STANDARD AND MULTI-MATERIAL TOPOLOGY OPTIMIZATION
DESIGN FOR AUTOMOTIVE STRUCTURES

by

CHAO LI

A thesis submitted to the Department of Mechanical and Materials Engineering
In conformity with the requirements for
the degree of Doctor of Philosophy

Queen’s University
Kingston, Ontario, Canada
(June, 2015)

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Abstract

Lightweight design, drawing an increasing attention for structural design in automotive industry, is recognized as an efficient and immediate way to improve fuel efficiency and reduce CO₂ emissions. Topology optimization, by determining an optimum geometry and material distribution of a structure at an early design stage, serves as the cornerstone for not only increasing the performance of products but also streamlining the entire structural design process.

In this thesis, the theory, algorithm, implementation and application of both of the traditional single-material topology optimization and an advanced multi-material topology optimization are presented, which can solve real-world engineering problems in the automotive industry. This research will advance structural optimization methods in academic research, and it is also expected that the developed method and tool would make a profound impact in the design of automotive parts and assemblies in the field.

In Chapter 2 and Chapter 3, the traditional single-material topology optimization is explained, and it is applied to the design of an automotive engine cradle and a cross-car-beam (CCB). The computational method helped an automotive tier-1 supplier company produce better engineering products while reducing time and cost of the design process.

In Chapter 4, a multi-material topology optimization methodology and its numerical tool are presented. This innovative approach can effectively deal with multiple, dissimilar materials in structural design. Advanced mathematical algorithms, numerical implementation, and practical
applications are discussed in detail, and effectiveness and efficiency of the methodology is demonstrated with a variety of engineering problems.

Detailed discussions are included in Chapter 5, and recommendations for future work are discussed in Chapter 6.
Co-Authorship


Acknowledgements

I cannot believe I am sitting in the library writing my thesis, exciting and relieved. Four years passed as a blink. August 8th, 2011, the first day I landed in Canada and came to Kingston, was like yesterday. The beautiful and friendly city has left me so much memory worth being remembered for a life time. Everyone I know is truly amazing and helpful, without whom, my PhD study would not be completed in success. They deserve all my heartfelt thanks for this wonderful journey.

My utmost thanks would be given to my supervisor Prof. Il Yong Kim, whose knowledge, wisdom, guidance and support have enabled me to explore the best of myself, not only to complete my research, but to pursue a better life. I believe that the primary responsibility of a supervisor does not limit to ‘tell’ a student ‘what’ to do, but ‘help’ a student understand ‘why’ to do and ‘how’ to do. Prof. Il Yong Kim is this outstanding supervisor who helped me grow as a ‘thinker’. He is one of the best supervisors I have ever met and I felt blessed and lucky to be mentored by him.

I would express my sincere gratitude to Dr. Justin Gammage and Mr. Balbir Sangha, from GM Canada, for offering the opportunity of doing the on-site research in the Canadian Regional Engineering Center (CREC). The achievement during the 7 months there was fruitful.

I would give my sincere thanks to Mr. Cheng Zeng and Mr. Sacheen Bekah, from KIRCHHOFF Van-Rob Inc., for offering the opportunity to do internship for about 10 months in my first year of PhD study. Thank you for the trust of providing me with various interesting and challenging
engineering tasks. The generosity of Mr. Oscar Jia for offering me the ride to work every day is greatly appreciated.

Special thanks would be given to my colleagues from Structural and Multidisciplinary System Design (SMSD) group. Their creativity, enthusiasm, and team work spirit have impressed me and influenced me considerably.

I would not have gone this far without the endless and selfless love, encouragement and support from my whole family, who are far away in China. I cannot thank more to them and feel truly sorry for not being enough with them.

Again, thank you all for the incredible experience!
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<th>Description</th>
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<tbody>
<tr>
<td>AHSS</td>
<td>Advanced High Strength Steel</td>
</tr>
<tr>
<td>BIW</td>
<td>Body-In-White</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer-Aided Design</td>
</tr>
<tr>
<td>CAE</td>
<td>Computer-Aided Engineering</td>
</tr>
<tr>
<td>CAFE</td>
<td>Corporate Average Fuel Economy</td>
</tr>
<tr>
<td>CCB</td>
<td>Cross-Car-Beam</td>
</tr>
<tr>
<td>DOE</td>
<td>Design of Experiment</td>
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<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>ESLM</td>
<td>Equivalent Static Load Method</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
</tr>
<tr>
<td>MBB</td>
<td>Messerschmitt-Bölkow-Blohm</td>
</tr>
<tr>
<td>MIG</td>
<td>Metal-Insert-Gas</td>
</tr>
<tr>
<td>MPG</td>
<td>Miles per Gallon</td>
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<tr>
<td>OC</td>
<td>Optimality Criteria</td>
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<tr>
<td>OEM</td>
<td>Original Equipment Manufacturer</td>
</tr>
<tr>
<td>RAMP</td>
<td>Rational Approximation for Material Properties</td>
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<tr>
<td>SCSWS</td>
<td>Steering Column and Steering Wheel System</td>
</tr>
<tr>
<td>SIMP</td>
<td>Solid Isotropic Microstructure with Penalization</td>
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Chapter 1

General Introduction

1.1 Research Background

With a rising cost of natural resources and growing concerns on the environment, energy conservation and protection of the environment have become an important issue worldwide. Both governments and customers demand more energy-efficient, cost-effective, and environmentally friendly products. For example, the Corporate Average Fuel Economy (CAFE) regulations in the United States mandate an improvement of the average fuel economy of cars and light trucks up to 54.5 mile per gallon (mpg) by Model Year 2025. In Canada, the fuel economy target of passenger cars is 40.8 mpg by 2016 and 56.2 mpg by 2025; in the European Union, the target fuel economy of passenger cars is 42.3 mpg by 2015 and 57.9 mpg by 2025; in South Korea, 39.5 mpg by 2015 and 58.8 mpg by 2020; and in China, 37 mpg by 2015 and 56 mpg by 2025 [1]. Facing tremendous pressure from governments and customers, improvement in fuel efficiency of the next generation vehicles is a top priority for automakers.

There are several ways to improve fuel efficiency of a vehicle, such as improving the powertrain efficiency or using a substitute energy, but one of the most promising approaches is to reduce the weight of a vehicle. It has been demonstrated that with every 100 kg weight reduction, the fuel consumption will decrease by approximately 0.4 L per 100 km [2] and carbon dioxide (CO₂) emissions will decrease by 8-11 g/km [3]. Therefore,
lightweight design of automobiles is an efficient and immediate way to improve fuel efficiency and reduce CO₂ emissions.

1.2 Current Lightweight Strategies

Current lightweight strategies can be categorized into three main approaches.

1.2.1 Lightweight Materials

The traditional low carbon steel can be substituted by lightweight materials, such as aluminum, magnesium, plastics, and composites. The properties of different materials, in terms of stiffness, strength, durability, corrosion, and formability, should be discreetly taken into consideration. In automotive industry, traditional materials like low carbon steel and iron have been gradually replaced by various lightweight materials over past decades. A report by Ducker Worldwide [4] predicted that 26.6% of all body and closure parts for light vehicles would be made of aluminum by 2025 in North America, whereas in 2015 only 6.6% is aluminum.

1.2.2 Advanced Manufacturing Process

Advanced manufacturing and production processes can be employed to reduce the extra weight of joints. It is also important to develop methods that can join dissimilar materials together. For example, the use of structural adhesives or overcasting can reduce the weight of mechanical joints, and replacing metal-insert-gas (MIG) welding with laser welding can avoid the introduction of extra material of solder, which will result in weight reduction. Friction stir welding can join different dissimilar materials, like steel and aluminum.
1.2.3 Design Optimization

Design optimization determines the optimum geometric feature while satisfying required performance criteria. This is achieved by allocating materials only at needed locations with optimum amounts. The structure then becomes highly efficient since it has only the minimum amount of materials required at the right location. Topology optimization finds most efficient structural layouts in conceptual and preliminary design stages, and size and shape optimization can fine-tune optimum designs in the detailed design stage. For example, size and shape optimization can determine the optimum design parameters of a tailored-welded-blank structure or a tailored-rolled-tube structure.

It is important to note that nowadays aforementioned three approaches are no longer independent of each other. With the introduction of more lightweight materials to the design and manufacture of automotive structures, it is important to simultaneously consider optimum choice of materials, effective use of various joining techniques, and optimum design of structural geometries. For this purpose, design optimization can play a pivotal role.

1.3 Structural optimization

Structural optimization is in rapid development due to the continued advancements in high performance computing and finite element method theory over the last several decades. The typical structural optimization techniques—topology optimization, size optimization, and shape optimization—are utilized in many industries including aerospace, automotive, civil, and the biomedical fields.
1.3.1 Size optimization

Size optimization (Figure 1.1 (a) [5]) is a relatively simple approach. Almost every geometry-related variable (e.g., the thickness of a bracket, radius of a tube, or the stiffness of a spring) can be parameterized as a size design variable for optimization. Thickness optimization of structural members has been intensively studied in the design of automotive parts.

1.3.2 Shape optimization

Shape optimization (Figure 1.1 (b) [5]) is used to find the optimal structural shape through changing the prescribed shape variables, such as the shape of a hole, shape of an exterior surface, or shape of a strengthening member. Generally, shape optimization is performed by a series of local shape changes to satisfy the performance requirements. Similar to size optimization, this approach can only explore a specific subset of the overall design space. The number of holes and the overall layout of a structure are unchanged by shape optimization. A typical example of shape optimization is an optimum shape design of an airfoil which considers structural and aerodynamic performance characteristics.

1.3.3 Topology optimization

Topology optimization (Figure 1.1 (c) [5] and Figure 1.2) determines an optimal profile and material distribution of a structure by considering a large design space. In other words, topology optimization is used to determine the features such as the number, location, and shape of holes and the connectivity of the domain [5]. Because it can explore the entire design space, topology optimization is used in the early design phase. The results from
topology optimization can then be used as the input geometry (or baseline design) for subsequent size and shape optimization processes. By performing topology, shape, and size optimization, we can determine the best possible design satisfying the performance targets on weight, stress, stiffness, modal frequency and others.

In standard topology optimization, the design variable is existence of material in each design pixel (or voxel), and therefore only one type of material can be considered. This traditional topology optimization cannot deal with design problems with multiple possible material choices, and mathematically rigorous and numerically effective and efficient approach for multi-material design is developed in this thesis.

![Figure 1.1 Methods of structural optimization](image)

**Figure 1.1 Methods of structural optimization [5]**
(a) Initial design space, loading and boundary condition

(b) Topology optimization result with single material

**Figure 1.2 Standard Topology Optimization**
1.4 Motivation

Theories and numerical techniques for size and shape optimization have been well developed. Without proper choice of optimal topology, however, size and shape optimization can make only limited performance improvement. Topology optimization starts with a non-biased design, which means there is no need for input from a designer, and the method determines a set of optimum load paths within the structure. Hence, topology optimization is recognized as the most fundamental design process for structural optimization, and it should be implemented in an early design stage, such as the conceptual and preliminary design stages. The output from topology optimization can then be used in the following detailed design stage, which can make a significant contribution to the performance and quality of structural products.

Topology optimization has been traditionally used for structural design with only a single material, which determines the existence of material within each design cell for a prescribed material type. For a design problem with multiple materials, however, this traditional topology optimization method cannot be used. As aforementioned, the use of multiple materials for weight reduction is an important task in automotive industry, and the current optimization technology is ill equipped for this challenge. Hence, it is imperative to develop an effective multi-material topology optimization algorithm and numerical tool that can deal with multiple materials for real-world design problems.

1.5 Primary Contribution

Not only will this research make a significant advancement in topology optimization academically, but it will also provide automotive industry with a methodology and tool that
can produce innovative structural designs for lightweight design. Both of the single-material and the multi-material topology optimization presented in this thesis can be used in other industries such as railway and aerospace.

Primary contributions of the work in this thesis are summarized as:

(1) A systematic lightweight design process is proposed, and its effectiveness and efficiency are demonstrated by means of an automotive engine cradle and an automotive cross-car-beam (CCB) examples.

(2) An innovative simulation and topology optimization framework for modal frequency performance of a steering column and steering wheel system (SCSWS) is developed.

(3) An advanced multi-material topology optimization algorithm is developed, and it is applied to various examples and real-world design problems.

(4) A numerical tool of the multi-material topology optimization is created which can solve real-world engineering problems in automotive industry.

1.6 Structure Overview

This thesis is organized in the manuscript format. Chapter 2 and Chapter 3 have been published separately in peer-reviewed journals, and Chapter 4 has been submitted to a peer-reviewed journal and currently under review. Because of the consistency and coherence of the research, minor repetitions are inevitable between different manuscripts (chapters), particularly in the section of Introduction of each chapter.

The structure of the thesis is organized as follows:
Chapter 1 provides an overview of the entire work discussed in this thesis, including the background, motivation, and contribution.

Chapter 2 presents an effective and efficient lightweight design process framework for an automotive engine cradle from conceptual design to detailed design by using topology, shape and size optimization. A version of this Chapter has been published on the peer-reviewed journal of *Structural and Multidisciplinary Optimization* as Li C, Kim IY, Jeswiet J., Conceptual and detailed design of an automotive engine cradle by using topology, shape, and size optimization, *Structural and Multidisciplinary Optimization*, 2015; 51(2): 547-564, DOI: 10.1007/s00158-014-1151-6.


Chapter 4 presents an innovative two-material topology optimization method through algorithm development, numerical implementation, and engineering applications. A version of this Chapter has been submitted to the peer-reviewed journal of *International Journal for Numerical Methods in Engineering*.

Chapter 5 includes a general discussion and summary of the research presented in previous chapters.
Chapter 6 discusses the limitations of the research and the potential work that could be carried out in the future.
REFERENCES


Chapter 2

Conceptual and Detailed Design of an Automotive Engine Cradle by

Using Topology, Shape, and Size Optimization

An automotive engine cradle supports many crucial components and systems, such as an engine, transmission, and suspension. Important performance measures for the design of an engine cradle include stiffness, natural frequency, and durability, while minimizing weight is of primary concern. This chapter presents an effective and efficient methodology for engine cradle design from conceptual design to detailed design using design optimization. First, topology optimization was applied on a solid model which only contains the possible engine cradle design space, and an optimum conceptual design was determined which minimizes weight while satisfying all stiffness constraints. Based on topology optimization results, a design review was conducted, and a revised model was created which addresses all structural and manufacturability concerns. Shape and size optimization was then performed in the detailed design stage to further minimize the mass while meeting the stiffness and natural frequency targets. Lastly, the final design was validated for durability. The initial design domain had the mass of 82.6 kg; topology optimization in conceptual design reduced the mass to 26.7 kg; and the detailed design task involving shape and size optimization further reduced the mass to 21.4 kg.
2.1 Introduction

With new government regulations on automobiles, automotive manufacturers are under tremendous pressure to improve vehicle fuel efficiency and reduce carbon emissions drastically. As an example, automakers have to significantly improve vehicle mileage through a series of steps beginning in 2016 through to 2025 due to the mandated Corporate Average Fuel Economy (CAFE). There are several ways how to improve vehicle fuel efficiency, such as improving the efficiency of power train, but one of the most promising methods is to reduce the vehicle weight. It has been shown that with every 100 kg weight reduction, the consumption of fuel will decrease by approximately 0.4 liters per 100 km [1] and the emissions of CO2 will decrease by 8 g to 11 g per kilometer [2].

An engine cradle is a critical component of a car with multiple functions. The engine cradle structurally supports the engine, transmission, and suspension; distributes high chassis loads over a wider area; and reduces vibrations and shocks that are generated by the engine, transmission, and roads. Engine cradles first appeared in the late 1960s and were used to balance the riding comfort and handling ability for luxury cars. An engine cradle is typically manufactured from welded steel stampings and is bolted to the vehicle body. By adopting engine cradles, energy-absorption and force-distribution have also considerably improved. Further, an engine cradle makes it possible to build the steering, engine, and transmission assemblies in one location, and to install the finished product in the completed vehicle in another location. This modular manufacturing process allows for the reduction of assembly time and cost. The use of engine cradles also allows mechanics to remove broken parts easily, which results in the reduction of repair time and maintenance cost.
Due to these benefits, engine cradles are used in various car segments, even in the compact cars like Ford Focus, Toyota Corolla, and Volkswagen Lavida. Moreover, on account of their advantages, a rear subframe that has a similar structure as the engine cradle has been developed and widely adopted for the purpose of carrying the rear suspension and transmission systems.

There are, however, two major drawbacks of adopting an engine cradle: 1) due to the addition of an engine cradle, the weight of the vehicle increases, which results in inferior fuel efficiency and increased carbon emissions; and 2) the design cycle time also becomes longer—including CAD modeling, numerical analysis, physical testing, and re-design—and this increases the cost of product development. It is therefore important for automakers to have the ability to determine lightweight engine cradle designs with a short design cycle time.

The current practice in automotive industry is not yet fully systematic or efficient. A conceptual design is proposed based on the designer’s experience, which is often subjective and sub-optimal. In the detailed design stage, a series of FEA (finite element analyses) are performed with respect to several performance measures such as stiffness, natural frequency, and durability (i.e. fatigue). If the target requirements cannot be met by fine tuning of the design parameters, the conceptual design is revisited and new design concepts are explored. This design iteration inevitably puts a significant strain on already tight product development schedule and finances of the company.

A number of academic studies can be identified which attempt to improve the performance of an engine cradle design. Triantos and Michaels [3] proposed the development of an

These studies allowed for the improvement of engine cradle performance in one way or another, but their contributions are limited because the application of the proposed approaches took place in the detailed design stage, that is, after the overall design concept is already selected. Truly effective designs could be determined, if a most promising engine cradle design is sought in the conceptual design stage. Topology optimization is an effective method for conceptual structural design. Due to significant computing power improvement, we can nowadays solve topology optimization with high resolution—and for simple geometries, topology optimization can determine optimum designs which really do not require further shape and size optimization.

Theory of topology optimization is already well established for a number of performance measures such as compliance, natural frequency, and stress. Topology optimization determines optimal structural geometry by distributing material over the design domain—the optimum geometry then can be considered as the most efficient load path network. There are a very large amount of studies in topology optimization theory. Bendsøe and Sigmund [9] summarized fundamentals of topology optimization in their book. Kim and
Kwak [10] proposed design space optimization in which the design domain (or the number of design variables) is also considered as a design variable. Kim and Weck [11] applied the concept of “adaptivity” to Genetic Algorithm-based topology optimization and convergence criterion of topology optimization [12]. Forsberg and Nilsson [13] developed an internal energy density method, avoiding the use of sensitivities, to apply topology optimization to nonlinear problems. Topology optimization was also applied to the simulation of the human bone adaptation process by Kim and his colleagues [14-15]. Lee and Park [16] utilized the equivalent static method in nonlinear topology optimization.

