outer region toward the wall, supplying the downstream flow. These eddies expand rapidly in all directions and then combine, decreasing the average passage-frequency as detachment approaches. Pressure-gradient relief begins near the intermittent transitory detachment as the detaching shear layer grows at a rate proportional to $\delta^2$ [1]. These large-scale structures supply turbulence energy to the near-wall detaching flow. Velocity fluctuations in the
backflow region are greater than or comparable to the mean velocity. Intermittent backflow occurs at the same distance from the wall as the maximum shearing-stress location.

The structure of separated turbulent flows is drastically different than that of attached flows; for separated flows, the largest turbulent stresses occur in the centre of the separated shear layer. These stresses are mostly caused by large-scale structures as the pressure fluctuations they produce influence the low-velocity backflow zone. In separated flows, there is significant interaction between the pressure and velocity fluctuations; this is because the backflow is re-entrained into the outer-region flow.

1.2.2 Unsteady separation

Unsteadiness —whether imposed or self-induced— influences detached flows. Although turbulent flows are inherently unsteady by nature, the term unsteady here refers
to organised time-dependent motion. Large-scale motions produced in unsteady separated flows do not contribute significantly to turbulent shear stresses but change the mean flow and produce low-frequency pressure fluctuations, making these flows difficult to model.

An important parameter to characterize unsteadiness is the reduced frequency \( k \) [14]

\[
k = \frac{\pi f L_{PG}}{U},
\]

where \( f = 1/T \), \( L_{PG} \) is the characteristic length of the pressure gradient (the chord of an airfoil, for instance), and \( U \) is the characteristic velocity. The reduced frequency represents the ratio between the convective and unsteady timescales. [15].

The response of the flow to the unsteadiness is greatly dependent on the reduced frequency, \( k \). Leishman [16] developed threshold values to describe unsteadiness in his work on helicopter aerodynamics. A reduced frequency \( k = 0 \) corresponds to a nominally steady flow. The flow is quasi-steady for \( 0 < k < 0.05 \); acceleration effects are insignificant, and the ensemble-averaged velocity profiles do not differ from a corresponding steady case. As the reduced frequency grows, acceleration effects begin to dominate the flow characteristics. Sources of ambiguity for the threshold values include the: length scale \( L \), which is highly dependent on the specific problem and arbitrary in the case of a flat plate TBL. The velocity scale \( U \) can also be difficult to define; in rotor dynamics, for instance, it changes continuously. Specific threshold values of reduced frequencies are, therefore, not universal.

In his review, Simpson [1] summarised general features of unsteady pressure-induced separations from various experimental and numerical studies: Large amplitude- and phase-variations are created in the flow for a flat plate boundary layer undergoing
unsteady APG-induced separation. As the free stream velocity increases during the cycle, $\gamma_{pu}$ (the fraction of time the flow velocity moves downstream) increases as backflow fluid is washed downstream. As the freestream velocity nears its maximum value in the cycle, increasing adverse pressure gradients at downstream locations cause progressively larger backflows near the wall, and $\gamma_{pu}$ remains high upstream. Near the wall, the ensemble-averaged velocity is significantly faster than the backflow region’s freestream velocity. The phase angle of the periodic backflow is nearly independent of wall distance $y$ near the wall. The turbulence structure progressively lags behind the ensemble-averaged flow oscillation.

Unsteady effects can produce dynamic hysteresis - when a physical quantity assumes two values at corresponding phases of a periodic cycle [17].

McCroskey [18] has studied differences between steady and unsteady separated flows. McCroskey reviewed unsteady flows over pitching airfoils and concluded that dynamic stall occurs on any airfoil or lifting device subject to sufficiently fast time-dependent motions such as pitching or plunging, which create an angle of attack larger than the static stall value. Flow characteristics for these unsteady motions differ drastically from corresponding steady-flow conditions. Additionally, they observed that an angle-of-attack oscillating about the static stall-angle causes significant hysteresis effects [18].

Lissaman [19] also studied the effect of hysteresis on aerodynamic performance. They observed that the size and shape of the separation region formed over an airfoil is Reynolds-number dependent and noted that the separation region is shorter for low Reynolds-numbers than for high Reynolds-number cases. They categorised the various separation regions as ‘long’ or ‘short’ and found that the characteristics of
the generated hysteresis effects depend on the separation bubbles’ size. They also observed that the shape and magnitude of the hysteresis loops vary nonlinearly with the amplitude and reduced frequency of the oscillation and the mean angle-of-attack of the airfoil. This behaviour was also observed in the experiments of Selig et al. [20].

The wind tunnel experiments on a pitching airfoil performed by Williams et al. [21] revealed that the lift coefficient exhibits dynamic hysteresis when transient separation occurs. The amount of hysteresis observed depended on the pitching frequency and pattern, and on the flow separation. Further, they observed that the hysteresis loop changed shape as frequency increased. However, it was present at both high and low pitching rates, supporting the conclusion that the occurrence of dynamic stall is not necessary for dynamic hysteresis to be present.

Schatzman & Thomas [15] performed experiments investigating a TBL with an unsteady APG at reduced frequency $k = 0.12$. Their results showed that when an APG is strong enough to cause an inflectional velocity profile, the flow becomes dominated by an embedded shear layer. They found that the shear layer is closely related to the inviscid instability of the outer inflection point. Upon investigating the scaling parameters for the embedded shear-layer, they noted the similarity of both the mean and phase-averaged velocity-profiles. This informed their proposition that the embedded shear-layer might be a characteristic of all turbulent boundary-layers with APG.

Particularly pertinent to this work is the study by Ambrogi et al. [13]. They performed high-fidelity LES of a spatially developing turbulent boundary-layer subject to a space- and time-dependent pressure-gradient; unsteadiness was prescribed by an imposed oscillating suction–blowing velocity-profile at the top of the computational
domain. The flow was found to separate and reattach to the wall periodically due to the alternating adverse and favourable pressure gradients. They investigated a range of reduced frequencies at a Reynolds number $Re_\ast = 1,000$ based on the boundary-layer displacement thickness $\delta_\ast$ at the inflow. They have identified that the transient flow separation process is influenced significantly by the reduced frequency $k$. These results are discussed in detail in Chapter 3.

### 1.2.3 Numerical Studies

Unsteady separated flows are complex and challenging to predict due to unsteady, non-linear, and turbulent flow features discussed in the previous section. The accuracy of the Reynolds Averaged Navier-Stokes (RANS) equations performance in predicting these flows depends on the closure models used. This section will discuss previous evaluations of the performance of various turbulence models in predicting unsteady pressure-induced separated flows.

In the past, many studies have evaluated the performance of RANS calculations in predicting steady separations as a standard test for evaluating models. (For example, [22–34]). Findings indicate that turbulence models typically have difficulty predicting complex flow-structures in regimes substantially different from the thin shear-layers used to calibrate the underlying turbulence models [24]. For separated flows, the general characteristics of the flow are predicted qualitatively. However, major discrepancies exist in the separation region, particularly near the wall [24]. Both separation and reattachment points are predicted incorrectly, and the various models do not give consistent trends or results.

Fewer studies have evaluated the performance of turbulence models in unsteady
separations. Garnier et al. [35] studied a periodically excited flow over a rounded backwards-facing step at various excitation frequencies and an uncontrolled case. They performed calculations using LES and four different URANS models. They demonstrated that the URANS models identified an optimal frequency for flow-control close to the value predicted by the LES; however, all the models significantly underestimated the amplitude of the separation bubble, and the models could not accurately reproduce uncontrolled flow.

Ekaterinaris and Menter [36] evaluated the accuracy of one- and two-equation turbulence models in unsteady separated flows over oscillating airfoils in light- and deep-stall regimes. The light-stall case had an angle of attack that varied as \( \alpha = 11^\circ\text{deg} + 4.2^\circ\sin(t) \) with a reduced frequency \( k = 0.1 \). This case was characterised by a moderate trailing-edge separation which developed at the cycle’s peak. The flow was separated for a large portion of the oscillation’s downstroke, and a recirculatory region of about half a chord length occurs. For the light-stall case, they found that Spalart-Allmaras (SA) model [37], \( K-\varepsilon \) and \( K-\omega \) SST model underpredict the extreme values of the unsteady loads and the magnitude and length of the separation region. The deep stall case with an angle-of-attack \( \alpha = 15^\circ + 4.2^\circ\sin(t) \) is characterized by massive flow separation. The flow remains separated for a large part of the downstroke, and significant hysteresis effects occur. The lift hysteresis was predicted reasonably well by all of the models however, the Spalart-Allmaras (SA) model [37], \( K - \varepsilon \) [38] and \( K - \omega \) SST model [39] underpredicts the magnitude of separation resulting in smaller extreme values of drag and pitching moment. The turbulence models also predict a more rapid reattachment than the experimental results, and the loads computed with the models deviated significantly from the experimentally measured values.

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Schobeiri [40] performed a study investigating the cause of numerical and modelling errors in predicting the efficiency and performance of high-pressure turbines. They conducted simulations using commercial software with a known shear stress transport (SST) turbulence model and an updated transition model. They observed quantitative and qualitative discrepancies between the RANS results and experimental results. They calibrated their numerical results to experimental results to propose modifications for predicting this flow type.

Nürnberg and Greza [41] investigated unsteady transitional flows in turbomachinery components using RANS, with the SA model coupled with a transition correlation. They simulated boundary layer flow with a separation bubble on a flat plate under steady and periodically unsteady mean flow conditions. They observed that the separation point predicted by RANS matched the experiment reasonably well; however, the separation bubble is slightly smaller than the experimental results. Further, the downstream reattachment of the flow is predicted significantly further downstream than the experimental measurement.

The results of Park and You [3] are particularly relevant to the present work. They investigated a turbulent boundary layer under unsteady APGs to evaluate the performance of the $K-\omega$ and SA turbulence models in unsteady separated flows. An APG-FPG was generated by applying a profile of freestream wall-normal velocity $V_\infty$ at the top of the domain, whose amplitude varies in time to obtain dynamic separation and reattachment in the flow. Their case differs from the present work in that their pressure gradient distribution remains Adverse-to-Favorable, rather than alternating between APG-FPG, zero pressure gradient and FPG-APG. As a result, the flow is separated throughout all phases. They observed that URANS calculations could
predict the formation of the separation bubble and the phase response of the shear layer qualitatively well. However, the phase-response of the skin friction coefficient (and, therefore, the separation and reattachment) is inaccurate. URANS predicts an earlier separation point and a longer recirculation bubble than the DNS. They attributed the near-wall errors in the RANS predictions to the anisotropy of the Reynolds Stress. The results of this work were used to validate the current study and are further discussed in 3.3.

1.3 Objectives and plan of the thesis

A drawback of studies that attempt to evaluate turbulence models in non-equilibrium flows, such as unsteady separations, are the numerous sources of error present. In many cases, it is difficult or impossible to separate the modelling errors from numerical ones due to the grid resolution or numerical method, differences in the boundary conditions, or experimental errors.

This work attempts to overcome this issue and consistently evaluate the three most commonly used one- and two-equation turbulence models by comparing the model predictions to a high-fidelity LES [13] while using the same numerics as the LES, and consistent boundary conditions. Additionally, the grid resolution used in the URANS was close to an LES mesh that yielded grid-converged results. By matching, as far as possible, the LES configuration and parameters, the modelling errors remain the only source of error between the URANS predictions and the LES reference data.

The thesis is organized as follows: first, in Chapter 2, we introduce the problem formulation, including the governing equations, boundary conditions, turbulence models, details on the numerical simulation and the temporal variations of the various cases.
studied. In Chapter 3, the results of the RANS calculations are presented and compared to the high-fidelity simulations. Conclusions and recommendations for future work will close the thesis in Chapter 4.
Chapter 2

Methodology

2.1 Governing equations

Two-dimensional incompressible flow is governed by the conservation of mass (continuity) and momentum (Navier-Stokes) equations:

\[
\frac{\partial u_i}{\partial x_i} = 0 \quad (2.1.1)
\]

\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (2.1.2)
\]

where the Cartesian coordinates in the streamwise and wall-normal directions are denoted by \( i = 1, 2 \) (or \( x, y \)). The velocity components are denoted by \( u_i \) (or \( u, v \)). \( p \) is the hydrodynamic pressure, \( \rho \) is the (constant) density and \( \nu \) the kinematic viscosity.
2.2 Reynolds-Averaged Navier-Stokes equations

Due to the wide range of length- and time-scales occurring in turbulent flows, solving the instantaneous Navier-Stokes (NS) equations for a given flow field is computationally expensive. Rather than solving for the instantaneous quantities, a flow variable $f$ is usually decomposed into mean and fluctuating parts.

The time average is defined as

$$\overline{f} = \frac{1}{\tau} \int_{0}^{\tau} f(x, t) dt$$

(2.2.1)

where $\tau$ is the integration time. Because of the periodicity (in time) of the problem considered here, we also use phase averaging. Phase-averaged quantities (denoted by angle brackets or capital letters) are defined as

$$F = \langle f \rangle = \frac{1}{N} \sum_{n=0}^{N} f(t + nT)$$

(2.2.2)

with $N \to \infty$, and $T$ is the period. A prime is used to indicate the fluctuation around the phase-averaged value: $f' = f - F$.

2.2.1 Unsteady RANS

Taking the phase average of (2.1.1) and (2.1.2) yields the Unsteady Reynolds-Averaged Navier Stokes (URANS) equations Eq. (2.2.4) and Eq. (2.2.3), which are
solved in this study. They take the form

\[
\frac{\partial U_i}{\partial x_i} = 0 \quad (2.2.3)
\]

\[
\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_j} (U_i U_j) = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle u'_i u'_j \rangle \quad (2.2.4)
\]

The phase-averaged Reynolds stresses \( \langle u'_i u'_j \rangle \) are parametrized using an eddy-viscosity model:

\[
\langle u'_i u'_j \rangle = -2\nu_t S_{ij} - \frac{2}{3} \mathcal{K} \delta_{ij} \quad (2.2.5)
\]

Where \( \nu_t \) is the eddy viscosity, \( \mathcal{K} = \langle u'_i u'_i \rangle / 2 \) is the turbulent kinetic energy, \( \delta_{ij} \) is Kronecker’s delta, and \( S_{ij} \) is the phase-averaged rate-of-strain tensor:

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \quad (2.2.6)
\]

2.3 Turbulence modelling

The eddy-viscosity is unclosed and must be determined using a model. In the present work, one and two-equation turbulence models are evaluated. We tested the Spalart-Allmaras [37], the two-layer \( \mathcal{K} - \varepsilon \) [42] and \( \mathcal{K} - \omega \) SST model [39]. Detailed descriptions of the implementation of each model will follow.