Introducing topology optimization to the early design stage, such as the conceptual design stage, can help effectively filter out inferior designs and produce the most promising designs for the subsequent design stages. This could also reduce design cycle by minimizing trial-and-error iterations. Due to these benefits, topology optimization has been applied to the design of many automobile and aircraft parts such as chassis and body. Yang and Chahande [17] introduce an in-house topology optimization software program, TOP, and used it to optimize a simplified truck frame, a deck lid, and a space frame structure. Lee and Lee [18] performed topology optimization to find the optimal layout of an aluminum control arm to reduce the weight and improve the rigidity. Lee et al. [19] used topology optimization to determine the material distribution of a door panel, and based on the topology result, the panel was partitioned to multiple domains with different thickness values that can be manufactured using tailor welded blanks. Chiandussi et al. [20] utilized topology optimization to improve the structural performance of a rear suspension subframe. Wang et al. [21] employed topology optimization and size optimization to improve the overall stiffness of an existing car body. Torstenfelt and Klarbring [22] found
the conceptual design of the modular car product families using topology optimization. Waquas et al. [23] applied topology optimization to the design of aircraft components. Cavazzuti et al. [24] presented a methodology for automotive chassis design using topology optimization, topometry optimization, and size optimization. Lee et al. [25] employed the equivalent static loads method (ELSM) to determine the cross-section of the crashbox using topology optimization to maximize the absorbed energy. Luo and Di [26] achieved a lightweight optimal design of an in-wheel motor by using topology optimization and size optimization.

Although an engine cradle adds a significant amount of weight and lengthens the design cycle as aforementioned, the design of an engine cradle, especially in the conceptual design stage, is currently done heuristically in industry; this is because a systematic and efficient topology optimization approach has not been utilized for the design of engine cradles.

The objectives of this research are 1) to perform the conceptual design of a real-world engine cradle based on a high-fidelity finite element (FE) model, using large-scale topology optimization and 2) to further improve the design in the subsequent detailed design stage, using shape and size optimization. By means of this work, the entire design process will be done effectively and efficiently, and the important performance metrics such as stiffness and natural frequency will be considered in design optimization. Manufacturability will be considered after a promising concept is selected by topology optimization, and the final optimum design will be validated for durability requirements.

Section 2.2 presents the design optimization problem statement for the entire design process of an engine cradle. Section 2.3 shows the conceptual design task, which consists
of finite element modeling and topology optimization for mass minimization with stiffness requirements. In Section 2.4, detailed design is performed: first, the optimum result by topology optimization in the conceptual design stage is carefully interpreted and reviewed, and a revised model is created. Shape and size optimization is conducted which considers stiffness and natural frequency requirements while further minimizing the mass. Section 2.5 shows durability tests of the optimum design from the detailed design stage, in order to confirm the design satisfies all fatigue requirements. Section 2.6 summarizes the study.

2.2 Mathematical Problem Statement of Engine Cradle Design Optimization

The design optimization problem for the entire design process of conceptual design and detailed design is mathematically stated as

\[
\minimize \ W(\rho_i, T_j, S_k) \\
\begin{align*}
KU &= P \\
f_i &\geq F_i \\
u_{i}^x &\leq D_{i}^x \\
u_{i}^y &\leq D_{i}^y \\
u_{i}^z &\leq D_{i}^z \\
0 < \rho_{\min} &\leq \rho_i \leq 1 & i & = 1, 2, \ldots, m \\
T_{jL}^L &\leq T_j \leq T_{jU}^U & j & = 1, 2, \ldots, n, \\
S_{kL}^L &\leq S_k \leq S_{kU}^U & k & = 1, 2, \ldots, n_s
\end{align*}
\]  

(2.1)

where the objective is to minimize the weight \( W \), \( \rho_i \) denotes the relative density of the \( i \)-th design variable for topology optimization, \( T_j \) represents the \( j \)-th design variable for size (or thickness) optimization, and \( S_k \) represents the \( k \)-th design variable for shape optimization. \( P \) and \( U \) are the vectors of nodal force and displacement, respectively, and \( K \) is the global stiffness matrix. \( f_1 \) indicates the first natural frequency of the system, and \( F_1 \)
is the lower bound for the first natural frequency. $u_l^x$, $u_l^y$ and $u_l^z$ denote the x-, y- and z-directional displacements of the $l$-th loading point, respectively. $D_l^x$, $D_l^y$ and $D_l^z$ are the corresponding upper bounds for x-, y-, and z-directional displacements of the $l$-th loading point, respectively. $q$ is the total number of loading points. $\rho_{\text{min}}$ is the minimum allowable bound for the density, which is implemented in order to avoid numerical difficulties. $m$, $n_t$, $n_s$ are the total number of design variables for topology optimization, size optimization, and shape optimization, respectively. $(T_l^x, S_l^x)$ and $(T_l^y, S_l^y)$ are the lower bounds and upper bounds for the corresponding size and shape design variables, respectively.

2.3 Conceptual Design

2.3.1 Finite Element Modeling

A solid model that contains the entire geometric domain of an engine cradle is shown in Figure 2.1. There are attachment areas which are used for the installation of other sub-systems, such as the engine and suspension. These areas must be fixed throughout the optimization process, and therefore they are excluded from the design domain for any optimization. The yellow areas in Figure 2.2 show these non-designable areas. All other domains are subject to topology optimization. The entire engine cradle domain (Figure 2.1) was meshed into nearly 7.9 million finite elements with about 1.6 million nodes. Excluding the elements of non-designable areas, the total number of finite elements that are used as design variables for topology optimization was close to 7.5 million; this is the green area in Figure 2.2. The material was aluminum, and the mass of this entire engine cradle domain, including the non-designable areas, was 82.6 kg.
Figure 2.1 Solid space for engine cradle
Figure 2.2 Non-designable domains (yellow)
Topology optimization was used in this conceptual design stage. The objective of the topology optimization is to determine the most effective design that minimizes the mass of the structure. Important performance requirements that must be considered through the entire design process are the stiffness, natural frequency, and durability. Typically, it is most difficult to satisfy stiffness targets among others. Hence, we chose the stiffness requirement as the design constraint for topology optimization in this conceptual design stage, while leaving other performance requirements for next-phase optimization tasks.

As system-level design requirements, maximum allowable displacements are imposed on the front suspension link, rear suspension link, powertrain mount, steering gear attachment, and lower link bracket. These constraints are applied at 10 loading points, and for each point, forces in all x-, y- and z-directions are applied and their corresponding target stiffness values are specified. Figure 2.3 shows all three forces at each of the 10 loading points, and Table 2.1 shows their numerical values.

Stiffness is not a parameter that can be measured directly. As stiffness \( \kappa \) is determined by the ratio of the applied force \( R \) to the displacement \( \delta \)

\[
\kappa = \frac{R}{\delta} \tag{2.2}
\]

the stiffness targets can be transformed into displacement targets that can be readily measured under individual external forces:

\[
\delta = \frac{R}{\kappa} \tag{2.3}
\]
Figure 2.3 Loading condition
Table 2.1 Stiffness requirements for topology optimization in the conceptual design stage

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Case</th>
<th>Direction</th>
<th>Target (mm)</th>
<th>ID</th>
<th>Load Case</th>
<th>Direction</th>
<th>Target (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Driver Side</td>
<td></td>
<td></td>
<td></td>
<td>Passenger Side</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>Front Susp. Link</td>
<td>X</td>
<td>2.871E-01</td>
<td>R6</td>
<td>Front Susp. Link</td>
<td>X</td>
<td>3.151E-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>3.799E-02</td>
<td></td>
<td></td>
<td>Y</td>
<td>3.573E-02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>7.600E-02</td>
<td></td>
<td></td>
<td>Z</td>
<td>9.500E-02</td>
</tr>
<tr>
<td>R2</td>
<td>Rear Susp. Link</td>
<td>X</td>
<td>1.269E-01</td>
<td>R7</td>
<td>Rear Susp. Link</td>
<td>X</td>
<td>1.252E-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>3.449E-02</td>
<td></td>
<td></td>
<td>Y</td>
<td>3.587E-02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>5.284E-02</td>
<td></td>
<td></td>
<td>Z</td>
<td>5.329E-02</td>
</tr>
<tr>
<td>R3</td>
<td>Powertrain Mnt.</td>
<td>X</td>
<td>3.176E-02</td>
<td>R8</td>
<td>Powertrain Mnt.</td>
<td>X</td>
<td>3.139E-02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>5.068E-02</td>
<td></td>
<td></td>
<td>Y</td>
<td>4.982E-02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>2.770E-02</td>
<td></td>
<td></td>
<td>Z</td>
<td>2.752E-02</td>
</tr>
<tr>
<td>R4</td>
<td>Steering Gear</td>
<td>X</td>
<td>1.755E-01</td>
<td>R9</td>
<td>Gear</td>
<td>X</td>
<td>3.801E-02</td>
</tr>
<tr>
<td></td>
<td>Attachment</td>
<td>Z</td>
<td>3.308E-01</td>
<td></td>
<td>Attachment</td>
<td>Z</td>
<td>3.533E-01</td>
</tr>
<tr>
<td></td>
<td>Lower Link</td>
<td>X</td>
<td>4.009E-02</td>
<td></td>
<td></td>
<td>X</td>
<td>4.144E-02</td>
</tr>
<tr>
<td>R5</td>
<td>Bracket</td>
<td>Y</td>
<td>2.152E-02</td>
<td>R10</td>
<td>Lower Link</td>
<td>Y</td>
<td>2.340E-02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>6.400E-02</td>
<td></td>
<td></td>
<td>Z</td>
<td>6.437E-02</td>
</tr>
</tbody>
</table>
2.3.2 Topology Optimization

The problem statement of the topology optimization for this engine cradle conceptual design is formulated as

$$\begin{align*}
\text{minimize} \quad & W(\rho) = \sum_{i=1}^{m} \rho_i v_i \\
\text{subject to} \quad & K(\rho)U = P \\
& u_i^x \leq D_i^x \\
& u_i^y \leq D_i^y \\
& u_i^z \leq D_i^z \quad l = 1,2,\ldots,q \\
& 0 < \rho_{\text{min}} \leq \rho_i \leq 1 \quad i = 1,2,\ldots,m
\end{align*}$$

(2.4)

where the objective is to minimize the weight, $\rho_i$ denotes the relative density of the $i$-th design variable and $v_i$ represents the volume of $i$-th element. $P$ and $U$ are the vectors of nodal force and displacement, respectively, and $K$ is the global stiffness matrix. $u_i^x$, $u_i^y$ and $u_i^z$ denote the x-, y- and z-directional displacements of the $l$-th loading point, respectively. $D_i^x$, $D_i^y$ and $D_i^z$ are the corresponding upper bounds for the x-, y- and z-directional displacements of the $l$-th loading point, respectively. $q$ represents the total number of loading points, and its value in this study is 10. $\rho_{\text{min}}$ is the minimum allowable bound for the density, which is implemented in order to avoid numerical difficulties, and $m$ is the total number of design variables (around 7.5 million).

Two primary methods in the density-based topology optimization are the solid isotropic microstructure with penalization (SIMP) method, see Bendsøe and Sigmund [9] and the rational approximation of material properties (RAMP) method [27]. Both methods convert
a continuous optimization model into a discretized model in which a penalty factor is used to drive the intermediate density to converge to either 0 (void) or 1 (solid).

The SIMP model has been proved to be efficient in solving a variety of engineering problems. The penalization is achieved by the following formulation:

\[ E = \rho_i^p E_0 \]  

(2.5)

where \( E \) represents the penalized Young’s modulus of the \( i \)-th design variable (or design voxel), \( E_0 \) indicates the real Young’s modulus of the base material, and \( p \) is the penalty factor whose value is greater than one.

Most topology optimization algorithms require the design sensitivity, which is expressed as the derivative of a response functional with respect to each design variable (i.e. density of each element, \( \rho_i \)). Topology optimization typically deals with a very large number of design variables and a small number of functionals (objective function and constraints) for compliance-related problems. The adjoint variable method is then the most efficient approach for calculating the sensitivities of compliance.

There are two numerical difficulties accompanied with topology optimization: checkerboards pattern and solution dependency on mesh resolution. The former phenomenon refers to the emergence of checkerboard-like patterns on the design domain, which erroneously overestimates the stiffness of the structure. This design is also impractical from the view point of manufacturing. The latter problem is regarding ever changing solutions as the mesh resolution increases. These solutions are also impractical.
The cause and solutions for both problems are well explained in the book by Bendsøe and Sigmund [9].

2.3.3 Numerical Results of Topology Optimization in the Conceptual Design Stage

Topology optimization is performed using OptiStruct 11.0 [28]. Topology optimization results are highly dependent on the choice of the penalty factor (for intermediate density control) and sensitivity filtering factor (for checkerboards control). By using an effective penalty factor, which is expressed as $p$ in Equation (2.5), we can minimize unfavorable intermediate-density elements in the final optimum results, and the choice of an optimum sensitivity filtering factor can restrain the emergence of checkerboards in optimization.

There are two more control parameters that affect the optimum results: minimum member size control and manufacturability control. Compared to bulky structures, thin structures can usually support loads more efficiently with minimum mass. Minimum member size control is a technique to keep such thin structures [29]. There are a number of manufacturing processes used for mass production in the automotive industry, but extrusion is particularly favorable because of its low cost and fast processing time. Extrusion requires the elongated structure has the “same (i.e. constant)” cross-sectional shape throughout the direction of extrusion (longitudinal direction). In the context of topology optimization, the problem becomes a 2D optimization of a 3D structure represented by a 3D finite element model, which means that all longitudinally-aligned elements with the same coordinate on the cross-sectional plane must have the same density value. This can be easily implemented in topology optimization.
We have tested several different combinations of these parameters and chose the best set.

Table 2.2 compares the optimization setting with no parameter control (Case A) to that with proper parameter control (Case B). The sub-domains where the extrusion constraint is applied in Case B are shown Figure 2.4.

**Table 2.2 Comparison of two optimization settings**

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No parameter control</td>
<td>Proper parameter control</td>
</tr>
<tr>
<td>Penalty factor ((p\text{ in Equation (2.5)}))</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Checkerboard control</td>
<td>No</td>
<td>1</td>
</tr>
<tr>
<td>Minimum member size control</td>
<td>No</td>
<td>8</td>
</tr>
<tr>
<td>Manufacturability constraint</td>
<td>No</td>
<td>Extrusion on 2 sub-domains</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Figure 2.4)</td>
</tr>
</tbody>
</table>

**Figure 2.4 Extrusion constraints**
Topology optimization was conducted on a Linux workstation (AMD Opteron Processor 6180, 800 MHz, 129065 MB RAM, 16 cores), with the CPU time of 120 hours. Figure 2.5 shows the optimization results by the two cases. Obviously, the optimum solution by Case B is more favorable because it has less intermediate-density elements, clearer configuration, and better manufacturability. This design has the mass of 26.7 kg, and its detailed features are shown in Figure 2.6. Six iteration snapshots during the entire optimization history are shown in Figure 2.7, and the history of the objective function is presented in Figure 2.8.

It is clear from those figures that elements with intermediate density values are not completely removed even with parameter control (Case B). Only elements with either 0 or 1 density value are physically meaningful, and therefore we implemented a threshold on density and removed intermediate-density elements as explained in the following section.
Figure 2.5 Topology optimization results with different control parameters

(a) Without control parameters (Case A)

(b) With proper control parameters (Case B)
Figure 2.6 Details of the topology structure with control parameters (Case B)
Figure 2.7 Topology optimization history for Case B (only 6 iteration snapshots shown here)

Figure 2.8 The history of the objective function (mass) during the topology optimization (Case B)
2.4 Detailed Design

2.4.1 Result Re-interpretation

A density threshold of 0.3 was chosen, and only those elements whose density values are greater than or equal to 0.3 were exported as a CAD model, as shown in Figure 2.9. A careful design review of this model revealed three design concerns: i) disconnected structural parts, ii) rough surfaces, and iii) very thin parts. Disconnected parts are generated due to the complete removal of elements whose density values are smaller than 0.3, and obviously these structural features cannot carry mechanical loads. Roughness of the surfaces has no benefits and only causes problems such as stress concentration and manufacturing difficulty. Very thin structural elements help improve structural stiffness; however, it is hard to manufacture these features, even with casting, and they are prone to fractures even under small mechanical shocks.

In order to address these concerns, we carefully examined each part of the optimum solution and created a revised engine cradle design, as shown in Figure 2.10. The extruded parts are modeled with thin walls for size optimization. The other parts are to be manufactured as casted parts, and therefore they are modeled based on casting feasibility and joining relationships with other sub-systems. The extruded parts are meshed with shell elements, and all the other parts are meshed with solid tetrahedral elements. The model contains 0.3 million nodes and 0.9 million elements.
Figure 2.9 Geometry extraction from topology optimization result (Case B)

Figure 2.10 Revised design
2.4.2 Finite Element Modeling

The optimum solution shown in Figure 2.5 (b) and Figure 2.6 satisfies all 30 stiffness constraints of Equation (2.4); however, there is no guarantee that the revised model generated by the design review task (Figure 2.10) still satisfies these 30 stiffness constraints. We conducted a finite element analysis of the revised model in Figure 2.10 and determined its stiffness values; the results show that the revised model now violates 8 stiffness constraints. The first objective of the size and shape optimization in this detailed design stage is then to make the design satisfy all 30 constraints. In addition, we added a natural frequency constraint in this detailed design stage, because it can be readily considered in size and shape optimization. With further minimization of mass as the objective function, we can then formulate the size and shape optimization as:

\[
\begin{align*}
\text{minimize} & \quad W(T_i, S_j) \\
\text{subject to} & \quad \mathbf{K} \mathbf{U} = \mathbf{P} \\
& \quad \mathbf{u}_i^x \leq D_i^x \\
& \quad \mathbf{u}_i^y \leq D_i^y \\
& \quad \mathbf{u}_i^z \leq D_i^z \\
& \quad f_i \geq F_i \\
& \quad 4\text{mm} \leq T_1 \leq 10\text{mm} \\
& \quad 4\text{mm} \leq T_2 \leq 12\text{mm} \\
& \quad 4\text{mm} \leq T_3 \leq 20\text{mm} \\
& \quad 0 \leq S_j \leq 1, \quad j = 1, 2 \\
& \quad -1 \leq S_j \leq 1, \quad j = 3, 4 \\
& \quad 0 \leq S_j \leq 2, \quad j = 5, 6, 7, 8
\end{align*}
\]

where the objective is still to minimize the weight \(W\), \(T_i\) is the \(i\)-th size (or thickness) optimization design variable, and \(S_j\) is the \(j\)-th shape optimization design variable. \(P\) and \(U\) are the vectors of nodal force and displacement, respectively, and \(K\) is the global stiffness
matrix. \( u_i^x \), \( u_i^y \) and \( u_i^z \) denote the x-, y- and z-directional displacements of the \( l \)-th loading point, respectively. \( D_i^x \), \( D_i^y \) and \( D_i^z \) are the corresponding upper bounds for those displacements. The total number of loading points is 10 in this study. \( f_1 \) indicates the first natural frequency of the system, and \( F_1 \) is the lower bound for the first natural frequency, which is equal to 210 Hz in this study.

Figure 2.11 shows the three design variables for size optimization. The thickness \( T_1 \) is applied to all extrusion edges which are represented in brown. In the same way, \( T_2 \) and \( T_3 \) are applied to the elements in green and yellow, respectively. The upper and lower bounds of the design domain are listed in Table 2.3.