2.3.1 Spalart-Allmaras Model

The Spalart-Allmaras model [37] is a one-equation eddy-viscosity model developed for aerodynamic applications using a combination of empiricism, dimensional analysis, selective dependence on molecular viscosity and Galilean invariance. In this model, the eddy viscosity is determined by a transport equation.
We solve the transport equation for the modified eddy viscosity $\tilde{\nu}$:

$$\frac{\partial \tilde{\nu}}{\partial t} + U_j \frac{\partial \tilde{\nu}}{\partial x_k} = C_{b1} S \tilde{\nu} - c_{w1} f_w \left( \frac{\tilde{\nu}}{d} \right)^2 + \frac{1}{\sigma} \frac{\partial}{\partial x_k} \left[ (\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_k} \right] + \frac{c_{b2}}{\sigma} + \frac{\partial \tilde{\nu}}{\partial x_k} \frac{\partial \tilde{\nu}}{\partial x_k}. \quad (2.2.7)$$

Here, $\nu_t = \tilde{\nu} f_{\nu1}$ and the closure coefficients and auxiliary functions are:

$$C_{b1} = 0.1355; \quad C_{b2} = 0.622; \quad C_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma}$$

$$C_{\nu1} = 7.1; \quad \sigma = \frac{2}{3}; \quad f_{\nu1} = \frac{\chi^3}{\chi^3 + C_{\nu1}^3}$$

$$f_{\nu2} = 1 - \frac{\chi}{1 + \chi f_{\nu1}}; \quad c_{w2} = 0.3; \quad c_{w3} = 2; \quad \kappa = 0.41$$

$$f_w = g \left[ \frac{1 + c_{w3}}{g^6 + c_{w3}^6} \right]^{\frac{1}{6}}; \quad \chi = \frac{\tilde{\nu}}{\nu}; \quad g = r + c_{w2}(r^6 - r); \quad r = \frac{\tilde{\nu}}{S \kappa^2 d^2}$$

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right); \quad S = \sqrt{2 \Omega_{ij} \Omega_{ij}}; \quad \tilde{S} = S + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{\nu2}$$

where $d$ is the distance from the wall for the nearest solid boundary; this study does not include a transition correction term.

The model has proved quite successful for aerodynamic flows. However, it is subject to limitations such as the incapability of accounting for $\nu_t$ decay in isotropic turbulence [37]. Furthermore, the TKE is not known, and only the Reynolds shear stresses can be recovered.
2.3. TURBULENCE MODELLING

2.3.2 $K - \varepsilon$ Model

In the $K - \varepsilon$ model $\nu_t$ is given by:

$$\nu_t = C_\mu \left( \frac{K^2}{\varepsilon} \right) \quad (2.2.8)$$

The model equations are

$$\frac{\partial K}{\partial t} + \frac{\partial (U_k K)}{\partial x_k} = \frac{\partial}{\partial x_k} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial K}{\partial x_k} \right] - P_K - \varepsilon \quad (2.2.9)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (U_k \varepsilon)}{\partial x_k} = \frac{\partial}{\partial x_k} \left[ \left( \nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_k} \right] + \frac{\varepsilon}{K} \left( C_{\varepsilon_1} f_1 P_K - C_{\varepsilon_2} f_2 \varepsilon \right) \quad (2.2.10)$$

Where $P_K$ is the turbulent energy production:

$$P_K = -\langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} \quad (2.2.11)$$

and Constants $C_\mu$, $C_{\varepsilon_1}$, $C_{\varepsilon_2}$, $\sigma_K$ and $\sigma_\varepsilon$ take the values 0.09, 1.44, 1.92, 1.0 and 1.3.

To integrate the model equations to the wall, a special treatment is necessary. We use the two-layer model [42]. In this formulation, the standard $K - \varepsilon$ model (described by the previous equations) is combined with a simple one-equation model to resolve the flow near the wall. The domain is divided into two regions sketched in Fig. 2.1 where the one equation model is employed in the sublayer, buffer layer and part of the fully turbulent layer (Region I in Fig. 2.1). The standard $K - \varepsilon$ model is employed away from the wall (Region II).

The one equation model requires only the solution of the TKE transport equations
2.3. TURBULENCE MODELLING

Figure 2.1: Diagram of regions where the one equation model is used for the two-layer $\mathcal{K} - \varepsilon$ model

The rate of energy dissipation is given by:

$$\varepsilon = \left( \frac{\mathcal{K}^{3/2}}{l_\varepsilon} \right)$$  \hspace{1cm} (2.2.12)

and the eddy viscosity is obtained from:

$$\nu_t = l_\mu C_\mu \sqrt{\mathcal{K}}.$$  \hspace{1cm} (2.2.13)

The length scales $l_\mu$ and $l_\varepsilon$ account for the damping effects in the near-wall region and depend on the local turbulence Reynolds number $R_y = R\sqrt{\mathcal{K}y}$

$$l_\mu = C_\mu y \left[1 - \exp(-R_y/A_\mu)\right]; \quad l_\varepsilon = C_\varepsilon y \left[1 - \exp(-R_y/A_\varepsilon)\right];$$  \hspace{1cm} (2.2.14)
where

\[ C_l = KC_{\mu}^{-3/4}, \quad A_\varepsilon = 2C_l \quad \text{and} \quad A_\mu = 70. \]  

(2.2.15)

### 2.3.3 \( K - \omega \) Model

The second two-equation eddy-viscosity model used in the present work is the \( K - \omega \) SST model. This model differs from the \( K - \varepsilon \) model in that the eddy viscosity \( \nu_t \) is determined from \( K \) and the turbulence frequency \( \omega \) rather than \( \varepsilon \). The turbulent transport equations are:

\[
\frac{\partial K}{\partial t} + \sum_{k} u_k \frac{\partial K}{\partial x_k} = \frac{\partial}{\partial x_k} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial K}{\partial x_k} \right] + P_K - \beta^* K \omega
\]  

(2.2.16)

\[
\frac{\partial \omega}{\partial t} + \sum_{k} u_k \omega \frac{\partial \omega}{\partial x_k} = \frac{\partial}{\partial x_k} \left[ \left( \nu + \frac{\nu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_k} \right] + \alpha S^2
\]

\[
- \beta \omega^2 + 2(1 - F_1) \sigma_\omega \frac{1}{\omega} \frac{\partial K}{\partial x_k} \frac{\partial w}{\partial x_k}
\]

(2.2.17)

\[
\nu_t = \frac{a_1 k}{\max(a_1 w, SF_2)}
\]

(2.2.18)

where the closure coefficients and auxiliary relations are:

\[ F_2 = \tanh \left( \max \left( \frac{2\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right) \right)^2 \]

\[ P_K = \min \left( \tau_{ij} \frac{\partial U_i}{\partial x_j}, 10\beta^* k \omega \right) \]

\[ F_1 = \tanh \left\{ \min \left[ \max \left( \frac{2\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{d^2 \omega} \right), \frac{4\sigma_\omega k}{CD_K \omega d^2} \right]^4 \right\} \]

\[ CD_{k\omega} = \max \left( 2\rho \sigma_\omega^2 \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-20} \right) \]

Each of the constants is a blend of an inner (1) and outer (2) constant, blended
via:

\[ \phi = \phi_1 F_1 + \phi_2 (1 - F_1) \]  

(2.2.19)

where \( \phi_{1,2} \) is the value of any arbitrary constant in the inner and outer layer, respectively. The constants take the values:

\[ \alpha_1 = \frac{5}{9}; \quad \alpha_2 = 0.44; \quad \beta_1 = \frac{3}{40}; \quad \beta_2 = 0.0828 \]  

(2.2.20)

\[ \beta^* = \frac{9}{100}; \quad \sigma_{k1} = 0.85; \quad \sigma_{k2} = 1; \quad \sigma_{\omega 1} = 0.5; \quad \sigma_{\omega 2} = 0.856 \]  

(2.2.21)

2.4 Problem Formulation

This work aims to use high-fidelity simulation results to validate RANS models for predicting unsteady separations. To do so, we replicate the problem studied by Ambrogi \textit{et al.} [13] in two-dimensions. We consider a spatially developing turbulent boundary layer. The spanwise and streamwise domain lengths of the URANS simulations are 64 \( \delta_o^* \) and 600 \( \delta_o^* \), respectively to match the LES, where \( \delta_o^* \) is the displacement thickness at the inflow. The domain length allows sufficient relaxation of the boundary layer towards equilibrium [13]. The mesh for the URANS calculations has \( N_x \times N_y = 1024 \times 160 \) grid points; this resolution was chosen based on the grid-convergence study of Ambrogi \textit{et al.} [13] They examined a coarser three-dimensional grid (using \( N_x \times N_y \times N_z = 1152 \times 129 \times 152 \)) and found that the difference in the mean velocity is less than 2\% when compared with their fine grid of \( N_x \times N_y \times N_z = 1536 \times 192 \times 256 \). Reynolds stresses were also examined, which showed good agreement [13].
reasons, and considering the fact that the LES grid is significantly finer than what would be used in an URANS study of this problem, we chose a number of points close to those of the coarse grid of [13].