Shape optimization uses perturbation vector approach [30], and the shape design variables are defined as the linear combination of the perturbation vectors as follows:

\[
X = X_0 + \sum_{i=1}^{n} PV_i \cdot S_i
\]

(2.7)

where \( X \) is the new vector of nodal coordinates, \( X_0 \) is the vector of initial nodal coordinates, \( PV_i \) is the perturbation vector associated to the \( i \)-th shape design variable \( S_i \). \( n_s \) is the total number of perturbation vectors and shape design variables. Figure 2.12 Shape optimization design variables shows the shape optimization design variables, and Table 2.4 presents the design domain for each design variable.
Figure 2.11 Size optimization design variables

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Lower Bound (mm)</th>
<th>Upper Bound (mm)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>4.00</td>
<td>10.00</td>
<td>Thickness of extrusion edges</td>
</tr>
<tr>
<td>$T_2$</td>
<td>4.00</td>
<td>12.00</td>
<td>Thickness of extrusion ribs</td>
</tr>
<tr>
<td>$T_3$</td>
<td>4.00</td>
<td>20.00</td>
<td>Thickness of attachments</td>
</tr>
</tbody>
</table>
Figure 2.12 Shape optimization design variables

Table 2.4 Design domain for shape optimization

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Perturbation Vector (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>0.00</td>
<td>1.00</td>
<td>3.00</td>
</tr>
<tr>
<td>S₂</td>
<td>0.00</td>
<td>1.00</td>
<td>3.00</td>
</tr>
<tr>
<td>S₃</td>
<td>-1.00</td>
<td>1.00</td>
<td>5.00</td>
</tr>
<tr>
<td>S₄</td>
<td>-1.00</td>
<td>1.00</td>
<td>5.00</td>
</tr>
<tr>
<td>S₅</td>
<td>0.00</td>
<td>2.00</td>
<td>5.00</td>
</tr>
<tr>
<td>S₆</td>
<td>0.00</td>
<td>2.00</td>
<td>5.00</td>
</tr>
<tr>
<td>S₇</td>
<td>0.00</td>
<td>2.00</td>
<td>5.00</td>
</tr>
<tr>
<td>S₈</td>
<td>0.00</td>
<td>2.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>
2.4.3 Shape and Size Optimization and Numerical Results

The objective of the shape and size optimization is to fine tune the local structure to further minimize the weight while satisfying all 30 stiffness constraints and the first natural frequency requirement. Size and shape optimization was conducted on Windows PC (AMD Phenom II x6 1090T Processor, 3200MHz, 7392 MB RAM, 6 cores). After 6 iterations with the CPU time of about 1.5 hours, the optimization converged with all constraints satisfied. The final mass of the engine cradle was 21.4 kg. The optimum values of the design variables are listed in Table 2.5.

Table 2.5 Optimal design variables

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Optimal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₁ (mm)</td>
<td>10.00</td>
</tr>
<tr>
<td>T₂ (mm)</td>
<td>11.20</td>
</tr>
<tr>
<td>T₃ (mm)</td>
<td>17.92</td>
</tr>
<tr>
<td>S₁</td>
<td>1.00</td>
</tr>
<tr>
<td>S₂</td>
<td>0.00</td>
</tr>
<tr>
<td>S₃</td>
<td>-1.00</td>
</tr>
<tr>
<td>S₄</td>
<td>-1.00</td>
</tr>
<tr>
<td>S₅</td>
<td>2.00</td>
</tr>
<tr>
<td>S₆</td>
<td>2.00</td>
</tr>
<tr>
<td>S₇</td>
<td>2.00</td>
</tr>
<tr>
<td>S₈</td>
<td>2.00</td>
</tr>
</tbody>
</table>
2.5 Validation of the Optimum Design for Durability

The structural integrity of a product should be maintained during its life time, and therefore durability is an important performance requirement. Durability tests are performed by applying several working stresses excited by different loading conditions like the lateral force, longitudinal force, steering gear force, etc. Alternating peak loads are the usually the main cause for failure. Due to such loads, micro-cracks gradually expand and later become macroscopic cracks; these cracks propagate, and ultimately a sudden mechanical fracture happens. Engine cradles must bear the combination of many loading conditions within the required life cycles. The durability requirements of the engine cradle in this study are listed in Table 2.6.

Table 2.6 Durability requirements

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Test</th>
<th>Force (KN)</th>
<th>Cycle Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Lateral Force</td>
<td>6.70</td>
<td>&gt;= 200 000</td>
</tr>
<tr>
<td>R2</td>
<td>Longitudinal Force</td>
<td>6.70</td>
<td>&gt;= 200 000</td>
</tr>
<tr>
<td>R3</td>
<td>Steering Gear Force</td>
<td>6.90</td>
<td>&gt;= 200 000</td>
</tr>
<tr>
<td>R4</td>
<td>Stabilizer Force</td>
<td>4.00</td>
<td>&gt;= 200 000</td>
</tr>
<tr>
<td>R5</td>
<td>Engine Force Rear</td>
<td>4.50</td>
<td>&gt;= 200 000</td>
</tr>
</tbody>
</table>
The cycle targets exceed $10^4$ times, and this problem can be explained by high cycle fatigue [31]. The Stress-Life (S-N) approach is frequently used, but this approach only focuses on the cyclical stresses that are predominantly within the elastic range. There is a possibility that the engine cradle is damaged with a plastic strain, which can then cause failure less than $10^4$ cycles. Hence, we chose the Strain-Life (E-N) approach which can deal with plastic deformations occurring under the given cyclic loading.

The complete E-N curve relates the total strain (including both elastic and plastic strain) to the number of cycles. The elastic strain $\varepsilon_e$, related to Hooke’s law, can be derived from Basquin formula

$$\varepsilon_e = \frac{\sigma}{E} = \frac{\sigma_f}{E} \cdot (2N)^b$$  \hspace{1cm} (2.8)

And the plastic strain $\varepsilon_p$ can be retrieved via Coffin-Manson’s formula:

$$\varepsilon_p = \varepsilon_f \cdot (2N)^c$$  \hspace{1cm} (2.9)

Finally, the complete E-N curve is obtained by the sum of the elastic and plastic strain terms:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma_f}{E} \cdot (2N)^b + \varepsilon_f \cdot (2N)^c$$  \hspace{1cm} (2.10)

where $\varepsilon$ is the complete strain, $\varepsilon_e$ is the elastic strain, $\varepsilon_p$ is the plastic strain, $E$ is the Young’s modulus, $\sigma_f$ is the fatigue strength coefficient, $\varepsilon_f$ is the fatigue ductility coefficient, $b$ is the fatigue strength exponent, and $c$ is the fatigue ductility exponent.

First, a stress analysis was executed to obtain the stress contour of the entire structure for each load case. The stress contours for the 5 loads are shown in Figure 2.13. This stress
information was then utilized in the durability analysis which was implemented in nCode Design Life 8.0 [32]. The E-N curve for the material is displayed in Figure 2.14. The cycle numbers for all nodes in the structure were beyond “cutoff” (or minimum cycle limit), and part of the results are shown in Table 2.7 as sample results. Thus, the current model successfully passed the durability validation.
Figure 2.13 Stress contours for different loading conditions
Figure 2.14 Strain life with elastic and plastic lines (nCode Design Life 8.0)

Table 2.7 Durability results (Part)

<table>
<thead>
<tr>
<th>Node</th>
<th>Damage</th>
<th>Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>311486</td>
<td>Beyond cutoff</td>
<td>Beyond cutoff</td>
</tr>
<tr>
<td>311487</td>
<td>Beyond cutoff</td>
<td>Beyond cutoff</td>
</tr>
<tr>
<td>311488</td>
<td>Beyond cutoff</td>
<td>Beyond cutoff</td>
</tr>
<tr>
<td>311489</td>
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<td>Beyond cutoff</td>
</tr>
<tr>
<td>311490</td>
<td>Beyond cutoff</td>
<td>Beyond cutoff</td>
</tr>
<tr>
<td>311491</td>
<td>Beyond cutoff</td>
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<td>311492</td>
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<td>311495</td>
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<td>311496</td>
<td>Beyond cutoff</td>
<td>Beyond cutoff</td>
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<tr>
<td>311497</td>
<td>Beyond cutoff</td>
<td>Beyond cutoff</td>
</tr>
</tbody>
</table>
It should be noted that our optimum design “passed” the durability test at once; however, it is possible that in other applications the optimum design created by topology, shape and size optimization does not meet the durability criteria. In this case, shape and size optimization with the fatigue life as an inequality constraint should be run by considering the overall shape change and local shape change (e.g. fillet radius). Generally, topology optimization is ill-equipped for fatigue criteria because fatigue failures are heavily affected by local shapes.

It is also meaningful to point out that there are other factors that affect fatigue life such as manufacturing and machining operations, welding quality, surface treatment, surface roughness, temperature, and corrosion. We do not have numerical methods that accurately consider all these factors. Hence it is required to conduct experimental validation tests of fatigue before proceeding to mass production.

2.6 Conclusions

This chapter presented an effective and efficient design process framework for automotive engine cradles from conceptual design to detailed design by using design optimization. Figure 2.15 shows the progress of design optimization tasks through the entire design process. The initial design domain had the mass of 82.6 kg; topology optimization in conceptual design produced an optimum design with the mass of 26.7 kg; and the detailed design task involved shape and size optimization and further reduced the mass to 21.4 kg. The topology optimization considered 30 stiffness constraints, and the size and shape optimization implemented the same 30 stiffness constraints and an additional first natural
frequency constraint. As the last task, it was confirmed that the final optimum design satisfies all durability constraints.

We solved the optimization problem in Equation (2.1) over two stages: topology optimization and then shape and size optimization. The result is more than just finding a “better” solution than the original design. We solved a practical, real-world optimization problem in this chapter, and we are unable to discuss global optimality of our solution, which will require the discussion of convexity (or all possible local minima) of the design space and the satisfaction of KKT (Karush–Kuhn–Tucker) conditions. However, we utilized state-of-the-art optimization methods and tools and ensured all convergence criteria are properly met, and therefore it can be stated that our solution is one of the best practical optimum designs that satisfy Equation (2.1).

The benefits of the proposed design process also include a shortened design cycle. Our industrial partner, Van-Rob Inc., stated that the period of designing an automotive engine cradle to the first-round technical review typically takes 10-12 weeks; however, by introducing topology optimization, the period has been shortened to 5 weeks, which is a massive time saving for product development in the fiercely competitive auto industry. Further, compared to the traditional trial-and-error method, the geometrical configuration of the lightweight design derived from topology optimization is more objective, reliable, and economic. It was also a valuable experience for the designers to interpret the topology optimization result and create a revised model which addresses all concerns of the “raw” result from topology optimization. It should be noted that the methodology is not
constrained only for engine cradle development, but it can be extended to the design of other products such as body-in-white structures, cross car beams, or control arms.

2.7 Limitation

In our research, we did not consider crashworthiness although it is one of the ultimate performance criteria for automakers. There are two main challenges for crashworthiness from the viewpoint of design optimization of a component. First, crash safety regulations are generally imposed onto the entire vehicle, except for bumper components. The problem here is that it is hard to calculate crashworthiness of an entire vehicle based on crashworthiness of separate components, because there are strong interactions among components, sub-assemblies, and assemblies. A supplier can consider the behavior of a particular component (in this research, engine cradle), but cannot consider interaction with other components.

It should be noted that we did not make any assumption or approximation in the definition of the loads. Our analysis and optimization was performed in collaboration with a Tier-I supplier, who does the actual design work based on explicit requirements from its customer (i.e. an OEM). The OEM as the customer gave the loading conditions to its supplier, and once a final design is submitted to the OEM, it will perform crash analysis considering the engine cradle and other components which are developed by many other suppliers.

Second, crash simulations have very high computing expense and the numerical analysis is highly nonlinear. Therefore, it is not yet practical to include crashworthiness of an entire vehicle in an optimization loop, which will require tens or hundreds of crash analyses. Some techniques, such as the equivalent static load method (ESLM), make approximation
of crash behavior, and allow for the inclusion of some simplified crash behavior in an optimization. Topology optimization for some crashworthiness studies have been seen nowadays by using LS-TaSC and Genesis/ESLM together, and it is one of our future directions. Once again, the Tier-I supplier is not required to consider crashworthiness, because crash analyses will be done by an OEM considering many components from many different suppliers. However, it would be advantageous for the supplier if it could consider crashworthiness in its product development.

The objective of this chapter was to consider important performance metrics of an engine cradle for a Tier-I supplier who actually design and manufacture the product. Hence, crashworthiness was not considered in the current research, but left as future work which will be conducted in collaboration with the OEM. Although it would be infeasible to include crashworthiness of an entire vehicle in a design optimization loop, we would be able to gain some meaningful insights by means of multiple crash simulations, and the results will be incorporated back into our engine cradle design.
Figure 2.15 Structural design process of an engine cradle
ACKNOWLEDGMENT

The authors would like to express their thanks to the group of CAD & CAE department and the members of Research & Development department from Van-Rob Inc. Their suggestions are acknowledged and much appreciated.

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[8] Xu J. T., Song Y. D., Ding J. B. and Ding S. Q., Analysis of a Front Sub-Frame Fatigue Strength Based on Miner Theory. 2010 International Conference on
Computer Application and System Modelling, 2010; pp. 191-193. DOI: 10.1109/ICCASM.2010.5623056


An automotive cross car beam (CCB) supports instrument panels including the HVAC (heating, ventilation, and air conditioning) system, knee airbags, steering column and steering wheel system (SCSWS), and the central console. Avoiding resonant frequencies and improving driving comfort is a major performance requirement in the design of a CCB. Due to the nature of mass production in the automotive industry, the consideration of manufacturability is important, and the current practice in the industry does grant detailed information on the SCSWS to the CCB designer. The objective of this chapter is to perform a complete topology, shape, and size optimization of a CCB by using a lightweight material, considering two practical manufacturing processes (extrusion and casting), and assuming the realistic situation where only limited information on the SCSWS is available to the CCB designer. First, a simplified finite element (FE) model of the SCSWS was developed, and it was calibrated using optimization such that important behavior of the simplified FE model agrees with that of the real SCSWS. Topology optimization was performed to determine the optimal material distribution for the parts that connect the SCSWS and CCB. Then a geometry reinterpretation of the favorable topology result was performed to address the concerns from the viewpoint of cost and manufacturability. A sensitivity study was conducted subsequently to determine size optimization design variables with significant effect on frequency performance. Finally, size and shape
optimization were performed together to further optimize the details of the CCB structure. The weight of the optimal aluminum design was reduced by nearly 40% compared to the steel design while the important performance requirements are met.

3.1 Introduction

Government regulations have become more and more stringent in the automotive industry especially on fuel efficiency and carbon emissions. For example, the US regulation, the Corporate Average Fuel Economy (CAFE), requires an automaker increase its corporate average fuel efficiency to 54.5 miles per gallon (MPG) for passenger cars and light-duty trucks by Model Year 2025. It is nearly as twice the fuel economy of the future vehicles as that of 2012. It has been proved that with every 100kg weight reduction, the consumption of fuel will be decreased by approximately 0.4 liters per 100 km [1] and the emissions of CO$_2$ will be reduced by 8g to 11g per kilometer [2]. Therefore, the lightweight design of automobile is an efficient and immediate way to improve the fuel efficiency and reduce CO$_2$ emissions.

Currently, automotive lightweight strategies can be categorized into three approaches: 1) Lightweight Material: the traditional low carbon steels can be replaced by high specific strength materials or lighter materials, such as high strength steel, aluminum, magnesium, and reinforced plastic composites. 2) Advanced Manufacturing: advanced manufacturing and production processes are developed to reduce the weight of automobile. For example, using structural adhesives will reduce the weight of joints, and to replace MIG welding with laser welding could avoid introducing extra material of solders. 3) Design Optimization: design optimization techniques can be used to minimize the material usage
without sacrificing performance requirements. For example, optimum designs of tailored-welded-blanks and tailored-rolled-tubes can be determined by size optimization.

The process of a product design should be logical, effective, and efficient. Figure 3.1 [3] gives an example of the entire structural design process of an automotive engine cradle. The design process starts with a design space which encloses the possible design domain. Design and non-design areas are intentionally separated considering the joining or manufacturing requirements. Finite element (FE) modeling is performed to discretize the continuous geometry and define loads and constraint conditions. Topology optimization is then conducted to obtain the optimal material distribution. CAD re-interpretation is executed next to generate a new geometry based on topology optimization results, considering manufacturability and tooling costs. Size and shape optimization is used to fine tune the local geometry. Finally, necessary performance tests are conducted to validate the final optimal design.

Even using the most efficient optimization methods, however, weight saving is not large enough if only pure steel is used, because of the ever stringent weight reduction requirements. It has therefore become an inevitable strategy that lightweight materials such as aluminum or magnesium are used more, despite their high costs. Reference [4] gave a direct cost of weight savings as aluminum and advanced high strength steel (AHSS) added to a pickup truck and as more aluminum is applied, the cost will increase rather nonlinearly. Another important aspect is that as the use of multiple materials increases, the complexity of the design increases, and it becomes more difficult to determine optimum designs.
Conceptual Design Stage

Detailed Design Stage

1) Design space modeling 82.6kg
2) Design and non-design domain separation
3) Loads and constraints modeling
4) Topology optimization 26.7kg
5) CAD reinterpretation
6) Size and shape optimization 21.4kg
7) Durability validation test

Figure 3.1 Structural optimization design process [3]
A CCB functions as a structural support that holds all functional components housed in the cockpit area, including the electronic parts (such as the HVAC system and the central console) and safety parts (such as the knee bolsters and the airbags). In particular, the SCSWS is mounted onto the CCB structure. If the natural frequency of the SCSWS / CCB overlaps with the excited frequency coming from the engine, transmission, or other sources, resonance will happen; this would cause discomfort to the driver because of high-amplitude vibrations transmitting through the steering column to the steering wheel. An important performance requirement for the CCB system is hence to maintain the first vertical and first lateral natural frequencies above prescribed target values such that resonance will not happen.

A CCB is a standard part to be equipped in a vehicle. The production of vehicles is gradually increasing except during the economic recession period from 2008 to 2009. Figure 3.2 shows the production of vehicles from 2001 to 2012 and the total number of productions has passed 80 millions in 2012. The production statistics is referred to ‘Organisation Internationale des Constructeurs d’Automobiles’ (OICA) [5]. Due to the nature of mass production in the automotive industry, every penny saved in cost will make a considerable profit and every pound reduced in weight will have a significant contribution in improving fuel efficiency and reducing carbon emissions of the entire fleet of cars.
One of the important design requirements for CCBs is that it must be designed in consideration of various surrounding components in the cockpit area, in particular the SCSWS. In most cases, however, SCSWS and CCB are designed and manufactured by different suppliers, and the Original Equipment Manufacturer (OEM) collects and controls all intellectual property rights. This is the typical industry practice, and the problem for the CCB designer, as a supplier to its customer (i.e. OEM), is that the designer has to perform the design work without having CAD or FE models of the SCSWS. This is because the OEM is reluctant to share detailed information among suppliers. Typically the CCB designer is only given a few key parameters, such as natural frequencies, locations of attachment points, and joining methods. It is therefore important to have an effective
method that allows the CCB designer to produce optimum designs when only limited
information on SCSWS is available. This chapter discusses a method to tackle this problem
in Section 3.2.

There is no universal principle to design an automotive part in a ‘most economic’ way. The
usual practice in the automotive industry depends on the knowledge and experience of
designers. Numerous trial-and-errors are performed to seek to find a ‘favorable’ design,
rather than the ‘optimum’ design, which meets the performance requirements. One of the
most evident drawbacks is that a considerable amount of time and cost are consumed by
the numerous re-designs and re-analyses. Therefore, structural optimization techniques
have been seen to be applied in the automotive design stage. Yildiz [6] developed a new
optimization approach based on Taguchi’s robust design approach and particle swarm
optimization algorithm and applied it to optimum design of a vehicle part. Yildiz and
Solanki [7] developed a new particle swarm-based optimization method to apply to multi-
objective crashworthiness optimization of a vehicle and demonstrated its effectiveness and
validity. Yildiz [8-10] utilized other heuristic optimization methods like genetic algorithm
or immune algorithm to apply onto the design of vehicle components. Durgun and Yildiz
[11] used Cuckoo search algorithm to perform the structural design optimization of vehicle
components. Craig et al. [12] created a response surface based screening method to
optimize the crashworthiness of an automobile body structure. Sinha [13] conducted
reliability-based multi-objective optimization on the vehicle structural crashworthiness
design considering occupant safety with the weight and the front door velocity as the
objectives. Jansson et al. [14] utilized the response surface methodology and space
mapping technique to optimize the draw-in for an automotive sheet metal part. Yim et al.
constructed approximate functions to optimize the sections properties of a thin wall beam including the area, the area moment of inertia and torsional constant. Baskar et al. [16] used Taguchi method to optimize an automotive door hinge system to improve the robustness of the vertical rigidity. Kim and Park [17] developed equivalent static loads method to consider nonlinear dynamic loads at each time step with the generation of the same response field. The method was applied to crashworthiness optimization of an automobile frontal structure under a front impact.

Topology optimization is a powerful tool that determines the optimum design in the conceptual design stage, and from the holistic view point of design, it produces most promising designs that can be used as initial designs in subsequent shape and size optimizations over the design procedure. Topology optimization can help improve the performance of products and also reduce the design period considerably.

Both the theory and application of topology optimization have been advanced in the past 30 years. It is a very useful tool in finding the optimal material distribution within a given design space under prescribed loads and boundary conditions. Many researchers and engineers utilized topology optimization in their work: Yang and Chahande [18] introduced an in-house topology optimization software, TOP, and used it to optimize a simplified truck frame, a deck lid, and a space frame structure. Cavazzuti et al. [19] presented a methodology for automotive chassis design using topology optimization, topometry optimization, and size optimization. Forsberg and Nilsson [20] developed an internal energy density method, avoiding the use of sensitivities, to apply topology optimization in nonlinear problems. Lee and Park [21] developed the equivalent static load method in
nonlinear topology optimization. Wang et al. [22] employed topology optimization and size optimization to improve the overall stiffness of an existing automobile body. Waqas et al. [23] applied topology optimization to the design of aircraft components. Kim and Kwak [24] developed a design space optimization method that extends the fixed design domain problem to an alterable design space by overcoming the numerical discontinuity. Boyle and Kim [25] compared different prosthesis shapes considering micro-level bone remodeling and stress shielding criteria using three-dimensional design space topology optimization. Maute and Frangopol [26] utilized topology optimization to design the micro-electro-mechanical systems. Kim et al. [27] proposed a numerically efficient and effective convergence criterion for topology optimization, and Kim et al. [28] applied topology optimization to vibration problems to maximize damping effect on the structure. Li and Kim [3] achieved an optimum lightweight design of an engine cradle systematically and efficiently by using topology, size, and shape optimization. Yildiz and Saitou [29] developed a topology optimization approach based on genetic algorithm for continuum structures and applied to multi-component topology optimization of a vehicle floor frame. Luo and Tan [30] employed size optimization and topology optimization for the in-wheel motor to decrease the unsprung mass. Kim and Jang [31] performed topology optimization using GENESIS to obtain the optimum layout of the frame structure of a flatbed trailer and then utilized Taguchi method to further minimize the mass. Lee and Lee [32] performed topology optimization on an aluminium control arm of a suspension to get the optimal layout and utilized shape optimization to tune the details of the geometry. Lee and et al. [33] achieved a better design of an automotive door in stiffness and natural frequency by using topology, size and shape optimizations and design of experiments (DOE). Yildiz and
et al. [34] created an initial design concept of an engine mount bracket based on the optimal structural layout from topology optimization. Yildiz [35] performed topology optimization to achieve the optimal design of vehicle components. Jiang and et al. [36] obtained a conceptual and basic design of a Mobile Harbor crane based on topology optimization and shape optimization. Zhu and et al. [37] performed topology optimization on an assembled aircraft structure to avoid the failure of the fastener joints. Oktay and et al. [38] utilized topology optimization to achieve an optimum design of a wing of an unmanned aerial vehicle under aerodynamic loads. Zhu and et al. [39] improved the stiffness and the strength of the stretch-forming die through topology optimization under gravity and surface loads. Bogomolny and Amir [40] achieved an optimal design of a reinforced concrete structure modeled with elastoplastic material by using topology optimization.

Several optimization studies are identified for the design of a CCB. Hamid [41] minimized the weight of a CCB under the constraints of the peak force and the intrusion in the side impact of a car. Lam et al. [42] replaced a steel CCB with an aluminum design, by using size optimization to satisfy the frequency and crashworthiness performance. Only size optimization was used in this study, and the thickness of the aluminum structure had to increase by 40%. Rita [43] discussed important issues in the early phase of a CCB concept development, including the geometry and material choices, assembly solutions, and safety regulations. Rahmani [44] utilized the particle swarm optimization (PSO) algorithm to solve a multidisciplinary optimization problem of an existing aluminum CCB. These studies introduced topology optimization to the design of a CCB in one way or another and produced optimum designs; however, the impact is limited because those studies did not look at the entire design process—including topology, shape, and size optimizations from
conceptual design to detailed design—especially when lightweight materials are to be used. A second important limitation is that manufacturability was not considered in the design process. For example, extrusion is one of the most favorable manufacturing processes in the automotive industry because of its low cost in mass production, but none of the studies considered manufacturing processes, including extrusion. Thirdly, these studies did not discuss how to perform design optimization without having detailed information on the SCSWS, which is a typical situation for the actual CCB designer.

The objective of this chapter is to perform a complete topology, shape, and size optimization of a CCB by using a lightweight material, considering two practical manufacturing processes, and assuming the realistic situation where only limited information on the SCSWS is available to the CCB designer. A steel CCB will be redesigned by using aluminum in order to achieve weight savings, while keeping most of the original geometries and satisfying the natural frequency performance requirements. In the preliminary design stage, topology optimization considering manufacturability will be performed. Two manufacturing process will be compared: extrusion (which has small design freedom for a low cost) versus casting (which has large design freedom for a high cost). Based on the evaluation of weight reductions and manufacturability, a most favorable design will be chosen for the subsequent detailed design stage. A sensitivity study will then be conducted and most influential design variables will be selected. An advance tailored-rolled-tube design will be considered, and size optimization and shape optimization will be conducted to further reduce weight while meeting the performance targets.
3.2 Simplification of the Steering Column and Steering Wheel System (SCSWS)

As aforementioned, an OEM typically outsources many parts to various suppliers, and a supplier typically provides products and services to various OEMs. Because of severe competition and confidentiality, OEMs are reluctant to share detailed technical information with their suppliers. A SCSWS is mounted on the CCB, and one of the most important performance requirements for the CCB is to maintain minimum required natural frequencies of the “steering column.” It is therefore essential for the CCB designer to be able to predict the behavior of the SCSWS during the design optimization of the CCB. The SCSWS is, however, normally designed and built by a different supplier, and its detailed information (including the FE model) is not passed along to the company that designs the CCB. This is the first challenge that a CCB designer faces, and this section discusses how to build an approximate SCSWS model, only based on the available information.

The OEM typically provides the supplier (i.e. CCB designer) with the first natural frequency (\( \omega_1 \)) and second natural frequency (\( \omega_2 \)) of the “actual” SCSWS and its mounting locations on the CCB. Here, \( \omega_1 \) and \( \omega_2 \) are associated with the first vertical mode shape and the first lateral mode shape of the SCSWS, respectively. Note that we intentionally show only a generic representation \( \omega_1 \) and \( \omega_2 \) in this chapter, instead of their actual numerical values, due to confidentiality. In all actual analysis and optimization, however, the actual numerical values were used.

We built a simplified FE model of the SCSWS by using beam elements, spring elements, rigid elements, and concentrated mass elements, as shown in Figure 3.3. Spring and concentrated mass elements were used to simulate the steering wheel; beam elements were
used to simulate the steering column; and the rigid elements (triangles in the figure) were used to represent rigid connections between the SCSWS and the CCB.

The next step was to solve an inverse problem: finding the optimum geometrical parameters of the simplified SCSWS so that the first and second natural frequencies of the steering column become as close to $\omega_1$ and $\omega_2$ as possible, respectively. This is an optimization problem for model calibration, and the design variables were lengths and diameters of the beam elements, stiffness values of the spring elements, and the magnitude of the concentrated mass element. The optimization resulted in an SCSWS model whose first and second natural frequencies are very close to $\omega_1$ and $\omega_2$, where $\omega_1$ is the true first natural frequency and $\omega_2$ is the true second natural frequency of the “actual” SCSWS. The optimization converged after 26 iterations.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Mode Shape</th>
<th>Targets (Hz)</th>
<th>Error (%) of Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1st Vertical Freq.</td>
<td>$\omega_1$</td>
<td>0.082</td>
</tr>
<tr>
<td>2nd</td>
<td>1st Horizontal Freq.</td>
<td>$\omega_2$</td>
<td>0.101</td>
</tr>
</tbody>
</table>

Table 3.1 shows the differences between the computed natural frequencies of the calibrated SCSWS model and the target values ($\omega_1$ and $\omega_2$): the differences were only 0.082% for $\omega_1$ and 0.101% for $\omega_2$. Note that all these values are for the SCSWS only, and in the next section, we will consider natural frequencies of the SCSWS and CCB assembly.
Figure 3.3 Simplified finite element model of the steering column and steering wheel system (SCSWS). The spring elements and concentrated mass element in the circle represent the steering wheel.
Table 3.1 Simplified SCSWS

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Mode Shape</th>
<th>Targets (Hz)</th>
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<td>1st Horizontal Freq.</td>
<td>$\omega_2$</td>
<td>0.101</td>
</tr>
</tbody>
</table>

3.3 Mathematical Problem Statement of CCB Design Optimization

We mounted the calibrated SCSWS model from the previous section onto the conventional CCB finite element model, which will be used as the initial design for optimization described in this section. A numerical modal analysis of the entire assembly of the calibrated SCSWS model and the conventional CCB model determined the first vertical frequency and the first lateral frequency, which are denoted as $F_1$ and $F_2$, respectively. The used generic representation $F_1$ and $F_2$, instead of their actual numerical values, due to confidentiality. These two values are constants and will be used as minimum (or lower) bounds for all future optimization.

The design optimization problem for the entire design process of conceptual design and detailed design is mathematically stated as
minimize \( \sum_{i=1}^{m} \rho_i v_i + \sum_{j=1}^{r} \Delta w_j(T_j) + \sum_{k=1}^{q} \Delta w_k(S_k) \)

subject to

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} &= 0 \\
f_1 &\geq F_1 \\
f_2 &\geq F_2 \\
0 &< \rho_{\text{min}} \leq \rho_i \leq 1 & i &= 1, 2, \ldots, m \\
T_j^L &\leq T_j \leq T_j^U & j &= 1, 2, \ldots, r \\
S_k^L &\leq S_k \leq S_k^U & k &= 1, 2, \ldots, q
\end{align*}
\]  

(3.1)

where the objective is to minimize the weight of the structure. \( \sum_{i=1}^{m} \rho_i v_i \) represents the weight from topology optimization, \( \sum_{j=1}^{r} \Delta w_j(T_j) \) represents the weight change of the specific parts from size optimization based on the topology optimization result, and \( \sum_{k=1}^{q} \Delta w_k(S_k) \) represents the weight change of the specific parts from shape optimization based on the topology optimization result. \( \rho_i \) denotes the relative density of the \( i \)-th design variable for topology optimization and \( v_i \) is the volume of the \( i \)-th element; \( T_j \) represents the \( j \)-th design variable for size (thickness) optimization; and \( S_k \) represents the \( k \)-th design variable for shape optimization. \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the global mass matrix, global damping matrix, and global stiffness matrix, respectively. \( \ddot{\mathbf{u}} \), \( \dot{\mathbf{u}} \) and \( \mathbf{u} \) denote the nodal acceleration vector, the nodal velocity vector, and the nodal displacement vector, respectively. \( f_1 \) indicates the first vertical frequency of the SCSWS / CCB assembly that will be examined during optimization, and \( F_1 \) is its lower bound. \( f_2 \) indicates the first lateral frequency of the SCSWS / CCB assembly that will be examined during optimization, and \( F_2 \) is its lower bound. \( \rho_{\text{min}} \) is the minimum allowable bound for the density, which is implemented in
order to avoid numerical difficulties. \( m, r, q \) are the total number of design variables for topology optimization, size optimization, and shape optimization, respectively. \((T^i, S^i)\) and \((T^u, S^u)\) are the lower bounds and upper bounds for the corresponding size and shape design variables, respectively.

### 3.4 Topology Optimization for CCB

Figure 3.4 shows two manufacturing methods and their respective possible design domains that were identified in consultation with the OEM, for future assembly process and estimated costs. Figure 3.4 (a) shows the extrusion scenario: four extruded parts within four design domains (in red) are to be attached to a circular hollow tube (in blue). Figure 3.4 (b) represents the casting scenario: a casted part within a design domain (in red) is to be mounted on the circular hollow tube; the two small corners in blue are a non-design domain, and they must remain unchanged during optimization. All other parts in grey are inherited from the conventional steel CCB design. Note that the right-side portion (i.e. passenger side) is hidden in the figure due to confidentiality, although it was fully modelled and considered in all numerical analyses and optimizations. Figure 3.5 shows detailed design domains for topology optimization of the two manufacturing scenarios.

Figure 3.6 shows a complete assembly where the simplified SCSWS is mounted on the aluminum CCB with the extrusion process. The FE model for the extrusion design was meshed with 538,127 elements and 568,677 nodes, and for casting design, the FE model contained 202,166 elements and 72,715 nodes.
Figure 3.4 Two manufacturing processes and their respective possible design domains of a CCB. The right-side portion (i.e. passenger side) is hidden in the figure due to confidentiality, but it was fully modelled and considered in all numerical analyses and optimizations.
a) Design space of the extrusion design for topology optimization

b) Design space of the casting design for topology optimization

Figure 3.5 Design space for topology optimization
It is important to note that the SCSWS / CCB assembly is not fixed on a rigid foundation, but it is mounted onto the body-in-white (BIW) structure at 15 attachment points. Figure 3.7 shows the attachment points. Only 11 points appear in the figure, because the other 4 attachment points are located on the right-side portion, which is hidden. In order to accurately consider mechanical characteristics of the connection points, we added bushing elements with zero length at the mounting points. The properties of the bushing elements were determined by physical tests conducted by the OEM.
Figure 3.7 Locations of the CCB attachment points to its surrounding environment (i.e. body-in-white structure)

The problem statement of the topology optimization for this problem formulated as:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{m} \rho_i v_i \\
\text{subject to} & \quad M\ddot{u} + C\dot{u} + Ku = 0 \\
& \quad f_1 \geq F_1 \\
& \quad f_2 \geq F_2 \\
& \quad 0 < \rho_{\min} \leq \rho_i \leq 1 \quad i = 1, 2, \ldots, m
\end{align*}
\] (3.2)

where the objective of topology optimization is to minimize the weight in terms of the density of each element. \(\rho_i\) denotes the relative density of the \(i\)-th design variable for topology optimization \(v_i\) is the volume of the \(i\)-th element; \(M\), \(C\), and \(K\) are the global mass
matrix, global damping matrix and global stiffness matrix. \( \ddot{u}, \dot{u} \) and \( u \) denote the nodal acceleration vector, the nodal velocity vector, and the nodal displacement vector, separately. \( f_1 \) indicates the first vertical frequency of the SCSWS / CCB assembly that will be examined during optimization, and \( F_1 \) is its lower bound. \( f_2 \) indicates the first lateral frequency of the SCSWS / CCB assembly that will be examined during optimization, and \( F_2 \) is its lower bound. \( \rho_{\text{min}} \) is the minimum allowable bound for the density, which is implemented in order to avoid numerical difficulties, and \( m \) is the total number of density design variables.

With respect to the topology optimization, two major constraints are the 1st order natural frequency of the vertical mode shape and the 1st order natural frequency of the lateral mode shape, as stated in Eq. (2). In addition, we also considered various numerical issues, the penalty factor, checkerboard control, minimum member size control, and manufacturability. Topology optimization was conducted using Altair OptiStruct11.0 [45]. Topology optimization results highly depend on the values of two control parameters: the penalty factor and the checkerboard control factor. An appropriate penalty factor can effectively remove elements with intermediate density values which are not practical from the view point of manufacturing. A rational sensitivity filtering factor can restrain emergence of checkerboard patterns and also help keep thin structural members which are efficient from the view point of load transfer. In OptiStruct, minimum member size control is a technique to keep such thin structural members [46]. In addition, we implemented “extrusion manufacturing constraint” for the topology optimization with the extrusion scenario (Figure 3.5 (a)).
Another aspect to consider in topology optimization for extrusion is the degree of design freedom. One possibility is that the four design domains in Figure 3.5 (a) can have four different optimum designs (i.e., 4 free designs), and the second is to produce only two optimum designs so that the two domains facing each other along the axis of the tube must have the same design (i.e. 2 pairs of free designs). Although it is expected the “4 free designs” option will generate better performance due to its larger design freedom, the “2 pairs of free design” option can reduce tooling costs and maintenance costs.

Table 3.2 shows input parameters for topology optimization with four scenarios. All three cases in Case A are for extrusion with different options in parameter control and design freedom, and Case B is for casting.

<table>
<thead>
<tr>
<th>Description</th>
<th>Case A-1</th>
<th>Case A-2</th>
<th>Case A-3</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Control</td>
<td>No</td>
<td>Optimum</td>
<td>Optimum</td>
<td>Optimum</td>
</tr>
<tr>
<td>Penalty factor</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Checkerboard control</td>
<td>No</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum member size control</td>
<td>No</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Manufacturability constraint</td>
<td>4 free designs</td>
<td>4 free designs</td>
<td>2 pairs of free designs</td>
<td>1 free design</td>
</tr>
</tbody>
</table>
Figure 3.8 shows the topology optimization for Case A-1 and Case A-2. It is clear that the optimal topology layout from Case A-2 is more favorable because it has less intermediate-density elements, clearer configuration, and better manufacturability. Topology optimization of Case A-2 was conducted on a Windows PC (AMD Phenom II x6 1090T Processor, 3200MHz, 7392 MB RAM, 6 cores), with the CPU time of 57 minutes and 18 seconds. Six iteration snapshots during the entire optimization history of Case A-2 are shown in Figure 3.9.