2.4.1 Boundary conditions

The bottom of the computational domain uses a no-slip condition and at the outflow, a convective condition [43] is used. In the spanwise direction, the LES uses periodic boundary conditions that are unnecessary in the present case. At the inflow, the LES uses a sequence of slices ($yz$-planes) obtained from a separate calculation to specify the velocity components. In our case, we have phase-averaged the velocity in the $yz$-plane and assigned the phase-averaged velocity at the inflow. The model quantities were obtained analogously by calculating the phase-averaged turbulent-kinetic energy and Reynolds stresses, and extracting the eddy viscosity from

$$
\nu_t = \frac{\epsilon}{\omega}, \quad \text{and} \quad \nu_t = \frac{\epsilon^2}{K}.
$$

respectively, for the $K - \epsilon$ and $K - \omega$ models.
At the top of the domain, a $V_\infty$ profile is prescribed:

$$v(x, L_y, t) = V_{top}(x, t) = v_{top}(x)g(t)U_o$$  \hspace{1cm} (2.4.2)$$

Where $U_o$ is the inflow freestream velocity and the spatial variation is

$$v_{top}(x) = \alpha \sin[\beta(x - x_{ref})]\exp[-\gamma(x - x_{ref})^2]. \hspace{1cm} (2.4.3)$$

The streamwise freestream velocity is obtained by imposing a zero-vorticity condition on the top boundary [7, 8, 44]. The spatial variation matches the suction-blowing velocity profile used by Na and Moin [7] and is the same for all cases studied.

The temporal modulation $g(t)$ differentiates the cases studied. In case A, $g(t)$ is given by

$$g(t) = -\sin \frac{2\pi t}{T}, \hspace{1cm} (2.4.4)$$

where $T$ is the oscillation period, and the frequency $f = 2\pi/T$. A favourable pressure gradient (FPG) is followed by an adverse pressure gradient (APG) for the first half of the cycle, whereas an FPG follows an APG during the other half. If we define a phase angle $\phi = 360^\circ \times t/T$, there are two phases ($\phi = 0^\circ, 180^\circ$) where a zero pressure gradient (ZPG) is imposed between the extreme phases. The maximum APG occurs at $\phi = 270^\circ$ while the maximum FPG occurs at $90^\circ$. The freestream velocity components $U_\infty$ and $V_\infty$ at four phases in the cycle are shown in Fig. 2.3.

Three reduced frequencies ($k = fL_{PG}/U_o$, defined in the Section Section 1.2) have been considered (Note that the definition of the reduced frequency is dependent on the length and velocity scale of the problem and therefore $k$ values are not universal). For $k = 0.2$, the flow is quasi-steady, as the convective time scale is much lower than
2.4. PROBLEM FORMULATION

Figure 2.3: Freestream velocity components, $U_{\infty}$ and $V_{\infty}$, at four phases in the cycle. Case A.

the imposed unsteady time scale. In the $k = 1$ case the two scales are similar. Finally, the high-frequency case $k = 10$ corresponds to a flutter-like rapid oscillation.

Case B matches the study by Park and You [3], in which the temporal variation is given by

$$g(t) = 1 + 0.1 \sin 2\pi \frac{t}{T}. \quad (2.4.5)$$

In this case, an APG is followed by an FPG throughout the cycle, and only the magnitude of the pressure gradient varies. Separation is always present, and the flow does not go through a ZPG phase. At $\phi = 90^\circ$ the APG is at its maximum amplitude, and reaches its minimum at $\phi = 270^\circ$. In this case, a reduced frequency $k$ of 1 has been used to match approximately the one used in Park and You [3].

In addition to the three unsteady cases, steady calculations with constant (in time) pressure gradients have been performed. Steady cases use the $V_{\text{top}}$ profiles at
2.5. NUMERICAL METHOD

Numerical integration of (2.2.3) and (2.2.4) is performed using the LES-3D code [45]. LES-3D uses second-order-accurate central differences on a staggered mesh and a time-advancement that is also second-order in time. The code has been validated extensively and previously applied to similar problems [44–46].

2.5.1 Temporal Integration

The equations are advanced in time using the fractional-step method [47, 48]. A second-order accurate semi-implicit time advancement is used, with a Crank-Nicolson scheme for wall-normal diffusive terms and a low-storage third-order Runge-Kutta
scheme for the others.

The Runge-Kutta scheme consists of three substeps. At each substep: first, a predictor step is carried out; then a Poisson equation is used to determine the pressure, and finally, a corrector step is performed that yields the final, divergence-free velocity field. In the predictor step, the pressure gradient at the previous time-step is used:

\[
\frac{U_{i}^{\text{pred}} - U_{i}^{q-1}}{\Delta t} = \beta q \frac{\partial}{\partial y} \left[ (\nu + \nu_t) \frac{\partial U_{i}^{\text{pred}}}{\partial y} \right] + \beta q \frac{\partial}{\partial y} \left[ (\nu + \nu_t) \frac{\partial U_{i}^{q-1}}{\partial y} \right] \\
+ \gamma q H_{i}^{q-1} + \zeta q H_{i}^{q-2} - 2\beta q \frac{\partial p_{i}^{q-1}}{\partial x_1}, \quad i = 1, 2, 3
\] (2.5.1)

Here \(q = 1, 2, 3\) denotes the step number. \(u_{i}^{\text{pred}}\) is the resulting predicted velocity field, \(\delta t\) is the local sub-step timestep and \(\alpha_k, \beta_k, \gamma_k\) and \(\zeta_k\) are coefficients which vary with substep \(q\). \(H\) contains the explicitly treated convective and remaining viscous terms [48]:

\[
H_i = \frac{\partial}{\partial x} \left[ (\nu + \nu_t) \frac{\partial U_i}{\partial x} \right] - \frac{\partial U_i U_j}{\partial x_j}
\] (2.5.2)

The coefficients \(\alpha_k, \beta_k, \gamma_k\) and \(\zeta_k, k = 1, 2, 3\) are constants selected so that the total time advancement between \(t^n\) and \(t^{n+1}\) is third-order accurate for the convective term
and second order for the viscous term.

\[
\begin{align*}
\gamma_1 &= \frac{8}{15}, & \gamma_2 &= \frac{5}{12}, & \gamma_3 &= \frac{3}{4} \\
\zeta_1 &= 0, & \zeta_2 &= -\frac{17}{60}, & \zeta_3 &= -\frac{5}{12}, \\
\alpha_1 &= \beta_1 = \frac{4}{15}, & \alpha_2 &= \beta_2 = \frac{1}{15}, & \alpha_3 &= \beta_3 = \frac{1}{6}, \\
\sum_{k=1}^{3} (\alpha_k + \beta_k) &= \sum_{k=1}^{3} (\gamma_k + \zeta_k) = 1
\end{align*}
\]

The predicted velocity is not divergence free. To enforce mass conservation we solve the Poisson equation

\[
\frac{\partial^2 \delta p}{\partial x_k \partial x_k} = \frac{1}{\Delta t} \frac{\partial U_{i,\text{pred}}}{\partial x_k} \quad (2.5.3)
\]

using a fast cosine transform in the streamwise directions, followed by direct inversion of the resulting tri-diagonal matrix in the wall-normal direction. Finally, the velocity and pressure are corrected using

\[
\begin{align*}
U_{i,q} &= U_{i,\text{pred}} - 2\beta_i \Delta t \frac{\partial \delta p}{\partial x_i} \quad (2.5.4) \\
p^q &= p^{q-1} + \delta p.
\end{align*}
\]

### 2.5.2 Spatial Discretization

We use a staggered grid, in which pressure, scalar quantities and turbulence modelling quantities (such as \(\nu_t, K, \varepsilon\) and \(\omega\)) are calculated on the cell face, denoted by red Fig. 2.5. Velocity components are calculated at the centers of the cell faces denoted by blue for \(u_i\) and green for \(u_j\). The staggered grid is employed for its ability
2.5. NUMERICAL METHOD

Figure 2.5: Illustration of the arrangement of the staggered grid on the $x - y$ plane. $i, j$ are used as position indexes. From [2].

to prevent the odd-even decoupling of pressure and its reduction of numerical error.