The topology optimization result of the “2 pairs of free designs” option with proper parameter control (i.e. Case A-3) is shown in Figure 3.10. Six iteration snapshots during the entire optimization history are shown in Figure 3.11, and the history of the objective function (mass within the design space) is presented in Figure 3.12.
a) Without control parameters (Case A-1)  

b) With control parameters (Case A-2)

Figure 3.8 Topology results comparison
Figure 3.9 Topology optimization history for Case A-2 (only 6 iteration snapshots shown here)
Figure 3.10 Topology result with identical parts (Case A-3)
Figure 3.11 Topology optimization history for Case A-3 (only 6 iteration snapshots shown here)
Figure 3.12 The history of the objective function during the topology optimization (Case A-3)

Topology optimization for the casting version adopted the same control parameters as Cases A-2 and A-3 except for the manufacturability constraint, and the topology optimization result is shown in Figure 3.13. The weight reduction of the entire aluminum model compared to the original conventional steel design is listed in Table 3.3.
Figure 3.13 Topology Result (Case B)
Table 3.3 Topology optimization results for four scenarios. The baseline design is the conventional steel-based design, and is not a result of topology optimization.

<table>
<thead>
<tr>
<th>Description</th>
<th>Baseline</th>
<th>Case A-1</th>
<th>Case A-2</th>
<th>Case A-3</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steel (conventional)</td>
<td>Extrusion (4 free designs)</td>
<td>Extrusion (4 free designs)</td>
<td>Extrusion (2 pairs of free designs)</td>
<td>Casting (1 free design)</td>
</tr>
<tr>
<td>Parameter Control</td>
<td>No</td>
<td>Optimum</td>
<td>Optimum</td>
<td>Optimum</td>
<td>Optimum</td>
</tr>
<tr>
<td>Initial Mass (kg)</td>
<td>12.298</td>
<td>8.072</td>
<td>8.072</td>
<td>8.072</td>
<td>19.966</td>
</tr>
<tr>
<td>Mass after Topology Optimization (kg)</td>
<td>N/A</td>
<td>7.718</td>
<td>7.864</td>
<td>7.880</td>
<td>7.544</td>
</tr>
<tr>
<td>Weight Drop % (Compared to Steel design)</td>
<td>N/A</td>
<td>-37.24</td>
<td>-36.05</td>
<td>-35.92</td>
<td>-38.66</td>
</tr>
</tbody>
</table>
Although the optimum design of Case B (casting version) has a slightly lower mass than the optimum designs of all three Case A’s (extrusion version), the higher cost of the casting process more than offsets its slight advantage. Considering manufacturing costs (including tooling) and weight reduction, we concluded that Case A-3 (extrusion with 2 pairs of free designs under optimum parameter control) is the best design.

Even the best topology optimization results have some intermediate densities and sometimes checkerboard pattern, and therefore we cannot use the raw results without performing results re-interpretation. Based on the optimum design of Case A-3 (Fig 3.11), we implemented a density threshold of 0.7, “cleaned up” the geometry, and built a CAD model. The result is shown in Figure 3.14, and this will be used as the base design in the detailed design phase in Section 3.5.

Figure 3.14 New geometry after implementing a threshold of 0.7 and conducting results re-interpretation
3.5 Size and Shape Optimization for Detailed Design

In this section, we performed simultaneous size (i.e. thickness) optimization and shape optimization based on the result from previous section (Figure 3.14). The first step was to identify important and practical design parameters for size optimization, which will determine optimum thicknesses of the stamping parts. There are total 11 thickness design variables that can be modified, as shown in Figure 3.15.

Instead of using all 11 thickness values as design variables, we studied the influence of each thickness design variable on the performance, in order to down select most important thickness variables. A design of experiment (DOE) study was performed to
calculate the effect of each thickness design variable change on the natural frequency performance. The Latin HyperCube was adopted as the sampling method for the DOE study. We chose 100 samples for each thickness design variable between the range of 1.5mm and 10mm. Therefore, there were 100 runs in total and the statistical results, including the mean, the standard deviation, the absolute ratio of standard deviation to the mean, and the variance, are listed in

Table 3.4.
### Table 3.4 Statistical results of the DOE study

<table>
<thead>
<tr>
<th>Response/Size variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Abs(Standard Deviation/Mean)</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>10.1035</td>
<td>1.2050</td>
<td>0.0544</td>
<td>1.4521</td>
</tr>
<tr>
<td>First Vertical Freq.</td>
<td>38.8865</td>
<td>1.3493</td>
<td>0.0347</td>
<td>1.8207</td>
</tr>
<tr>
<td>First Lateral Freq.</td>
<td>42.5101</td>
<td>0.9839</td>
<td>0.0231</td>
<td>0.9680</td>
</tr>
<tr>
<td>BraceCenter</td>
<td>5.7521</td>
<td>2.4683</td>
<td>0.4291</td>
<td>6.0924</td>
</tr>
<tr>
<td>ColBkt1</td>
<td>5.7521</td>
<td>2.4666</td>
<td>0.4288</td>
<td>6.0843</td>
</tr>
<tr>
<td>ColBkt2</td>
<td>5.7521</td>
<td>2.4651</td>
<td>0.4286</td>
<td>6.0766</td>
</tr>
<tr>
<td>CwlSide</td>
<td>5.7521</td>
<td>2.4648</td>
<td>0.4285</td>
<td>6.0751</td>
</tr>
<tr>
<td>CwlTop</td>
<td>5.7521</td>
<td>2.4652</td>
<td>0.4286</td>
<td>6.0771</td>
</tr>
<tr>
<td>CwlTopPass</td>
<td>5.7521</td>
<td>2.4622</td>
<td>0.4280</td>
<td>6.0623</td>
</tr>
<tr>
<td>DrCap</td>
<td>5.7521</td>
<td>2.4653</td>
<td>0.4286</td>
<td>6.0777</td>
</tr>
<tr>
<td>PipeBrk</td>
<td>5.7521</td>
<td>2.4663</td>
<td>0.4288</td>
<td>6.0826</td>
</tr>
<tr>
<td>PipeCL</td>
<td>5.7521</td>
<td>2.4680</td>
<td>0.4291</td>
<td>6.0912</td>
</tr>
<tr>
<td>TubeDriver</td>
<td>5.7521</td>
<td>2.4622</td>
<td>0.4281</td>
<td>6.0626</td>
</tr>
<tr>
<td>TubePas</td>
<td>5.7521</td>
<td>2.4670</td>
<td>0.4289</td>
<td>6.0861</td>
</tr>
</tbody>
</table>
The quantitative effects of each thickness design variable on the frequency performance are shown in Figure 3.16 (a) and (b). A total of 5 design variables were identified that have significant effect on the first vertical frequency or first lateral frequency: BraceCenter, CwlSide, DrCap, PipeBrk and TubeDriver. The other 6 design variables were given predetermined thickness values and designated as fixed parameters.

Figure 3.16 (c) shows that one of the 5 design variables, TuberDriver, has the greatest influence on mass, and we in consultation with the industry partner decided to build this tube using the tailored-welded-tubes technique. For this, TubeDriver was partitioned into 6 pieces of welded tubes, and each piece can have an independent thickness design variable. Thus, the thickness optimization of the CCB was defined with 10 design variables, as shown in Figure 3.17.
(a) Significant effect of each thickness design variables to the first vertical natural frequency

(b) Significant effect of each thickness design variables to the first lateral natural frequency
(c) Significant effect of each thickness design variables to the mass

Figure 3.16 Significant effects of thickness design variables to the responses
Figure 3.17 Final 10 size (thickness) design variables
Figure 3.18 shows the 9 shape optimization parameters that are defined on the optimum design from topology optimization (Figure 3.14). Note that only two parts are shown in the figure, because the other two will have the same designs.

Figure 3.18 Shape design variables (one side)
Shape optimization uses the perturbation vector approach [47], and the shape design variables were defined as the linear combination of the perturbation vectors as follows:

\[ X = x_0 + \sum_{i=1}^{n_p} PV_i \cdot S_i \]  

(3.3)

where \( X \) is the new vector of nodal coordinates, \( x_0 \) is the initial nodal coordinates, \( PV_i \) is the perturbation vector associated with the \( i \)-th shape design variable \( S_i \). \( n_s \) is the total number of perturbation vectors and shape design variables. The perturbation vectors in this research were all set at 3mm, and all shape design variables were in the domain of \([-1, 1]\).

The size and shape optimization was then formulated as

\[
\begin{align*}
\text{minimize} & & W_0 + \sum_{i=1}^{10} \Delta w_i (T_i) + \sum_{j=1}^{9} \Delta w_j (S_j) \\
\text{subject to} & & M \ddot{u} + C \dot{u} + K u = 0 \\
& & f_1 \geq F_1 \\
& & f_2 \geq F_2 \\
& & 1.2 \text{mm} \leq T_i \leq 10 \text{mm}, \quad i = 1, 2, \ldots, 10 \\
& & -1 \leq S_j \leq 1, \quad j = 1, 2, \ldots, 9 
\end{align*}
\]  

(3.4)

where the objective is still to minimize the weight of the structure; \( W_0 \) represents the weight after geometry reinterpretation, which is the input model for shape and size optimization. \( \sum_{i=1}^{10} \Delta w_i (T_i) \) is the weight change of the specific parts from size optimization and \( \sum_{j=1}^{9} \Delta w_j (S_j) \) is the weight change of the specific parts from shape optimization. \( M, C, \) and \( K \) are the global mass matrix, global damping matrix, and global stiffness matrix, respectively. \( \ddot{u}, \)
\( \ddot{u} \) and \( u \) denote the nodal acceleration vector, the nodal velocity vector, and the nodal displacement vector, respectively. \( f_1 \) indicates the first vertical frequency of the SCSWS / CCB assembly that will be examined during optimization, and \( F_1 \) is its lower bound. \( f_2 \) indicates the first lateral frequency of the SCSWS / CCB assembly that will be examined during optimization, and \( F_2 \) is its lower bound. \( T_i \) is the \( i \)-th size (thickness) optimization design variable, and \( S_j \) is the \( j \)-th shape optimization design variable.

For shape and size optimization, two major constraints for satisfying the performance requirements are the 1st order natural frequency of the vertical mode shape, and the 1st order natural frequency of the lateral mode shape, as stated in Eq. (4). This optimization was conducted on Windows PC (AMD Phenom II x6 1090T Processor, 3200MHz, 7392 MB RAM, 6 cores). After 4 iterations with the CPU time of 11 minutes and 3 seconds, the optimization converged with all constraints satisfied. The optimization history of shape and size optimization is shown in Figure 3.19. The results are shown in Table 3.5 and Table 3.6 for size (thickness) design variables and shape design variables, respectively. All the values of all shape design variables were in the order of \( 10^{-6} \) or \( 10^{-5} \), which means the shape changes are negligible; this reveals that the topology optimization results were already very accurate.

The performance of the weight and natural frequencies from all the design stages are listed in Table 3.7.
Figure 3.19 The history of the objective function of the shape and size optimization
### Table 3.5 Size design variables (mm)

<table>
<thead>
<tr>
<th>Thickness Design Variable</th>
<th>Step-1</th>
<th>Step-2</th>
<th>Step-3</th>
<th>Step-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₁</td>
<td>N/A</td>
<td>6.6</td>
<td>6.754</td>
<td>6.8</td>
</tr>
<tr>
<td>T₂</td>
<td>N/A</td>
<td>5.3</td>
<td>4.408</td>
<td>4.4</td>
</tr>
<tr>
<td>T₃</td>
<td>N/A</td>
<td>2.4</td>
<td>1.982</td>
<td>2.0</td>
</tr>
<tr>
<td>T₄</td>
<td>N/A</td>
<td>5.6</td>
<td>5.233</td>
<td>5.2</td>
</tr>
<tr>
<td>T₅</td>
<td>N/A</td>
<td>4.6</td>
<td>3.966</td>
<td>4.0</td>
</tr>
<tr>
<td>T₆</td>
<td>N/A</td>
<td>4.6</td>
<td>3.961</td>
<td>4.0</td>
</tr>
<tr>
<td>T₇</td>
<td>N/A</td>
<td>4.6</td>
<td>7.930</td>
<td>8.0</td>
</tr>
<tr>
<td>T₈</td>
<td>N/A</td>
<td>4.6</td>
<td>2.435</td>
<td>2.4</td>
</tr>
<tr>
<td>T₉</td>
<td>N/A</td>
<td>4.6</td>
<td>9.356</td>
<td>9.4</td>
</tr>
<tr>
<td>T₁₀</td>
<td>N/A</td>
<td>4.6</td>
<td>8.988</td>
<td>9.0</td>
</tr>
</tbody>
</table>
Table 3.6 Shape design variables

<table>
<thead>
<tr>
<th>Shape Design Variable</th>
<th>Step-1</th>
<th>Step-2</th>
<th>Step-3</th>
<th>Step-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Design (Steel Version)</td>
<td>Topology Optimization (based on Case A-3)</td>
<td>Size and Shape Optimization (based on Step-2)</td>
<td>Round off Thickness (based on Step-3)</td>
</tr>
<tr>
<td>S₁</td>
<td>N/A</td>
<td>N/A</td>
<td>5.3593e-05</td>
<td>0</td>
</tr>
<tr>
<td>S₂</td>
<td>N/A</td>
<td>N/A</td>
<td>-5.9839e-05</td>
<td>0</td>
</tr>
<tr>
<td>S₃</td>
<td>N/A</td>
<td>N/A</td>
<td>-1.8282e-05</td>
<td>0</td>
</tr>
<tr>
<td>S₄</td>
<td>N/A</td>
<td>N/A</td>
<td>-1.8627-05</td>
<td>0</td>
</tr>
<tr>
<td>S₅</td>
<td>N/A</td>
<td>N/A</td>
<td>-6.9887-06</td>
<td>0</td>
</tr>
<tr>
<td>S₆</td>
<td>N/A</td>
<td>N/A</td>
<td>-8.2426-06</td>
<td>0</td>
</tr>
<tr>
<td>S₇</td>
<td>N/A</td>
<td>N/A</td>
<td>-1.6543e-05</td>
<td>0</td>
</tr>
<tr>
<td>S₈</td>
<td>N/A</td>
<td>N/A</td>
<td>-6.8542e-06</td>
<td>0</td>
</tr>
<tr>
<td>S₉</td>
<td>N/A</td>
<td>N/A</td>
<td>-7.4599e-06</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3.7 CCB performance comparison through the entire design stage

<table>
<thead>
<tr>
<th>Design Phase</th>
<th>Step-1</th>
<th>Step-2</th>
<th>Step-3</th>
<th>Step-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Design (Steel)</td>
<td>12.298</td>
<td>7.880</td>
<td>7.424</td>
<td>7.427</td>
</tr>
<tr>
<td>Size and Shape Optimization (based on Step-2 result)</td>
<td>N/A</td>
<td>0.42</td>
<td>-0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td>Round off Thickness (based on Step-3 result)</td>
<td>N/A</td>
<td>-0.92</td>
<td>-0.09</td>
<td>-0.07</td>
</tr>
</tbody>
</table>
3.6 Conclusions

From the view point of lightweight strategies, this research utilized all three approaches simultaneously: lightweight material, structural optimization, and advanced manufacturing process to solve a real engineering problem. A completed design optimization of an aluminum CCB from an already well-designed steel CCB by using topology, shape, and size optimization for weight reduction was discussed explicitly. The final design had a 39.61% of weight savings compared to the baseline steel design.

Finite element analyses and optimization was conducted in order to build and calibrate an approximate SCSWS model that represents the behavior of a real SCSWS with reasonable accuracy. This approximation process can effectively address the typical challenge that CCB supplier frequently faces: design optimization of a CCB needs to be performed, when detailed information on the SCSWS is not available.

The entire optimization process in this research achieved significant weight savings; moreover, the optimum design retained most of the original geometric parameters from the conventional steel design with no changes. This means that time and cost for redesigning the production line, the fabrication procedure, the assembly procedure, and packaging could be saved significantly.

The initial mass of the aluminum extrusion design was 8.03kg; and topology, size, and shape optimization reduced the mass to 7.43kg. The cost of aluminum is estimated to be $2.2 per pound [48]. Therefore, the design optimization in this research saved material cost by $2.91 per a unit of CCB with aluminum features. The number of units manufactured in
the automotive industry is very high—for example, hundreds of thousands—therefore, cost savings due to the optimization would be substantial.

The entire design process of the aluminum CCB is shown in Figure 3.20. We maximized the benefit of design optimization by doing topology optimization in the conceptual design stage and shape/size optimization in the following detailed design stage. Two manufacturing processes, extrusion and casting, were considered with various optimization control parameters and design freedom options. All the process was conducted based on the assumption that only limited information on the SCSWS is available, which is the typical situation for the actual CCB designer. For this, a simplified FE model of the SCSWS was built and calibrated.

It should be also noted that the use of topology, shape, and size optimization for the CCB system development also allowed for considerable savings in design time and associated costs. Our industrial partner has benefited greatly from the shortened design cycle. In addition, the geometry layout resulted from topology optimization is more objective and robust than that from the conventional trial-and-error. The methodology discussed in this chapter could also be spread to the development of other automotive parts like engine cradles or control arms.

3.7 Limitations

The limitation of this study is that crashworthiness of this CCB assembly was not considered due to two major challenges. First, crashworthiness of a separate individual assembly (in this research, cross car beam) could not truly reflect its deformation and energy absorption behavior since there are strong interactions among components and
assemblies. This is the key reason why crashworthiness is mainly considered by the OEM. Second, crashworthiness analysis of an automotive assembly involves geometry-nonlinearity, material-nonlinearity and contact-nonlinearity. To include crashworthiness in an optimization process is extremely time-consuming. Specially, topology optimization for solving highly nonlinear problem is not well developed yet. Some techniques, such as the equivalent static load method (ESLM), could linearize the nonlinear behavior, and have a potential to improve the computational efficiency. In the future, after the OEM performed crash test on a vehicle level, the feedback will be incorporated back into the model to further improve the comprehensive performance of the CCB.
Figure 3.20 Complete design optimization procedure of a CCB
ACKNOWLEDGEMENTS

The authors would like to thank the CAE group from Kirchhoff Van-Rob Inc. Their suggestions are acknowledged and much appreciated.

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Chapter 4

Two-material Topology Optimization Based on Optimality Condition

This chapter conducted mathematical sensitivity formulations of two-material topology optimization based on the adjoint variable method, and applied it to six structural optimization problems. Commercial software programs were used for finite element analysis including pre- and post-processing, and for multi-material topology optimization, an in-house code was developed. Three simple, classical examples were solved for compliance minimization, and then three case studies were performed by considering various volume fractions. The optimum solutions by two-material topology optimization were compared to optimum solutions determined by the standard single-material topology optimization. All results showed that our two-material topology optimization algorithm and computational tool can determine effective optimum solutions for various problem statements, and optimum designs by the two-material topology optimization have better performance than solutions by one-material topology optimization.

4.1 Introduction

Topology optimization was developed to determine important design features such as the number, location, and shape of holes, and the connectivity of domains [1]. Due to its effectiveness in exploring the design space, topology optimization is widely used for preliminary design, and optimum solutions from topology optimization are then used as baseline designs in subsequent detailed design stages.
The homogenization method is considered as a breakthrough of structural topology optimization because it transforms a complicated discrete form-finding problem (e.g. truss-like discretization) into a material distribution based continuous problem. The fundamental concept of the homogenization method is to determine existence of the material within each element of a structure. Cheng and Olhoff [2] first introduced a microstructure to determine an optimum thickness distribution of elastic plates. Bendsøe and Kikuchi [3] first applied the homogenization method to continuum structures with compliance as the objective function and maximum allowable volume as the constraint.

The density method [4] has also developed over the last several decades, and it has been successfully integrated into commercial software programs which can nowadays effectively solve large-scale, real-world engineering topology optimization problems. The density method assumes an explicit mathematical relationship between the relaxed elemental density value and the elasticity tensor. The density method is an effective and efficient method which does not require the homogenization process due to the introduction of microstructures. There are two widely used density-elasticity interpolation models. The first is the Solid Isotropic Material with Penalization (SIMP) model [5], which has been successfully applied to a large number of problems and proven to be efficient and easy to implement. The second is the Rational Approximation of Material Properties (RAMP) model [6]. A penalty factor is implemented into the relation between the element density and the elasticity in order to penalize intermediate densities, which drives the optimization process to converge to the density value of either zero or one. During the past three decades, tremendous progress has been made in the development of theories, various algorithms, and application of these methods to a variety of real-world engineering and scientific
problems. Kim and his colleagues developed topology optimization based on the genetic
algorithms [7], proposed a numerically efficient convergence criterion which significantly
reduces computing cost [8], and solved a real-world automotive chassis design problem
considering manufacturability [9]. Li and Kim also performed conceptual and detailed
designs by using topology, shape, and size optimization [10-11].

With the rising cost of natural resources and concerns about the environment, energy
conservation and protection of the environment has become a very important issue
worldwide nowadays. Both governments and customers demand more energy-efficient,
cost-effective, and environmentally friendly products. As an example, the Corporate
Average Fuel Economy (CAFE) regulations in the United States require the average fuel
economy of cars and light trucks produced by an automaker be 54.5 mpg (mile per US
gallon) by Model Year 2025. Similarly, there is a tremendous demand for improved fuel
economy in the aerospace and rail industry as well. Reducing the weight of a vehicle is a
most effective and promising solution that improves fuel efficiency and decreases carbon
emissions.

There are three mainstream strategies for reducing weight: The first is to use lightweight
materials. The traditional low carbon steel is replaced by lightweight materials, such as
high strength steel, aluminum, magnesium, and plastic composites. Different material
properties—in terms of stiffness, strength, durability, corrosion, and formability—and cost
should be carefully taken into consideration. In the automotive industry, traditional
materials like low carbon steel and iron have been gradually replaced by various
lightweight materials over past decades [12]. There are however disadvantages in the use of lightweight materials, and one of the major concerns is higher material cost [13].

The second strategy is to develop advanced manufacturing processes. For example, the use of structural adhesives or overcasting can reduce the weight of mechanical joints, and if MIG welding is replaced by laser welding, there is no need for solder, and weight can be reduced. As the third strategy for weight reduction, various design optimization methods can be used in the design process. Design optimization determines optimum geometric features that minimize weight while satisfying important performance requirements. Topology optimization finds most efficient structural layouts from the view point of weight and structural integrity in conceptual and preliminary design stages. Size and shape optimization can fine-tune optimum designs in the detailed design stage: for example, it can determine optimum design parameters of tailored-welded-blanks and tailored-rolled-tubes. It should be noted that design optimization alone—based on the traditional material—does not achieve enough weight savings, given that energy efficiency regulations become ever more stringent.

In order to maximize weight savings, those three lightweight approaches are adopted and used altogether nowadays, and this introduces new challenges for design optimization. If one or multiple lightweight materials are added to an automotive structure with traditional low carbon steel as an example, the design work now becomes a multi-material optimization problem. Therefore engineering designers have to deal with a much more complex optimization problem, which must determine an optimum placement of multiple materials as well as optimum geometric features of each respective materials. This design
problem has larger design freedom which would potentially allow for better optimum solutions, but at the same time, it is more difficult to obtain optimum solutions because of increased complexity, larger design space, and stronger interactions among various types of design parameters. Topology optimization has been traditionally used for the design with a single material, which basically determines the existence of material within each design cell for a prescribed material type. Currently available topology optimization theories and methods for this type of single material design are ill equipped for design problems that consider multiple materials.

A limited number of studies are found in multi-material topology optimization. Ramani [14] calculated a pseudo-sensitivity of each element based on the change of the objective and constraint functions over consecutive iterations, keeping the design variables as discrete parameters. Huang and Xie [15] proposed a ‘sensitivity number’ ranking scheme based on their bi-directional evolutionary topology optimization method. Saxena [16] used the genetic algorithm for multi-material topology optimization, which allowed for a binary representation of the design variables. Han and Lee [17] utilized the evolutionary topology optimization and changed the material type of elements based on strain energy values. Stegmann and Lund [18] expressed the element constitutive matrix as a weighted sum of constitutive matrices of candidate materials, and they solved composite shell structures using the approach. Yin and Ananthasuresh [19] used normal distribution functions to create a material interpolation model. Sensitivity calculation procedures in all these studies are partially or completely heuristic. This means that computing cost could be potentially very high for real-world problems which have large numbers of design variables. It is also particularly important for those heuristic or semi-heuristic approaches to choose
“optimum” control parameters for simulations, which would require a number of trial-and-errors. Yoon [20] developed a new patch stacking method to solve the multi-material topology optimization based on an element connectivity parameterization (ECP) method. This is a unique idea, but current limitations include use of higher number DOFs (degree-of-freedoms) than the typical FE-based optimization method and checkerboard problems. Stegmann and Erik used [21] a weighted sum interpolation of the element constitutive matrices to perform a topology optimization of general composite laminate shell structures. Gao and Zhang [22] solved structural topology optimization with multiphase material using mass as constraint rather than volume. Thomsen [23] presented a stiffness maximization of a structure which is composed of two isotropic materials by using the homogenization technique. The material of each cell was modeled by a composite using its concentration and orientation as design variables. Because the number of design variables of each cell has been increased to 4, the computing cost of the proposed method is high. All these previous studies typically solved simple two-dimensional classical problems, such as the 2D cantilever or the 2D MBB (Messerschmitt-Bölkow-Blohm) beams, based on the assumption of plane stress and using small numbers of design variables. Applicability and numerical efficiency for solving large-scale and complicated engineering problems have not been proven yet.

The objectives of this chapter are (1) to perform mathematical sensitivity formulation of two-material topology optimization based on the adjoint variable method and (2) to solve 2D and 3D topology optimization problems with two possible materials by using the sensitivity information. All numerical implementation was conducted by using commercial software programs as much as possible, which would allow industry to easily take up the
computational tool and utilize it for real-world problems. In order to verify accuracy and effectiveness of the developed method, we first solved three classical topology optimization problems, by considering two materials: 2D cantilever, half MBB, and 3D cantilever structures. The method was then used to solve three case studies, including a 3D Control Arm problem, which have more complex geometries, design and non-design domains, and different element types. We compared the optimum solutions by the two-material topology optimization with various volume fractions to the optimum solutions by the standard single material optimization.

4.2 Problem Definition

A system of an elastic structure for the classical structural topology optimization problem under a single static load case is graphically represented in Figure 4.1. The design domain \( \Omega \subseteq \{ \mathbb{R}^2, \mathbb{R}^3 \} \) is defined as a 2D or 3D open set, with the essential boundary condition \( \Gamma_u \) and the natural boundary condition \( \Gamma_t \). The design domain can contain permanent void domains (i.e. forbidden zones) and constant, non-designable domains (i.e. passive zones with full of material). The essential boundary condition and the natural boundary condition are mutually exclusive, and constitute the entire boundary (\( \partial \Omega = \Gamma \)): \( \partial \Omega = \Gamma = \Gamma_u \cup \Gamma_t \) and \( \emptyset = \Gamma_u \cap \Gamma_t \).
In this research, we considered topology optimization with mean compliance as the objective function for minimization and volume as the constraint:

\[
\begin{align*}
\text{minimize:} & \quad \int_{\Omega} b \cdot u d\Omega + \int_{\Gamma_t} t \cdot u d\Gamma, \\
\text{subject to:} & \quad \int_{\Omega} \sigma(u) : d(\delta u) d\Omega = \int_{\Omega} b \cdot \delta u d\Omega + \int_{\Gamma_t} t \cdot \delta u d\Gamma, \quad \text{for all} \ \delta u \in u_0, \\
& \quad \int_{\Omega} \rho d\Omega \leq V, \\
& \quad 0 \leq \rho_{\text{min}} \leq \rho \leq 1.
\end{align*}
\]

where \( \mathbf{u} \) is the displacement vector, \( \mathbf{b} \) indicates the body force vector, and \( \mathbf{t} \) denotes the traction vector. \( \sigma(u) \) represents the Cauchy stress tensor in terms of the displacement vector.
$u, \delta u$ is the virtual displacement, and $u_0$ is the kinematically admissible space. $\rho$ is the normalized density field, which is used as the design variable, and $\rho_{\min}$ is its lower bound. $d(\delta u)$ is the symmetric part of $\text{grad}(\delta u)$ and can be derived as:

$$\sigma(u) : \text{grad}(\delta u) = \sigma(u) : \text{symm}(\text{grad}(\delta u)) \quad (4.2)$$

$$\text{symm}(\text{grad}(\delta u)) = \frac{1}{2}(\text{grad}(\delta u) + \text{grad}^T(\delta u)) = \frac{1}{2}(\frac{\partial(\delta u_i)}{\partial x_j} + \frac{\partial(\delta u_j)}{\partial x_i}) = d(\delta u) \quad (4.3)$$

For linear elastic structures, the constitutive equation is $\sigma = C(\rho) : \varepsilon(u)$, where $\sigma$ is the Cauchy stress tensor, $\varepsilon(u)$ is the Euler-Almansi strain tensor, and $C(\rho)$ is the 4th-order elasticity tensor.

Topology optimization ultimately determines the existence of material within each design cell, and it has a binary (0-1 or void-solid) form of design variables. It is hard to solve this type of discrete problems numerically, and therefore it is relaxed to a continuous problem by introducing an interpolation scheme. We used the SIMP model which has been proved to be effective for many real-world applications:

$$C(\rho) = \rho^n C^0 \quad (4.4)$$

where $C(\rho)$ is the (variable) elasticity tensor, whose value changes according to the normalized density $\rho$. $C^0$ is the (constant) elasticity tensor of the base material. The
exponent $p \ (p > 1)$ is the penalty factor, and it drives the optimization to converge on either zero value of design variables or one, by penalizing intermediate density values.

4.3 Theory of Two-Material Topology Optimization based on Optimality Condition

4.3.1 Interpolation function with two materials

In this chapter, we considered two different types of materials for structural topology optimization. The interpolation function based on the SIMP model for bi-material topology optimization is formulated as:

$$C_{ijkl}(\rho_1, \rho_2) = \rho_1^p \rho_2^p C_{ijkl}^1 + (1 - \rho_1^p) \rho_2^p C_{ijkl}^2$$  \hspace{1cm} (4.5)

where $C_{ijkl}(\rho_1, \rho_2)$ is the index representation of the (variable) elasticity tensor, whose value changes according to the normalized densities $\rho_1$ and $\rho_2$. The first normalized density $\rho_1$ represents existence of material within each design cell; and the second one $\rho_2$ is used to represent the portion of Material 1 with respect to Material 2. Further explanation is given in Section 4.3.2. $C_{ijkl}^1$ and $C_{ijkl}^2$ are the index representations of the (constant) elasticity tensors of the Material 1 and Material 2, respectively. The exponent $p$ is the penalty factor, and it performs the same role as in the case of single-material topology optimization in Equation (4.4). Figure 4.2 shows an example of the relation between Young’s modulus and the two normalized density values for two different isotropic materials.
Figure 4.2 A representative relationship between Young’s modulus and two normalized density values for two materials ($\rho_1$ : Total Rel. Density; and $\rho_2$ : Rel. Density of Mat-1)
4.3.2 Two-material topology optimization statement of compliance minimization

In order to solve the optimization problem, we use the finite element method, and now bi-material topology optimization in the discrete form may be expressed as

\[
\begin{align*}
\text{minimize} & \quad C(\rho_1, \rho_2) \\
\text{subject to} & \quad K(\rho_1, \rho_2)U = F \\
& \quad \sum_{i=1}^{N} \rho_1^i v^i \leq V_{\text{Total,Max}} \\
& \quad \sum_{i=1}^{N} \rho_2^i v^i \leq V_{1,\text{Max}} \\
& \quad 0 < \rho_{\text{min}} \leq \rho_1^i \leq 1, \quad i = 1, 2, \ldots, i, \ldots, N \\
& \quad 0 \leq \rho_2^i \leq 1, \quad i = 1, 2, \ldots, i, \ldots, N
\end{align*}
\] (4.6)

where $\rho_1^i$ is the normalized density that determines the existence of the material (whether it be Material 1 or Material 2) within the $i$-th design cell, and $\rho_2^i$ is the relative density that determines the portion of Material 1 with respect to Material 2 within the $i$-th design cell. This relative density ($\rho_2^i$) has a valid physical meaning only if there is a material within the $i$-th design cell (i.e. $\rho_1^i = 1$). There are three physically meaningful cases for the $i$-th design cell: (1) $\rho_1^i = 1$ and $\rho_2^i = 1$ means that the cell is completely filled with Material 1; (2) $\rho_1^i = 1$ and $\rho_2^i = 0$ means that the cell is completely filled with Material 2; and (3) $\rho_1^i = 0$ means that the cell is empty. $\rho_1$ and $\rho_2$ are then the vector of the normalized density for material existence and the vector of relative density for the portion of Material 1, respectively. $v^i$ is the volume of the $i$-th design cell. $V_{\text{Total,Max}}$ is the maximum, total allowable volume of Material 1 and Material 2 combined, and $V_{1,\text{Max}}$ is the maximum allowable volume of...
Material 1. $C$ is compliance, which is used as the objective function for minimization. $K$ is the global stiffness matrix, $U$ is the nodal displacement vector, and $F$ is the external force vector. $N$ is the total number of the design variables.

### 4.3.3 Sensitivity derivation of compliance with respect to density variables

Topology optimization typically deals with a very large number of design variables and a small number of functionals (objective function and constraints). The adjoint variable method is hence the most efficient approach for calculating design sensitivities. The design sensitivity by means of the adjoint variable method based on the SIMP model is written as

\[
\frac{\partial C}{\partial \rho_j} = \frac{\partial F^T U}{\partial \rho_j} = \frac{\partial (F^T U - \lambda^T (KU - F))}{\partial \rho_j}
\]

\[
= (F^T - \lambda^T K) \frac{\partial U}{\partial \rho_j} - \lambda^T \frac{\partial K}{\partial \rho_j} U
\]

\[
= -\lambda^T \frac{\partial K}{\partial \rho_j} U \quad \text{when} \quad F^T = \lambda^T K \quad \lambda = U
\]

\[
= -U^T \frac{\partial K}{\partial \rho_j} U \quad i = 1, 2, ..., i, ..., N, \quad j = 1, 2
\]

where $F$ is the external force vector, $U$ is the nodal displacement vector, $K$ is the global stiffness matrix, and $\lambda$ is the adjoint variable vector. $N$ is the total number of the design variables.

The sensitivity expression for each of the two materials, using the adjoint variable method based on the SIMP model, is written as
\[
\frac{\partial C}{\partial \rho_i} = -U^T \frac{\partial K}{\partial \rho_i} U = -u_i^T \frac{\partial k_i}{\partial \rho_i} u_i = -p(\rho_i)^{p-1}[(\rho_i^k)^p u_i^T k_i^1 u_i + (1 - (\rho_i^k)^p) u_i^T k_i^2 u_i] \quad (4.8.1)
\]

\[
\frac{\partial C}{\partial \rho'_i} = -U^T \frac{\partial K}{\partial \rho'_i} U = -u_i^T \frac{\partial k_i}{\partial \rho'_i} u_i = -(\rho_i^p)^p u_i^T k_i^1 u_i - p(\rho_i^p)^{p-1} u_i^T k_i^2 u_i \quad (4.8.2)
\]

where \( K \) is the global element with the interpolated material property, \( k_i \) is the stiffness of the \( i \)-th element (or design cell), \( k_i^1 \) is the element stiffness of the \( i \)-th element with the properties of Material 1, and \( k_i^2 \) is the element stiffness of the \( i \)-th element with the properties Material 2. \( U \) is the global displacement vector, and \( u_i \) is the displacement vector of the \( i \)-th element. (Here, the \( i \)-th element is the same as the \( i \)-th design cell.)

It can be easily seen that \( \frac{\partial C}{\partial \rho_i} \) must be less than or equal to zero. For an actual numerical implementation, we should implement a lower limit \( \rho_{\text{min}} (\rho_{\text{min}} > 0) \) for each of \( \rho_i \) in order to avoid singularity; hence \( \frac{\partial C}{\partial \rho_i} \) is strictly non-positive and finite for non-singular structures.

For \( \frac{\partial C}{\partial \rho'_i} \), the values of \( u_i^T k_i^1 u_i \) and \( u_i^T k_i^2 u_i \) will determine the sign of \( \frac{\partial C}{\partial \rho'_i} \). If

- \( u_i^T k_i^1 u_i \geq u_i^T k_i^2 u_i \), then \( \frac{\partial C}{\partial \rho'_i} \leq 0 \);
- \( u_i^T k_i^1 u_i < u_i^T k_i^2 u_i \), then \( \frac{\partial C}{\partial \rho'_i} \geq 0 \).

For the actual numerical case studies in this chapter, we used steel for Material 1 and aluminum for Material 2; therefore, we had \( u_i^T k_i^1 u_i > u_i^T k_i^2 u_i \), and therefore \( \frac{\partial C}{\partial \rho'_i} \) was negative in all our
case studies. The negativity of \( \frac{\partial C}{\partial \rho_1} \) and \( \frac{\partial C}{\partial \rho_2} \) imply that as either density increases, compliance of the structure decreases which means stiffness of the structure increases.

4.3.4 Update of design variables based on Optimality Condition (OC)

The Lagrangian function for problem stated in Equation (4.6):

\[
L = C(\rho_1, \rho_2) + \Lambda^1 \left( \sum_{i=1}^{N} \rho_1 \rho_2^i v^i - V_i \right) + \Lambda^2 \left( \sum_{i=1}^{N} \rho_1^i v^i - V \right) + \lambda^T (KU - F) + \\
+ \sum_{i=1}^{N} \lambda_1^i (\rho_{\text{min}} - \rho_1^i) + \sum_{i=1}^{N} \lambda_2^i (\rho_1^i - 1) + \sum_{i=1}^{N} \lambda_3^i (0 - \rho_2^i) + \sum_{i=1}^{N} \lambda_4^i (\rho_2^i - 1)
\]  

(4.9)

where \( \Lambda^1, \Lambda^2, \lambda_1^1, \lambda_2^1, \lambda_3^3 \) and \( \lambda_4^4 \) are the Lagrange multipliers.