Second-order central differencing is used for all terms. For the convective term, a reverse weighting technique [49] is used to interpolate the velocity field, evaluating derivatives at staggered locations. The interpolation and averaging operators are:

$$|\phi|_i = \frac{(x_{i+1/2} - x_i)\phi_{i+1/2} + (x_i - x_{i-1/2})\phi_{i-1/2}}{x_{i+1/2} - x_{i-1/2}}$$  \hspace{1cm} (2.5.6)$$

and

$$\frac{\delta_1 \phi}{\delta_1 x\}_{i} = \frac{\phi_{i+1/2} - \phi_{i-1/2}}{x_{i+1/2} - x_{i-1/2}}$$  \hspace{1cm} (2.5.7)$$

Here, $\phi$ is a staggered variable, $x$ is a generic coordinate direction, $i$ is a position.
index and \([\phi]\) denotes interpolation. Multi-dimensional derivatives or interpolations are obtained by applying interpolation and derivative operators consecutively in each coordinate direction. The resulting discretized equations are:

\[
\frac{\delta_1 U_k}{\delta_1 x_k}|_{l,m} = 0 \quad (2.5.8)
\]

\[
\frac{\delta_1 U_i}{\delta_1 t}|_{l,m} + \frac{\delta_1 [U_i]_{l,m}[U_k]_{l,m}}{\delta_1 x_k}|_{l,m} = \frac{\delta_1}{\delta_1 x_k} \left\{ (\nu + \nu_t) \frac{\delta_1 U_i}{\delta_1 x_k} \right\}|_{l,m} - \frac{\delta_1 p}{\delta_1 x_i}|_{l,m} \quad (2.5.9)
\]

for spatial indices \(l\) and \(m\).

### 2.6 Post Processing

For Cases A and B, once the solution has reached a converged repetitive behaviour, the oscillation period is divided into 20 equally spaced phases. Snapshots are recorded at each 20\(^{th}\) of a cycle. This procedure is consistent with Ambrogi et al.[13] for simple comparison with phase-averaged data. Only one snapshot is required for the Steady Cases because the solution converges to a time-independent result.

#### 2.6.1 Quantification of Error

To quantify and compare the performance of the turbulence models in predicting the flow behaviour, we use the skin-friction coefficient, and define an “Integrated Error (IE)”.

The skin-friction coefficient \(C_f\) is defined as

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho U_o^2} \quad (2.6.1)
\]
where $\tau_w$ is the phase-averaged wall-stress:

$$\tau_w = \nu \left. \frac{\partial U}{\partial y} \right|_{y=0}.$$  \hspace{1cm} (2.6.2)

The IE is defined as

$$\text{IE} = \int_{L_{PG}} |C_{f,URANS} - C_{f,ref}| \, dx$$

$$\times \frac{C_{f,ref}(x = 0) \times L_{PG}}{}.$$  \hspace{1cm} (2.6.3)

Here, $C_{f,ref}$ is obtained from the LES or, in the Park case [3], the DNS data interpolated onto the URANS grid; the integral is normalized by the reference $C_f$ at the inflow, multiplied by the distance over which the pressure gradient acts, $L_{PG}$.
Chapter 3

Results

3.1 Steady calculations and validation

To validate the problem setup and methodology we first compare the results of steady calculations with the LES results of Ambrogi et al. [13], which have been extensively validated. The methodology we employed, as described in Section 2.4, eliminates common sources of error in numerical studies, such as grid resolution, numerical errors, and uncertainties in the boundary conditions. Thus, differences caused by URANS and the various turbulence models are isolated.

Figure 3.1 shows the spatial variation of the skin friction coefficient with $x/\delta_o^*$ for various pressure gradients. In the steady ZPG case the turbulence models compare well with the LES (as expected, since this flow is used to calibrate the constants). Still, differences emerge in the two strong pressure gradient cases, particularly in the FPG-APG case. The various turbulence models are consistent with one another in their performance. The differences between their predictions are small relative to the difference between the models and the LES results. As discussed earlier, the discrepancies are not due to numerical error or resolution, but intrinsic to the use of
3.1. STEADY CALCULATIONS AND VALIDATION

Figure 3.1: Variation of $C_f$ with $x/\delta_o^*$ for the steady calculations with freestream velocity corresponding to ZPG (top), FPG-APG middle) and APG-FPG (bottom).

URANS and the models themselves.

Figure 3.2 shows the variation of the time-averaged $C_f$ with $x/\delta^*$ for Case B. The DNS and RANS calculations of Park et al. [3] at $Re_{\delta_o} = 300$ are compared with the RANS calculations of the present study, which are, however, at a higher Reynolds number, $Re_{\delta_o} = 1,000$. We have chosen this Reynolds number to remain consistent with Case A. Furthermore, turbulence models are designed for high-$Re$ applications, and testing them at low $Re$ may lead to excessively negative conclusions.
Figure 3.2: Case B: Variation of time-averaged skin-friction coefficient $C_f$ with $x/\delta_o^*$. Figure

The computational domain in our calculations, is much longer than that used by [3]. At this Reynolds number the domain length and height chosen ensure that the numerical boundary conditions do not affect the results in the regions of interest [13]. Nonetheless, a comparison with the results in the literature can reveal major shortcomings of the model.

At the inflow, both this study and the DNS of [3] match the experimental correlations [50] for a ZPG TBL at the corresponding $Re\delta_o^*$. The RANS prediction of the separation region are similar (considering the differences in $Re$ and in computational setup) to the RANS results by Park et al. [3] for the same case. The turbulence models overpredict the length of the separation region, and the magnitude of the reversed flow is higher than the DNS results.
3.2 Unsteady flow, Case A

In this section, Case A will be the focus of discussion. The performance of turbulence models in Case B was discussed in [3] and will be revisited here within the framework of our simulation parameters (which, as was mentioned, differ in various ways from those in Ref. [3]).

3.2.1 Flow characteristics

The physical behaviour of the turbulent boundary layer with unsteady pressure gradients has been described in detail by [13]. To contextualize the ensuing results, however, this section will overview the flow characteristics of the three reduced-frequencies using the LES data.

The reduced frequency $k = 10$ is a high-frequency case, where the convective timescale is greater than the unsteady timescale. $k = 1$ is a case in which the unsteady and convective timescales are equal, and $k = 0.2$ is a quasi-steady case where the convective timescale is lower than the unsteady timescale. The imposed unsteady frequencies affect the physical characteristics of the flow, and the three cases exhibit distinct flow features.

Figure 3.3 shows the phase-averaged velocity contours for the three cases at four representative phases in the cycle. At $\phi = 0^\circ$, a zero pressure gradient is applied at the top of the domain. The boundary layer thickness increases along the streamwise direction in the high-frequency case ($k = 10$) and the quasi-steady case ($k = 0.2$). In both $k = 1$ and 0.2 cases the boundary layer is thicker than in the high-frequency case, as the near-wall fluid has more time to respond to the freestream perturbation. The $k = 1$ case has a unique behaviour: superposed on the boundary layer, there is
3.2. UNSTEADY FLOW, CASE A

(b) $\phi = 90^\circ$.

(c) $\phi = 180^\circ$.

(d) $\phi = 270^\circ$.