Sensitivities of the Lagrangian function with respect to \( \rho_1^i \) and \( \rho_2^i \) are derived as

\[
\begin{align*}
\frac{\partial L}{\partial \rho_1^i} &= -U^T \frac{\partial K}{\partial \rho_1^i} U + \Lambda^1 \rho_2^i v^i + \Lambda^2 v^i - \lambda_1^i + \lambda_2^i \\
\frac{\partial L}{\partial \rho_2^i} &= -U^T \frac{\partial K}{\partial \rho_2^i} U + \Lambda^1 \rho_1^i v^i - \lambda_4^i + \lambda_3^i
\end{align*}
\]  

(4.10)

Dual feasibility and complementary slackness conditions are found as

\[
\begin{align*}
\Lambda^1 &\geq 0 \\
\Lambda^1 \left( \sum_{i=1}^{N} \rho_1^i \rho_2^i v^i - V_i \right) &= 0 \\
\Lambda^2 \geq 0 \\
\Lambda^2 \left( \sum_{i=1}^{N} \rho_1^i v^i - V \right) &= 0 \\
\lambda_1^i &\geq 0 \\
\lambda_1^i (\rho_{\text{min}} - \rho_1^i) &= 0 \\
\lambda_2^i &\geq 0 \\
\lambda_2^i (\rho_1^i - 1) &= 0 \\
\lambda_3^i &\geq 0 \\
\lambda_3^i (0 - \rho_2^i) &= 0 \\
\lambda_4^i &\geq 0 \\
\lambda_4^i (\rho_2^i - 1) &= 0 \\
i &= 1, 2, ..., i, ..., N
\end{align*}
\]  

(4.11)
In order to satisfy the stationary condition, we rewrite Equation (4.10) as

\[ \frac{\partial L}{\partial \rho_i} = -U^T \frac{\partial k}{\partial \rho_i} U + \Lambda^1 \rho_i^2 v^i + \Lambda^2 v^i = -u_i^T \frac{\partial k}{\partial \rho_i} u_i + \Lambda^1 \rho_i^2 v^i + \Lambda^2 v^i = 0 \] (4.12.1)

\[ \frac{\partial L}{\partial \rho_2} = -U^T \frac{\partial k}{\partial \rho_2} U + \Lambda^1 \rho_2^2 v^j = -u_i^T \frac{\partial k}{\partial \rho_2} u_i + \Lambda^1 \rho_2^2 v^j = 0 \] (4.12.2)

The update scheme for each of the design variables is then written as

\[ (\rho_i^{(k+1)}) = \begin{cases} \max \left\{ (1 - \varsigma)(\rho_i^{(k)}), \rho_{\min} \right\} & \text{if } (\rho_i^{(k)}) (1, B_k^i)^\eta \leq \max \left\{ (1 - \varsigma)(\rho_i^{(k)}), \rho_{\min} \right\} \\ \min \left\{ (1 + \varsigma)(\rho_i^{(k)}), 1 \right\} & \text{if } \min \left\{ (1 + \varsigma)(\rho_i^{(k)}), 1 \right\} \leq (\rho_i^{(k)}) (1, B_k^i)^\eta \\ (\rho_i^{(k)}) (1, B_k^i)^\eta & \text{Otherwise} \end{cases} \] (4.13.1)

\[ (\rho_j^{(k+1)}) = \begin{cases} \max \left\{ (1 - \varsigma)(\rho_j^{(k)}), 0 \right\} & \text{if } (\rho_j^{(k)}) (2, B_k^j)^\eta \leq \max \left\{ (1 - \varsigma)(\rho_j^{(k)}), 0 \right\} \\ \min \left\{ (1 + \varsigma)(\rho_j^{(k)}), 1 \right\} & \text{if } \min \left\{ (1 + \varsigma)(\rho_j^{(k)}), 1 \right\} \leq (\rho_j^{(k)}) (2, B_k^j)^\eta \\ (\rho_j^{(k)}) (2, B_k^j)^\eta & \text{Otherwise} \end{cases} \] (4.13.2)

where

\[ B_k^i = \frac{u_i^T \frac{\partial k}{\partial \rho_1} u_i}{\Lambda^1 \rho_i^2 v^i + \Lambda^2 v^i}, \quad B_k^j = \frac{u_i^T \frac{\partial k}{\partial \rho_2} u_i}{\Lambda^1 \rho_j^2 v^j + \Lambda^2 v^j}. \]

where \( k \) is the iteration number, \( \eta \) is the damping ratio, and \( \varsigma \) is the moving limit. Both \( \eta \) and \( \varsigma \) control the change of design variables, and they can be adjusted to improve computational efficiency.

4.3.5 Update of lagrangian multipliers based on monotonicity

From Equation (4.12.1), we have
\[ v_i = -\frac{\partial C}{\partial \rho_i^i} \frac{1}{\Lambda^i \rho_2^i + \Lambda^2} \]  

(4.14)

and therefore the current total volume of the structure is

\[ V = \sum_{i=1}^{n} \rho_i^i v_i = \rho_1^i \frac{-\partial C}{\partial \rho_1^i} \frac{1}{\Lambda^1 \rho_2^i + \Lambda^2} . \]  

(4.15)

Both \( \frac{\partial V}{\Lambda^1} \) and \( \frac{\partial V}{\Lambda^2} \) are less than zero, which means the total volume of the two materials combined monotonically decreases with respect to \( \Lambda^1 \) and \( \Lambda^2 \) separately. An example of the relation between the total volume and the two Lagrangian multipliers is shown in Figure 4.3.
Figure 4.3 An example of the relation between the total volume and the two Lagrangian multipliers. ($\Lambda^1$: Lagrangian Multiplier-1, and $\Lambda^2$: Lagrangian Multiplier-2)
From Equation (12-2), we have

\[ v_i = \frac{-\partial C}{\partial \rho_i^2} \frac{1}{\Lambda^1 \rho_i^1} \]  

(4.16)

Hence, the current volume of Material 1 of the structure is

\[ V_i = \sum_{i=1}^{n} \rho_i \rho_i^1 v_i = \rho_i \rho_i^1 \frac{-\partial C}{\partial \rho_i^2} \frac{1}{\Lambda^1 \rho_i^1} = \rho_i^1 \frac{-\partial C}{\partial \rho_i^2} \frac{1}{\Lambda^1} \]  

(4.17)

and \( \frac{\partial V_i}{\Lambda} \) is less than zero, which means the volume of Material 1 monotonically decreases with respect to \( \Lambda^1 \). An example of the relation between the volume of Material 1 and the Lagrangian multiplier \( \Lambda^1 \) is shown in Figure 4.4.

![Figure 4.4](image_url)  

Figure 4.4 An example of the relation between the volume of Material 1 and the Lagrangian multiplier \( \Lambda^1 \)
Consider the monotonicity and computational efficiency, the bisection method or golden section method can be used to obtain $B_k^i$ and $B_k^j$. Because $B_k^i$ is related to two Lagrangian Multipliers $\Lambda^1$ and $\Lambda^2$ and $B_k^j$ is related to $\Lambda^1$ only, $\Lambda^1$ will be calculated first to determine $B_k^j$ and then it is used to get $B_k^i$.

4.3.6 Advanced sensitivity filtering for two-material topology optimization

There are typically two numerical problems in topology optimization. The first is mesh-dependency, which means that different optimal solutions are obtained according to different mesh sizes. As the mesh density increases, the optimum solution tends to have a higher number of “truss-like” structures, as shown in Figure 4.5 [1]. This mesh dependency problem reflects the issue of non-existence of solutions in the continuous problem. The second numerical difficulty is checkerboard patterns, as shown in Figure 4.6 [1]. Diaz and Sigmund [24] and Jog and Haber [25] proved that such a material distribution pattern is not an optimal microstructure, but it is a numerical noise caused by the errors in finite element formulation. A checkerboard pattern makes stiffness of the structure erroneously overestimated. Techniques used to achieve mesh-independent solutions also often address the checkerboard-pattern problem because a checkerboard is a type of mesh dependent phenomenon.
Figure 4.5 An example of dependence of the optimal solution on different mesh refinement. Discretized elements for a) 2700, b) 4800 and c) 17200 [1]

Figure 4.6 An example of checkerboard pattern [1]
Several methods have been proposed to overcome these numerical difficulties. Perimeter control [26] constrains the number of holes inside the structure, and density slope control [27] constrains the local slope of the element densities. Sensitivity Filtering [28] uses a weighted average of the sensitivities of neighbor elements. Because the sensitivity filtering approach directly uses the sensitivity information derived for optimization, there is no extra constraint brought into the optimization problem. A minimum member size of the structure, which is often considered as a manufacturing constraint, can be also controlled via the selection of the filtering radius.

A standard sensitivity filtering algorithm for minimum compliance topology optimization with two materials is formulated as:

\[
\frac{\partial \hat{C}}{\partial \rho_i} = \frac{1}{\rho_i} \sum_{e=1}^{\hat{Q}_e} \hat{H}_e \rho_e \frac{\partial C}{\partial \rho_i}, \quad \frac{\partial \hat{C}}{\partial \rho_e} = \frac{1}{\rho_e} \sum_{i=1}^{\hat{Q}_e} \hat{H}_e \rho_i \frac{\partial C}{\partial \rho_e}
\]

\[
\hat{H}_e = r_{\text{min}} - \text{dist}(i, e), \quad \{e \in Q \mid \text{dist}(i, e) \leq r_{\text{min}} \}
\]

(4.18)

where \( r_{\text{min}} \) is the sensitivity filtering radius, elements \( i \) and \( e \) are the adjacent elements within the radius. The index \( i \) is used to represent the target element for which the filtered sensitivity is sought, and the index \( e \) is used for adjacent elements of the target element. \( \text{dist}(i, e) \) denotes the distance between the target element \( i \) and the neighbor element \( e \).

The sensitivity filtering method that uses a fixed filtering radius can effectively solve simple problems such as 2D cantilever or MBB problems. In real-worlds problems that have complex geometries and multiple types of elements, however, the filtering radius-based approach could cause difficulties in numerical implementation. First, it is usually
difficult and time consuming to identify the neighboring elements within a prescribed filtering radius when dealing with 3D structures that have complex, non-uniform mesh, and multiple element types. Second, the number of neighboring elements within a filtering radius can vary significantly according to the density of mesh in that area.

In our study, we chose neighbor elements based on “layers” instead of a “radius”. Figure 4.7 shows the selection of neighbor elements of a target element for 2D and 3D geometries. Direct neighbor elements that share a side (for 2D) or a surface (for 3D) are shown in green, and indirect neighbor elements that share a vertex (for 2D) or an edge (for 3D) are shown in yellow. We considered both types to be neighbor elements. The modified filtered sensitivity scheme is formulated as:

\[
\frac{\partial \hat{C}}{\partial \rho^i} = \frac{1}{\rho_i} \sum_{e=1}^{\mathcal{Q}} \hat{H}_{e}^{L_i} \rho^i \frac{\partial C}{\partial \rho^i} \quad \frac{\partial \hat{C}}{\partial \rho^2} = \frac{1}{\rho^2} \sum_{e=1}^{\mathcal{Q}} \hat{H}_{e}^{L_i} \rho^2 \frac{\partial C}{\partial \rho^2}
\]

where \( L_{\text{max}} \) indicates a prescribed number of layers for filtering. \( \mathcal{Q} \) is the total number of the adjacent elements within the \( L_{\text{max}} \) layer(s) from the target element \( i \). \( L_{e} \) is the number of the layer where the respective neighbor element belongs to. For example, \( \hat{H}_{e} = 1 \) will be used for all neighbor elements in the first layer and \( \hat{H}_{e} = 2 \) for all neighbor elements in the second layer, if \( L_{\text{max}} = 2 \).
Figure 4.7 Adjacent (neighbor) elements of the target element by one layer
4.4 Numerical Implementation of Two-Material Topology Optimization

For simple design problems, it is possible and also practical to prepare in-house software programs for all tasks required for multi-material topology optimization, including pre-processor (including meshing), finite element analysis, post-processor, and optimization. For real-world engineering problems, on the other hand, it is impractical to write in-house codes for all these tasks because we have to deal with complex 3D geometries, loads, and boundary conditions. In addition, it is less effective for technology transfer from academia to industry if we stick to in-house codes—which are often limited to particular types of physics and problems—for numerical tasks that currently available commercial software can readily deal with.

For the actual case studies, we utilized a commercial software program for all finite element analysis tasks; and for multi-material optimization which commercial programs cannot solve, we implemented our own optimization platform. Figure 4.8 shows the flowchart and use of software programs for each task. The OC (Optimality Criteria) method was implemented and used.

We used Altair HyperMesh as a preprocessor to discretize the geometry, to assign material and element properties, and to define boundary conditions and loads. RADIOSS was used as the FE (finite element) solver, and HyperView was used as a postprocessor for visualization of results. All these modules are integrated onto our optimization platform which is written in a HyperWorks programming script language, HyperMath.
Figure 4.8 Flow chart of two-material topology optimization implementation on the platform of HyperWorks
4.4.1 Implementation of two-material topology optimization on three classical examples

Three simple classical examples were solved in order to verify the proposed algorithm and method: 2D cantilever, 2D MBB and 3D cantilever. Steel was used as Material 1, and aluminum was used as Material 2. This means that Young’s modulus of Material 1 is three times as high as that of Material 2. Table 4.1 shows the material properties. The upper limit volume fraction of the sum of the two materials was 50%, and the upper limit volume fraction of Material 1 was 25%. This means that the maximum allowable total volume ($V_{\text{Total,Max}}$) and the maximum allowable volume of Material 1 ($V_{1,\text{Max}}$) in Equation (4.6) are $V_{\text{Total,Max}} = 0.5 \times V_{\text{Design Space}}$ and $V_{1,\text{Max}} = 0.25 \times V_{\text{Design Space}}$, respectively, where $V_{\text{Design Space}}$ is the volume of the design space. The objective function for minimization was compliance as defined in Equation (4.6).

Table 4.1 Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus (MPa)</th>
<th>Poisson's Ratio</th>
<th>Density (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>2.10E+05</td>
<td>0.3</td>
<td>7.80E+03</td>
</tr>
<tr>
<td>Aluminum</td>
<td>7.00E+04</td>
<td>0.3</td>
<td>2.70E+03</td>
</tr>
</tbody>
</table>
4.4.1.1. Example I - 2D Cantilever

The first example is the classical 2D cantilever beam with a point load applied at the center of the right-side edge, as shown in Figure 4.9 (a). The cantilever beam was meshed with 4,800 2D plane stress elements. This topology optimization converged at the 165th iteration. The optimum solution is shown in Figure 4.9 (b), and the history of the objective function is shown in Figure 4.10. Figure 4.11 shows the material distribution of Material 1 and Material 2 at each iteration during the first 14 iterations; during this period, most drastic solution changes occurred.

Figure 4.9 Two-material topology optimization of the 2D cantilever problem
Figure 4.10 Objective function history of the 2D cantilever problem with two materials

Figure 4.11 Topology changes of Material 1 and Material 2 during the first 14 iterations, for the 2D cantilever problem
4.4.1.2. Example II - 2D MBB

The second example is the half MBB problem shown in Figure 4.12 (a). A half of the beam was meshed with 2,500 2D plane stress elements. The optimization converged at the 135\textsuperscript{th} iteration with the two volume constraints active, and Figure 4.12 (b) shows the optimum topology of this problem.

![Design space (half MBB) and Optimum two-material topology result](image)

\textbf{Figure 4.12 Two-material topology optimization of the half MBB problem}

4.4.1.3. Example III - 3D Cantilever

The third example is a 3D cantilever problem with a point load applied at the center of the edge between the right-side surface and the bottom surface, as shown in Figure 4.13 (a). This problem converged at the 64\textsuperscript{th} iteration with the two volume constraints active, and Figure 4.13 (b) shows the optimum design. Figure 4.14 shows the topology change history during the first 14 iterations, where Material 1 is shown in brown, and Material 2 in green.
a) Design space (3D-Cantilever)  

b) Optimum two-material topology result

Figure 4.13 Two-material topology optimization of the 3D cantilever

Figure 4.14 Topology results of the first 14 iterations of the 3D cantilever problem
4.4.2 Implementation of two-material topology optimization on three case studies with various volume fraction ratios

In this section, we solved 3 case studies and discussed the results according to various volume fraction (or mass fraction) ratios.

4.4.2.1. Case study I - Cclip

The first case study is a 3D shell structures, which is called the “Cclip” problem (Figure 4.15). This Cclip structure is constrained at three locations (A, B, and C): at node A, only the translational DOF in the Z direction is constrained. At node B, all three translational DOFs in X, Y, and Z directions are constrained, and at node C, only the translational DOF in the Y direction is constrained. Two point forces with the same magnitudes are applied at node E (+Y direction) and F (-Y direction). The Cclip is discretized into 1,120 mixed first-order triangular and quadrilateral shell elements with 1 mm thickness. Steel and aluminum are again chosen as the base materials for topology optimization for Material 1 and Material 2, respectively. Before applying our two-material topology optimization, we solved the problem using Altair OptiStruct; because OptiStruct can solve only the standard single material topology optimization problem, we used steel for optimization with volume fraction of 50%.

The problem was then solved by our two-material topology optimization program with the same problem statement as the above mentioned one-material topology optimization. Mathematically, this is stated as: the maximum allowable total volume \( V_{\text{Total Max}} \) and the maximum allowable volume of Material 1 \( V_{1\text{,Max}} \) in Equation (4.6) are
\[ V_{\text{Total,Max}} = 0.5 \times V_{\text{Design Space}} \quad \text{and} \quad V_{1,\text{Max}} = 0.5 \times V_{\text{Design Space}}, \]

respectively, where Material 1 is steel, Material 2 is aluminum, and \( V_{\text{Design Space}} \) is the volume of the design space. According to this statement, zero percent (0%) of volume is allocated for aluminum, and this means that the optimum solution by our two-material topology optimization should be equivalent to the solution by one-material topology optimization using OptiStruct. These two results are compared in Figure 4.16; the two optimum designs are nearly identical, and their objective function (i.e. compliance) values are very close with each other.

In order to study how the optimum solution changes according to the two volume fractions (i.e. \( V_{\text{Total,Max}} / V_{\text{Design Space}} \) and \( V_{1} / V_{\text{Design Space}} \)), we solved 6 optimization problems with various volume fractions, as shown in Table 4.2. All 6 problem statements were formulated so that the final weight is 27.825 g. Figure 4.17 shows all 6 optimum solutions by the 6 problem statements, and Figure 4.18 compares the final compliance values of the 6 cases.
Figure 4.15 Initial design space (finite element model) of the Cclip problem with boundary conditions and loads

![Figure 4.15 Image]

- a) OptiStruct for one-material topology optimization, Steel (50%). Optimum compliance = 4.146 Nmm
- b) Our in-house code for two-material topology optimization, Steel (50%) & Aluminum (0%). Optimum compliance = 4.152 Nmm

Figure 4.16 Comparison of the optimum solutions of the Cclip problem

Table 4.2 Volume Fraction Constraints for the Cclip problem

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Total material ( = \frac{V_{\text{Total Max}}}{V_{\text{Design Space}}} ) [%]</th>
<th>Material 1: Steel ( = \frac{V_1}{V_{\text{Design Space}}} ) [%]</th>
<th>Material 2: Aluminum ( = \frac{V_2}{V_{\text{Design Space}}} - \frac{V_1}{V_{\text{Design Space}}} ) [%]</th>
<th>Total weight of Material 1 and Material 2 combined [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>27.825</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>45</td>
<td>15</td>
<td>27.825</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>40</td>
<td>30</td>
<td>27.825</td>
</tr>
<tr>
<td>4</td>
<td>79</td>
<td>35</td>
<td>44</td>
<td>27.825</td>
</tr>
<tr>
<td>5</td>
<td>89</td>
<td>30</td>
<td>59</td>
<td>27.825</td>
</tr>
<tr>
<td>6</td>
<td>98</td>
<td>25</td>
<td>73</td>
<td>27.825</td>
</tr>
</tbody>
</table>
Figure 4.17 The optimum topology with two materials of each case (red-steel, green-aluminum). All 6 designs have the same weight.
Figure 4.18 Performance (i.e. compliance) of 6 optimum solutions of the Cclip problem

Although the optimum topology of all 6 cases have the same weight (27.825g), the performance of these optimum designs (i.e. compliance values) are different from each other, as shown in Figure 4.18. Case #1 has the same problem statement as the standard topology optimization that considers one material (steel). Case #2 has inferior performance compared to the one-material design. All other 4 cases show superior results. This means
that with the same weight two-material topology optimization is able to produce designs with better performance (i.e. higher stiffness values).