Figure 3.3: Contours of the phase-averaged velocity $\langle u \rangle$ from LES at reduced frequencies $k = 10$, 1, and 0.2.
a pocket of slow-moving fluid originating from the recirculation region at \( \phi \simeq 270^\circ \), which is advected downstream.

The acceleration phase of the flow, \( \phi = 90^\circ \) is shown in Figure 3.3(b). Here, an FPG is followed by an APG. As a result, in the first half of the domain the flow accelerates and the boundary layer becomes thinner. A region of faster fluid can be seen in the centre of the domain for each of the three cases; differences between the cases can be seen downstream of this acceleration region. The high-frequency case exhibits the thinnest boundary layer, followed by the quasi-steady case. The \( k = 1 \) case exhibits a region of slow-moving fluid (originally the recirculation bubble), which moves downstream almost as a solid body [51].

During the second ZPG phase, at \( \phi = 180^\circ \), the fluid exhibits similar behaviour to that shown in Fig. 3.3(a), where the boundary layer thickens for the \( k = 10 \) and 0.2 cases, while the separation bubble is washed out of the domain for the \( k = 1 \) case. Finally, during the separation phase of the cycle, shown in Fig. 3.3(d), the maximum APG is followed by the FPG. The adverse pressure-gradient causes the flow to slow down, the boundary layer detaches from the wall, and a recirculation region is formed. The size of the closed recirculation region and the magnitude of the reversed flow depend on the reduced frequency \( k \). For the \( k = 10 \) case, the pressure gradient does not have enough time to interact with the near-wall fluid, and the separation region is very small. In the \( k = 1 \) case the separated-flow bubble is larger in size and the reversed flow has larger magnitude because the timescale of the unsteadiness is greater; therefore, the APG-FPG interacts with the flow for a longer time. In the quasi-steady case, the period of unsteadiness is the largest, and the APG-FPG is applied for the longest time. As a result, the size of the separation
3.2. UNSTEADY FLOW, CASE A

Figure 3.4: Streamwise development of $C_f$ for the $k = 10$ case at four phases of the cycle.

region is also maximum, as is the magnitude of reversal. The scale of the separation region is comparable to that of the steady cases studied in this work.

3.2.2 Skin-friction coefficient

Figures 3.4, 3.5, and 3.6 show the streamwise development of the skin friction coefficient $C_f$ at four extreme phases of the flow ($\phi = 0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$) for the $k = 10$, 1 and 0.2 reduced frequencies respectively. For the high frequency, the models predict the skin friction fairly well. The largest error is observed during the first FPG phase ($\phi = 90^\circ$); the error is still significant during the following FPG phase, ($\phi = 180^\circ$). At this frequency the wall stress and freestream velocity (or
pressure gradient) are not synchronized, since the inner layer does not have time to adapt to the outer-layer changes. As a result, a phase lag exists between the maximum pressure gradient phases and the peak $C_f$ (Fig. 3.7), whose value is approximately 36°. The laminar Stokes layer, however, has a phase lag of 45°, consistent with our results. Thus, a ZPG-like behavior cannot be observed at the phases shown, but would be present at $\phi \simeq 45^\circ$ and 225° instead. The predictions from all models are quite similar.

At $k = 1$ more significant differences can be observed. In Section 3.2.1, the downstream shedding of the separation region was introduced, a phenomenon only observed at this reduced frequency. The error is significant in the FPG phases around

\[ \Phi = 0^\circ \]
\[ \Phi = 90^\circ \]
\[ \Phi = 270^\circ \]
\[ \Phi = 180^\circ \]
3.2. UNSTEADY FLOW, CASE A

Figure 3.6: Streamwise development of $C_f$ for the $k = 0.2$ case at four phases of the cycle.

$90^\circ$ (turbulence models are known to be inaccurate in accelerating flows [34]). The error is largest for the $K - \omega$ model. The error decreases substantially at later times, and the separation is predicted well, in contrast with what was observed for the steady cases discussed earlier (see Fig. 3.1). Errors are more significant in the return to equilibrium, in particular in the region of the advected slow-moving fluid. The $k = 1$ case also exhibits a small lag between freestream velocity and wall stress of approximately $9^\circ$. Very little difference can be observed between the various models.

Finally, 3.6 shows the development of the skin friction coefficient for the quasi-steady ($k = 0.2$) case. At this frequency, the performance of the turbulence models is similar to the $k = 1$ case; however, the absence of the shedding results in more
3.2. UNSTEADY FLOW, CASE A

Figure 3.7: Time evolution of $C_f$ (top row) and $U_\infty/U_o$ (bottom row) at the centre of the recirculation region ($x/\delta_o^* = 300$). 

accurate predictions downstream of the reattachment region. This is particularly visible during the $\phi = 0^\circ$ and $180^\circ$ phases. At the $\phi = 90^\circ$ phase, the performance of the turbulence models is similar to that for the $k = 1$. Again, the return-to-equilibrium is slower than in the reference data. Turbulence models are known to have difficulties predicting the return-to-equilibrium in many flows [24, 34], and the present results are consistent with this behaviour. The separation and reattachment points are predicted equally well by all models. However, the added complexity of the shedding present in case results in inferior performance downstream of the pressure-gradient region at this frequency.

In general, very little difference can be observed between the prediction of the various models. The FPG region and the return-to-equilibrium are the areas where the modelling error is maximum; the advection of the slow-moving fluid region observed
for \( k = 1 \), although predicted qualitatively well (see the next section) is one of the main sources of error. The agreement between URANS predictions and LES results is better for the unsteady cases than for the steady ones (a behaviour that is not entirely consistent with the results of [3]). This issue will be discussed at length later in Section 3.3.

### 3.2.3 Phase-averaged velocity

The global performance of the turbulence models may be analysed qualitatively by examining the contours of the phase-averaged velocity, shown in Figures 3.8, 3.9 and 3.10 at the four representative phases of the period. All turbulence models predict the general features of the flow correctly, with only small discrepancies with respect to the LES. The height of the separation region when the strongest APG-FPG is applied is predicted reasonably well, and the turbulence models can capture reattachment and the downstream shedding of the recirculation region. The downstream shedding of the recirculation region and the reattachment of the boundary layer are complex phenomena and the fact that the model can capture them is significant. The size and shape of the recirculation bubble vary slightly with model used, with the largest discrepancies in the \( k = 0.2 \) case followed by the \( k = 1 \) case. Also, the size and shape of the shed region are predicted differently between the various turbulence models. The acceleration phases of the cycle are those in which the largest errors occur, as will be shown later.

Figures 3.11 through 3.13 show the phase-averaged streamwise velocity profiles at three locations, \( x/\delta_o^* = 200, 300 \) and 450. The first is at the beginning of the pressure gradient region, the second and third in the middle and at the end of this region.
3.2. UNSTEADY FLOW, CASE A

Figure 3.8: $k = 10$ case. Contours of phase averaged velocity $\langle u \rangle$ at four phases of the cycle are shown in a clockwise direction: (a) $\phi = 0^\circ$, (b) $\phi = 90^\circ$, (c) $\phi = 180^\circ$ and (d) $\phi = 270^\circ$. 
Figure 3.9: $k = 1$ case. Contours of phase averaged velocity $\langle u \rangle$ at four phases of the cycle are shown in a clockwise direction: (a) $\phi = 0^\circ$, (b) $\phi = 90^\circ$, (c) $\phi = 180^\circ$ and (d) $\phi = 270^\circ$. 
Figure 3.10: $k = 0.2$ case. Contours of phase averaged velocity $\langle u \rangle$ at four phases of the cycle are shown in a clockwise direction: (a) $\phi = 0^\circ$, (b) $\phi = 90^\circ$, (c) $\phi = 180^\circ$ and (d) $\phi = 270^\circ$.  

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Figure 3.11: \( k = 10 \) case. Velocity profiles at \( x/\delta_0^* = 200, 300 \) and 450. Four phases are shown: \( \phi = 0^\circ, 90^\circ, 180^\circ \) and \( 270^\circ \).
Figure 3.12: $k = 1$ case. Velocity profiles at $x/\delta_o^* = 200, 300$ and 450. Four phases are shown: $\phi = 0^\circ, 90^\circ, 180^\circ$ and $270^\circ$. 
Figure 3.13: \( k = 0.2 \) case. Velocity profiles at \( x/\delta^* = 200, 300 \) and 450. Four phases are shown: \( \phi = 0^\circ, 90^\circ, 180^\circ \) and \( 270^\circ \).
At the first location, all models predict the velocity accurately for all frequencies and at all phases. This is expected because turbulence models are calibrated for zero-pressure gradient flows. Departures from the LES, however, begin to occur inside the recirculation zone and depend strongly on the flow phase (i.e., on the freestream pressure gradient) and on the frequency. At the high frequency the agreement between the URANS and the LES is still extremely good. At lower frequencies modelling errors begin to appear.

For $k = 1$ the $K-\varepsilon$ model is the least accurate. The error is particularly significant in the FPG and the preceding phases ($\phi = 0^\circ, 90^\circ$). The SA and $K - \omega$ give very similar results. At $\phi = 270^\circ$ the separation bubble is very small: because of the phase lag the strongest reverse flow occurs at $\phi = 260^\circ$. The $K - \varepsilon$ model predicts a higher velocity immediately above the separation region. Note that with the SA model the advection of the recirculation bubble is somewhat slower (Fig. 3.10).

For $k = 0.2$ the agreement is worse, reflecting the quasi-steady state of the flow. In particular, no model predicts the velocity profile in the recirculation correctly. Note that the largest discrepancy in the $C_f$ occurs in the advection region around $x/\delta^*_s \simeq 500$. The last location shown here is at the very beginning of this region.

The Reynolds shear stresses, $\langle u'v' \rangle$ are shown in Figures 3.14 through 3.16. As is the case for the velocity (which is of course driven by the Reynolds stresses, and must reflect their behaviour) they are in good agreement with the data for $k = 10$. The major discrepancy is observed near the wall, where all the models overpredict the stresses in a very thin layer ($y/\delta^*_s < 2$). The stress predicted by the models is strongly tied to the velocity gradient, and the decoupling of the near-wall from the outer layer at this frequency is predicted incorrectly. We observe the same phenomenon for $k = 1$,
Figure 3.14: \( k = 10 \) case. Profiles of the Reynolds shear stress \( \langle u'v' \rangle \) at \( x/\delta_o = 200, 300 \) and 450. Four phases are shown: \( \phi = 0^\circ, 90^\circ, 180^\circ \) and \( 270^\circ \).
Figure 3.15: $k = 1$ case. Profiles of the Reynolds shear stress $\langle u'v' \rangle$ at $x/\delta_o^* = 200$, 300 and 450. Four phases are shown: $\phi = 0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$. 
Figure 3.16: $k = 0.2$ case. Profiles of the Reynolds shear stress $\langle u'v' \rangle$ at $x/\delta^* = 200$, 300 and 450. Four phases are shown: $\phi = 0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$. 
especially in the recirculation and recovery regions.

In the quasi-steady \( k = 0.2 \) case, the error, as expected, is maximum in the recirculation region. Although the trends are predicted correctly, the shear stress magnitude is overpredicted and the thickness of the recirculation bubble is higher. This indicates that the dissipation and diffusion predicted by the models are too high. Modifications of these terms may, therefore, be required to improve the model's accuracy.

Note that both \( \phi = 0^\circ \) and \( \phi = 180^\circ \) are phases at which the pressure gradient is zero. Dynamic hysteresis is, however, present (see the discussion in [13]), and memory effects result in velocity profiles that are quite different from those in a fully developed, ZPG, case. The intermediate frequency is the one in which this phenomenon is most significant. The \( K - \varepsilon \) model is the one that predicts the hysteresis least accurately, at least in this region.

### 3.2.4 Integrated Error

In Section 2.6.1 we introduced the Integrated Error (IE) as an integral measure of the accuracy of a given model. Figure 3.18 shows the distribution of the Integrated Error (IE) throughout the cycle in the \( k = 1 \) case. Similar results were obtained for the other reduced frequencies (not shown). The performance of the turbulence models varies throughout the cycle and largely depends on the pressure gradient applied during each phase. The largest errors occur during the acceleration phases of the cycle, where the maximum FPG precedes the APG. This is consistent with the well-known difficulty that turbulence models encounter in flows with strong acceleration, as discussed in Section 1.2. In contrast, the turbulence models predict the separation
Figure 3.17: Velocity profiles at $x/\delta^* = 200$ for the steady and unsteady case ($k = 1$) at $\phi = 0^\circ$ and $180^\circ$. (a) $k = 10$; (b) $k = 1$; (c) $k = 0.2$.

phases of the cycle and the mild and zero pressure gradient phases with significantly smaller errors, again consistent with results in the literature. The model predictions, on the other hand, are surprisingly accurate in the strong APG-FPG phases of the flow, although some inaccuracies occur in the regions affected by the shedding of the recirculation region. The magnitude of the errors is larger for the steady cases than the unsteady ones, except at the ZPG phases.
3.2. UNSTEADY FLOW, CASE A

The better performance of the turbulence models in the unsteady case compared to the steady cases is counterintuitive due to the added complexity associated with unsteadiness. The better accuracy of the unsteady calculations can be explained by memory effects caused by the cyclical nature of the flow. In the unsteady case, the flow begins as a ZPG; the pressure gradient then intensifies until it is a strong FPG-APG and returns to ZPG before becoming APG-FPG. As a result, the phases characterized

Figure 3.18: Comparison of the IE for steady cases and unsteady $k = 1$ case for Spalart Allmaras (panel I), $k - \varepsilon$ (panel II) and $k - \omega$ (panel III)
3.3. COMPARISON OF CASES A AND B

Table 3.1: Time-averaged Integrated Error (IE) for the three turbulence models at the three reduced frequencies.

<table>
<thead>
<tr>
<th></th>
<th>$k = 10$</th>
<th>$k = 1$</th>
<th>$k = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>5.1%</td>
<td>6.8%</td>
<td>8.3%</td>
</tr>
<tr>
<td>$K_\omega$</td>
<td>6.9%</td>
<td>7.2%</td>
<td>7.7%</td>
</tr>
<tr>
<td>$K_\varepsilon$</td>
<td>5.9%</td>
<td>7.1%</td>
<td>7.9%</td>
</tr>
</tbody>
</table>

by mild and zero pressure gradients act as an 'anchor' that stops the solution from diverging too far from the correct one during the strong-pressure-gradients phases. The fact that the pressure gradient changes sign, in other words, causes memory effects that result in more accurate prediction of the flow during the strong pressure gradient cases than in the corresponding steady cases.

The time average of the integrated error in the pressure gradient region is summarized in 3.1. The turbulence models predict $C_f$ with the smallest error in the $k = 10$ case, and the largest IE for the $k = 0.2$ case. The fact that the error is most significant in the pressure gradient region for the lower reduced frequencies may also be explained by the different physical phenomena associated with each frequency. For the high-frequency case, the magnitude of flow reversal and the size of the separation bubble is the smallest, whereas in the low-frequency cases, the interaction with strong pressure gradients is longer and the height of the recirculation region is longer and larger in magnitude (discussed in Section 3.2.1). As a result, the errors in turbulence model predictions are greater.

3.3 Comparison of Cases A and B

As mentioned previously, Cases A and B are differentiated mainly by the temporal variation of their pressure gradients. Case A alternates between phases in which an
APG precedes an FPG (APG-FPG phases) and one in which the FPG is followed by the APG (FPG-APG phases). Thus, the flow separates during the APG-FPG phases, but reattaches when a ZPG is applied. In Case B the configuration is always one of APG-FPG, with varying magnitudes. The flow is separated throughout the entire cycle in this case. From a physical point of view, Case A could correspond to an airfoil pitching between angles-of-attack $+\alpha$ and $-\alpha$, while Case B is similar to a pitching airfoil oscillating between $\alpha_1 \approx \alpha_2$, where both angles are in the near-stall regime.

This difference explains the discrepancy between the conclusions reached here and those by [3]. They have found that URANS methods provide misleading information regarding the separation bubble; namely that results obtained with URANS predict earlier separation and a larger recirculation bubble compared with DNS.

Figure 3.19 compares the spatial development of $C_f$ during the maximum APG-FPG phase in (panel (a)), the time-averaged $C_f$ for Case A (panel (b)), and the results for Case B (c,d). In panel (c), Case B results from the present study at $Re\delta_5 = 1,000$ are shown with the DNS of [3] (at $Re\delta_5 = 300$). Panel IV(d) shows the turbulence model predictions and DNS of [3] only.

In Case A, the increased $C_f$ during the FPG-APG phases (the acceleration part of the cycle) balances the lower $C_f$ found when separation occurs, so that the time-integrated $C_f$ is close to that of a ZPG boundary layer. Because of modelling errors during the acceleration phases, all the models slightly overpredict the LES.

By contrast, in Case B the separation and the recirculation bubbles can be detected from the time-averaged data as well, Fig. 3.19(c,d). Notice that the recirculation region, in the unsteady case, is much less extended than both the steady case and
Figure 3.19: Spatial development of (a) $C_f$ for the maximum APG-FPG phase of Case A; (b) time-averaged $C_f$ for Case A; (c) time-averaged $C_f$ for Case B; (d) time-averaged $C_f$ for Case B from [3].

Case B, even at the maximum APG phase, Fig. 3.19(a). Figure 3.19(c) shows that, despite the different Reynolds number and domain size, the results from the present simulation are consistent with those of [3]. Finally, comparing 3.19(c) and (d), we observe that the behaviour of the URANS solutions is also very similar, independently of the Reynolds number.

The magnitude of the time-averaged error in the URANS prediction of $C_f$ is over
twice as large in Case B as in Case A; with 6.8% in Case A and 16.7% in Case B with the Spalart-Allmaras model. While the error with the $K - \omega$ model error was 7.2% in Case A and 16.7% in Case B. It is also interesting to notice that in Case B, the main error is near the separation and reattachment regions. In contrast, in Case A, it occurs mostly downstream of the recirculation bubble.

Comparing the two cases that are characterized by the temporal variation of the pressure gradients supports the previous conjecture: that the success of the turbulence models in predicting separation in Case A may be attributed to memory effects as the pressure gradients alternate from APG-FPG to FPG-APG.

This difference is because the cycle in Case A goes through two phases where the freestream velocity is nearly uniform as discussed in Section 3.2.4. In contrast, Case B remains APG-FPG throughout the entire cycle and does not have ZPG phases to 'anchor' the predictions.
Chapter 4

Conclusion and future work

We carried out URANS calculations of a flat plate boundary layer at $Re_{\delta^*} = 1,000$ with pressure-induced unsteady separation to evaluate the performance of turbulence models in predicting flows where unsteady separation occurs. The most commonly used one and two-equation models were considered: The Spalart Allmaras model, the two-layer $K-\varepsilon$ and the $K-\omega$ model. The grid has been validated for this problem by [13], and numerical methods and boundary conditions were matched to the LES of [13] to isolate errors caused by the turbulence models.

Separation is induced by freestream adverse and favourable pressure gradients matching the spatial variation of [7]. Two cases have been considered, which are differentiated by the temporal variation of the pressure gradients at the top of the domain. In Case A, the pressure gradients alternated from APG-FPG to FPG-APG with intermediate phases of ZPG. In this case, the flow alternates from phases of acceleration and separation. For Case A, three reduced frequencies were considered $k = 10, 1$ and 0.2. Case B considered flow with APG-FPG of varying magnitudes throughout the cycle. In this case, separation remains present at all phases of the cycle. A reduced frequency of $k = 1$ is considered for this case.
The results have been validated and compared to the high-fidelity LES of [13] and the DNS of [3].

4.1 Key findings

First, the turbulence models performed reasonably well in predicting complex unsteady flow phenomena. Each turbulence model predicts the general features of the flow correctly, with only small discrepancies from the LES. Notably, the size of the separation region when the strongest APG-FPG is applied is a near match with the correct solution. The fact that the models can capture complicated flow phenomena, such as the downstream shedding of the recirculation region and the reattachment of the boundary layer, is significant. While the turbulence models successfully capture the accelerating and separating behaviour of the flow in the outer layer of the domain, the largest errors occur closest to the wall, particularly in the downstream region where shedding of the separation bubble occurs.

The performance of the turbulence models depends strongly on the phase of the cycle (i.e., on the freestream pressure gradients applied at a given time). Turbulence models predict mild and zero pressure gradient phases with minimal differences from the LES, which is expected because the models have been calibrated for flows of this type. However, discrepancies between the models and LES results arise during the strong pressure gradient phases. In the recirculation bubble, the Reynolds stresses and, consequently, the velocity has the largest error.

The shear stress magnitude is overpredicted, particularly during phases of strong FPG-APG, indicating that the dissipation and diffusion predicted by the models are too high. Modifications of these terms may, therefore, be required to improve the
Counterintuitively, the turbulence models predict unsteady cases more accurately than steady ones. The reason for this phenomenon most likely lies in the type of time evolution of the pressure gradient. In the unsteady case, the flow begins as a ZPG; the pressure gradient intensifies until it is a strong FPG-APG and returns to ZPG before becoming APG-FPG. As a result, the intermediate phases, characterized by mild- and zero-pressure gradients, act as ‘anchors’ that stop the solution from diverging too far from the correct result when the strong pressure gradients are applied. The fact that the pressure gradient changes sign causes memory effects that result in more accurate flow prediction during the strong pressure gradient cases than in the corresponding steady cases.

This conjecture is confirmed by comparing the performance of the turbulence models in Case A and Case B. The turbulence models are more successful (Time-averaged IE = 6.8 %) in their predictions for Case A, where the pressure gradients alternate from FPG-APG to APG-FPG and pass through phases of ZPG. In Case B, (Time-averaged IE = 16.7%) separation remains present throughout the entire cycle, and ZPG phases do not occur to anchor the turbulence models. Both cases are physically relevant. Case A corresponds to a physical configuration in which a symmetric oscillation occurs; for instance, the pressure gradient on the tail of a fish would undergo similar APG-FPG cycles. A case in which the oscillation amplitude is smaller, such that the main flow characteristics do not change (an airfoil near stall, whose angle of attack varies by a few degrees only, for example) would correspond to Case B.
4.2 Future directions

Additional cases will be evaluated to investigate further the influence of memory effects on the performance of turbulence models. High-fidelity LES and URANS calculations will be performed for problems with the same formulation as Case A and B, with different temporal variations of pressure gradients. Case C will alternate from APG-FPG to ZPG before returning to APG-FPG. Its temporal variation takes the form:

\[ g(t) = 0.5 + 0.5 \sin \left( \frac{2\pi t}{T} \right); \]  \hspace{1cm} (4.2.1)

where \( T \) is the oscillation period and the frequency \( f = \frac{2\pi}{T} \). Case D will alternate from APG-FPG to an APG-FPG of 50% magnitude before returning to maximum APG-FPG. Its temporal variation takes the form:

\[ g(t) = 0.75 + 0.25 \sin \left( \frac{2\pi t}{T} \right); \]  \hspace{1cm} (4.2.2)

Evaluating the turbulence models’ performance in the two additional cases will further test the conjecture that the mild and ZPG phases act as an anchor which stops the turbulence models from diverging too far from the correct result.
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