4.4.2.2. Case study II - Long Cantilever

The second case study, “Long Cantilever” problem, has design and non-design domains as shown in Figure 4.19. All 6 DOFs of the nodes on the left edge are constrained, and a point force is applied at the tip in the negative Y direction. The cantilever is discretized into 1,768 quadrilateral shell elements with 1 mm thickness. The middle part, which is shown in green in the figure, is the design domain for optimization, and 1,064 shell elements are used. All remaining domains (in red) are non-design domains, and 724 shell elements are used. We performed the same set of optimizations as did in the Cclip problem. Figure 4.20 compares the optimum solution by the standard single material topology optimization using OptiStruct to the optimum solution by our two-material topology optimization. Note that only design domains are shown in the figure. The two designs are very similar with each other, and they have the same level of performance (i.e. compliance).

We then solved two-material topology optimization using 6 different volume fractions. The 6 sets of volume fractions presented in Table 4.2 were used, which means that all six cases in this example have the same values of $V_{\text{Total Max}}/V_{\text{Design Space}} \%$, $V_1/V_{\text{Design Space}} \%$, and $(V_{\text{Design Space}} - V_1)/V_{\text{Design Space}} \%$ in Table 4.2. The resultant weight was then 1.651g for all six cases, and optimum solutions are shown in Figure 4.21.
Figure 4.19 Initial design space (finite element model) of the Long Cantilever problem with boundary conditions and loads
a) OptiStruct for one-material topology optimization, Steel (50%). Optimum compliance = 19.947 Nmm

b) Our in-house code for two-material topology optimization, Steel (50%) & Aluminum (0%). Optimum compliance = 19.981 Nmm

Figure 4.20 Comparison of the optimum solutions of the Long Cantilever problem
Figure 4.21 The optimum topology with two materials of each case (red-steel, green-aluminum). All 6 designs have the same weight.
Figure 4.22 shows the performance (i.e. compliance) of these 6 optimum solutions by two-material topology optimization, in comparison to the optimum solution by one-material topology optimization that has the same weight. It should be noted again that Case #1 has the same problem statement as the one-material topology optimization with steel only. All actual two-material topology optimizations produced better results.
4.4.2.3. Case study III - Control Arm

The third case study is a 3D Control Arm problem with design and non-design domains (Figure 4.23). The control arm is meshed with 99,307 elements, which include hexahedral elements, pentahedral elements, and rigid elements. Points A and node B are the two center nodes of the rigid elements, and all three translational DOFs are constrained at these two nodes. Only the translational displacement in the Z direction is constrained at Point C. Three concentrated forces are applied at Point D, which is located at the center of rigid elements that connect the interior surface of the hole. We performed the same optimization tasks as did in the Cclip and Long Cantilever problems. Figure 4.24 compares the optimum result by OptiStruct with steel only to the optimum solution by two-material topology optimization with zero volume fraction for aluminum. Nearly identical results were obtained. The 6 volume fractions in Table 4.2 were used for 6 optimization runs with a two-material description. Figure 4.25 shows the solutions—note that all 6 designs have the same weight (22.85kg). All two-material designs (Case #2 – Case #6) again showed better results than the one-material design when the weight is the same (Figure 4.26).
Figure 4.23 Initial design space (finite element model) of the Control Arm problem with boundary conditions and loads
a) OptiStruct for one-material topology optimization, Steel (50%). Optimum compliance = 4.48 Nmm

b) Our in-house code for two-material topology optimization, Steel (50%) & Aluminum (0%). Optimum compliance = 4.47 Nmm

Figure 4.24 Comparison of the optimum solutions of the Control Arm problem
Figure 4.25 The optimum topology with two materials of each case (red-steel, green-aluminum). All 6 designs have the same weight.
Figure 4.26 Performance (i.e. compliance) of 6 optimum solutions of the Control Arm problem

4.5 Discussion

The first contribution of this research is that topology optimization was conducted with respect to two different types of dissimilar materials. Commercial software programs like OptiStruct, Genesis, or TOSCA can solve topology optimization with only single material. This chapter conducted mathematical sensitivity formulation of two-material topology optimization based the adjoint variable method, and applied it to various structural optimization problems. The results were compared to the optimum solutions determined
by the standard single-material topology optimization. All three case studies showed that two-material topology optimization can determine stiffer optimum designs than one-material topology optimization, for a prescribed weight constraint. With this finding, we can numerically confirm that two-material topology optimization can find lighter designs than one-material topology optimization, for prescribed stiffness constraints.

The second contribution is that this chapter demonstrated that all tasks for multi-material topology optimization could be conducted on a commercial software platform. We are doing collaborative research projects with many automotive and aerospace companies, and “technology transfer” is a very important aspect in the collaboration. For industry, it is of little use if all tasks are implemented in the form of in-house codes, because pre- and post-processors of in-house codes are not as powerful or user friendly as those by commercial programs and also because commercial FEA solvers can deal with a wide variety of engineering problems when most in-house codes cannot. We fully utilized HyperMesh, HyperView, and RADIOSS, and all our mathematical optimization algorithms were written in HyperMath. This means that our tool can solve problems that have complex geometries, and complicated boundary conditions and loads.

4.6 Limitations and Future Work

The most important limitation of this study is that a user has to explicitly allocate a volume fraction for each of Material 1 and Material 2. It would be more practical if there is only one final weight constraint (i.e. total weight of Material 1 and Material 2 combined together), and the optimizer automatically determines the optimum weight distribution between Material 1 and Material 2 while minimizing compliance. Its dual problem, which
minimizes the total weight with stiffness constraints would be even more practical for industry users. This requires more mathematical formulation and algorithm development, and it is reserved as our immediate future work.
ACKNOWLEDGEMENT

Authors would like to thank Justin Gammage, Balbir Sangha, and colleagues at General Motors of Canada for their technical advice and direction.

REFERENCES


Chapter 5
General Discussion

This thesis discussed the theory, algorithm, numerical implementation, and application of both standard single-material topology optimization and advanced multi-material topology optimization. The effectiveness, efficiency, and contributions of the developed methodology have been demonstrated through the study of a variety of engineering problems. This research will not only benefit the theoretical development of topology optimization, but will also have a significant effect on industrial applications. As opposed to previous research, which only discussed simple benchmark problems, this thesis distinguishes itself by closely integrating academic research and real-world engineering problems to advance the structural design process for industry.

In Chapter 2, an effective and efficient design process framework for an automotive engine cradle was presented by using topology, shape, and size optimization. Standard single-material topology optimization was introduced to the preliminary design stage to find the optimum material distribution for the desired detailed design. The proposed design framework benefited our industrial partner by cutting the original design time in half. Further, a design initiated from topology optimization is more objective and reliable, and this proposed design methodology could be easily extended to the design of products in other industries.
In Chapter 3, a lightweight design using lightweight material, structural optimization, and advanced manufacturing techniques for an automotive CCB was discussed. The design framework proposed in Chapter 2 was adopted for the development of this CCB, and promising weight reduction and cost savings were achieved. Additionally, an innovative simulation method was developed to approach the performance of a real SCSWS, which successfully addressed the dilemma that a CCB supplier usually confronts: how to achieve the optimum design of a CCB when detailed information about the SCSWS is not available.

In Chapter 4, topology optimization using two dissimilar materials was conducted to accommodate the increasing demands from industry. Due to the difficulty in theoretical development and numerical implementation, none of current commercial tools have solved multi-material topology optimization problems. This chapter, as a breakthrough, discussed the mathematical formulations of two-material topology optimization and successfully implemented it to address various structural optimization problems. The results showed that advanced two-material topology optimization could achieve more favorable designs than standard single-material topology optimization. Specialists from our industrial partner successfully tested our in-house tool and the technology-transfer effect between academia and industry was substantial and profound.

Although this thesis attempted to cover all the work I have done for my PhD study, some work was, unfortunately, not able to be presented for reasons of coherence, logic, and confidentiality.
Chapter 6

Future Work

The work discussed in this thesis serves as the cornerstone for the advancement of topology optimization and provides for a variety of promising research directions.

Current research on standard single-material topology optimization faces the dilemma of how to improve the performance of product design and reduce the computational cost. High fidelity and smooth-edged topology results with less, or even without any, geometric reinterpretation is the ultimate goal, but this goal is currently constrained by the computational capabilities and the financial tolerance of a company. Examining how to effectively maintain manageable computational costs would be a promising direction for future research.

Current research on multi-material topology optimization is limited in its ability to solve compliance-minimization problems, which require designers to pre-allocate volume fractions for different materials. To achieve a favorable design that satisfies all the performance requirements, optimization might be performed more than once for the purpose of tuning the volume fractions. Therefore, multi-material topology optimization for weight-minimization would be a promising and exciting direction for industrial engineers. Moreover, only metallic materials (steel and aluminum) were used in previous studies. By introducing more non-metallic materials, like plastic, polymer, or carbon fiber,
into multi-material topology optimization, a revolutionary change to engineering design is foreseeable.

Further, only a few manufacturing processes (e.g., extrusion and draw) have been incorporated into current single-material and multi-material topology optimization. Without considering manufacturability at an early design stage, topology results might suffer from inferior designs that need an extensive geometric reinterpretation, which impairs the strength of introducing topology optimization into the design framework. Hence, the question of how to effectively incorporate more manufacturing processes into topology optimization is drawing great research interest.

It is believed that more feasible research directions can be discovered and carried out to advance topology optimization, and the growing influence of topology optimization in structural design can be readily foreseen in various industrial fields.
Appendix A

In-house Tool of Multi-material Topology Optimization

The developed in-house tool for solving multi-material topology optimization problems is programmed on HyperMath platform and contains about 4,000 lines. As a courtesy, main code is shown as follows:

```
\\\\\\\\ Two-Material Topology Optimization\\\\\\\\
\\\\\\\\ Interface to .fem file defined by HyperMesh, ESE should be set as an output \\
\\\\\\\\ The Filepath and FEAfile names will be modified if needed \\
\\\\\\\\ Comments with *** means this line could be modified depending on different models \\
\\\\\\\\ If adjacent elements are needed for Sensitivity Filtering, then the attachedelements.txt is needed by using the .tcl macro. \\

ClearAll();
ReleaseMemory();
StartTime=CpuTime();

\\\\\\\\ Definitions for .fem File location and name \\
global Filepath;
global TOPfile;
global MATfile;
global FEAname;
Filepath="E:/Topology/ToDaoz/SampleModel_and_Code/";  // *** Location of .fem file Note: use backslash \ here, since it will be used in <NodesBetweenTwoMats.html> for converting to .hm
FEAname="cclip2.fem"  // *** .fem file name

\\\\\\\\ Control factors for volfrac and penalty factor \\
global volfrac;
global volfracy;
global penal;
global MAXiteration;

volfrac=0.857;  // *** Volume Fraction (Mat_1+Mat_2) of entire designable domain
volfracy=0.111;  // *** Volume Fraction (Mat_2 only) of designable domain
penal=3;  // *** Penalty Factor
MAXiteration=300;  // *** Max iteration for OC

\\\\\\\\ Initialization for the MAX element number and MAX node number \\
global MAXcomp;
```
global MAXprop;
global MAXmat;
global MAXnode;
global DsgnMATID;
global CompDesignID;
global RBE2NO;

  **MAXele**=5000;                                // *** Assign the maximal element NO. to read
  MAXnode=1207;                                  // *** Assign the maximal node NO. to read
  DsgnMATID=1;                                     // *** MAT ID for designable elements
  MAXcomp=30;                                      // *** Assign the maximal component NO. to read
  MAXprop=30;                                      // *** Assign the maximal property NO. to read
  MAXmat=30;                                       // *** Assign the maximal material NO. to read
  CompDesignID=[1];                                // *** Designable Component IDs. Will be changed depending on different FE model. Could be multiple IDs.
  RBE2NO=0;                                       // *** No. of RBE2 elements (will be extended to elements who does not have a strain energy) in the model.

  global EST;
global EAL;
global STEEL;
global AL;
  EST=2.1e5;
  EAL=7e4                           // *** Predefined INITIAL Young's Modulus for designable elements
  STEEL=7.9e-9;
  AL=2.7e-9                       // *** Predefined density for designable elements

  ///////////  Definations for .hml File (Codes) location and name  ///////////
  global Codepath;

  Codepath="E:\Topology\ToDaoz\SampleModel_and_Code\HyperMathCode\" // *** Location of .hml files
  FEMrewritten=Codepath+"FEMrewrittenAdv_2D.hml"
  // FEMrewrittenGrid=Codepath+"FEMrewrittenGrid.hml"
  FEMrewrittenElement=Codepath+"FEMrewrittenElement_2D.hml"
  // FEMrewrittenPropMat=Codepath+"FEMrewrittenPropMat.hml"
  // AdjacentEle=Codepath+"AdjacentEleAdv2.hml"
  AdjacentEle=Codepath+"AdjacentEleAdv2_2D.hml"
  //AdjacentEleTest=Codepath+"TestRun_AdjacentElement.hml"
  // AdjacentEleFiltered=Codepath+"AdjacentEleAdv2_Filtered.html"
  HyperMathOC=Codepath+"HyperMath_OC.html"
  Include(FEMrewritten)
  Include(FEMrewrittenElement)
  //FEMrewrittenTime=FEMrewrittenTime2-FEMrewrittenTime1;
  //print("FEMrewrittenTime.",FEMrewrittenTime);
  // Include(AdjacentEle)
  Include(AdjacentEle)
  Include(HyperMathOC)
  //OCTime=CpuTime();
  //print("OCTime.",OCTime-FEMrewrittenTime);
Density4plot=Codepath+"Density4plot.html";
Include(Density4plot)  // To generate .txt files for plotting in HyperView, and new .fem file with separate component number of ST and AL.

TotalTime=CpuTime();
print("TotalElapsedTime: ", TotalTime-StartTime);

///// Subroutine: FEMrewrittenAdv_2D.hml /////
///// FEMrewritten for the new ***TOP.fem file, for 2D shell structure /////

FEMrewrittenTime1=CpuTime();
 ///// To read once the original FEA file for gathering COMPONENT, PSHELL AND MAT1 info /////
CompNo=0;  // No. of total component
NodeTable={};  //+++ Store the each nodal coordinate
PropNo=0; PropTable={};  // No. of total Property, store original property in a table
PropMatID=0;  // Original Mat ID in the original Prop Card
MatNo=0; MatTable={};  // No. of total Material, store original material in a table
CompInfo=Zeros(MAXcomp,5);  // Component info: ComponentID, EleStrtA, EleEndA, EleStrtB, EleEndB  (Max number of row could be changed)
PropInfo=Zeros(MAXprop,2);  // Property info: PropertyID, MaterialID (Max number of row could be changed)
MatInfo=Zeros(MAXmat,1);  // Material info: MaterialID (Max number of row could be changed)
CompID=0; EleStrt=0; EleEnd=0;
global MaxEleID;
MaxEleStrt=0; MaxEleEnd=0; MaxEleID=0;
MaxEleStrtA=0; MaxEleStrtB=0; MaxEleEndA=0; MaxEleEndB=0;
GRIDPointer=0; ELEPointer=0;
PSOLIDPointer=0; PSHELLPointer=0; MAT1Pointer=0;

global FEAfile;
FEAfile=Filepath+FEAnam  // Name and location of original .fem file
fidFEAread=Open(FEAfile,'r+');  // Open the original .fem file
fidcomp=fidFEAread::read();

while fidcomp!=nil do
  tok, rem=StrTok(fidcomp);
  if StrCmp(tok,'$HMMOVE') then
    CompID=StrSubrange(fidcomp,9,16)*1;
    fidcomp=fidFEAread::read();  // Read next line including '1THRU100'
    EleStrt=StrSubrange(fidcomp,9,16)*1;  // Record the starting element ID in current component
    EleEnd=StrSubrange(fidcomp,25,32);  // Record the ending element ID in current component (Could be empty since there is only one element in this component)
  end if
end while;
if EleEnd==' ' 
   then
       EleEnd=0; 
       // If there is only one element in this component, assign 0 to EleEnd.
   else
       EleEnd=EleEnd*1;
   end

if CompInfo(CompID,2)==0 then 
   // To check if CompInfo(CompID,2) has already been filled.
   CompInfo(CompID,1)=CompID;
   CompInfo(CompID,2)=EleStrt;
   CompInfo(CompID,3)=EleEnd;
else 
   // If it has already been filled, then add another group of elements in
   CompInfo(CompID,4), CompInfo(CompID,5)
   CompInfo(CompID,4)=EleStrt;
   CompInfo(CompID,5)=EleEnd;
end

end

CompNo=Max(CompInfo(:,1)); 
// Record component number

if StrCmp(tok,'PSHELL') then
   PropNo=PropNo+1; 
   // Record property number
   PropID=StrSubrange(fidcomp,9,16)*1;
   PropMatID=StrSubrange(fidcomp,17,24)*1;
   PropInfo(PropNo,1)=PropID;
   PropInfo(PropNo,2)=PropMatID;
   PropTable(PropID)=fidcomp; 
   // ATTENTION: Here using PropID to index PropTable
end

if StrCmp(tok,'MAT1') then
   MatNo=MatNo+1; 
   // Record material number
   MatID=StrSubrange(fidcomp,9,16)*1;
   MatInfo(MatNo)=MatID;
   MatTable(MatNo)=fidcomp; 
   // ATTENTION: Here using MatNo to index PropTable
end

fidcomp=fidFEAread::read(); 
// At the end, CompNo=Max number of comp, PropNo=Max number of Prop, and
MatNo=max number of Mat
end

MaxEleStrtA=Max(CompInfo(:,2)); MaxEleStrtB=Max(CompInfo(:,4));
MaxEleEndA=Max(CompInfo(:,3)); MaxEleEndB=Max(CompInfo(:,5));
MaxEleStrt=Max(MaxEleStrtA,MaxEleStrtB);
MaxEleEnd=Max(MaxEleEndA,MaxEleEndB);
MaxEleID=Max(MaxEleStrt,MaxEleEnd); 
// Aftering renumbering, element starts with ID 1 and with increment 1. Therefore,
MaxEleID=Total number of elements

Close(fidFEAread);

//~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~//
To rewrite the original FEA .fem file into Top.fem File

To rewrite the ELEMENT part

CompDesignNo=Max(Size(CompDesignID)); // To get the Max number of designable comps

DsgnEleNbr=0;
NonDsgnEleNbr=0;

for i=1,CompDesignNo do
    CompID=CompDesignID[i];
    if CompInfo(CompID,4)>0 then // If: There are more than two groups of elements in this comp
        DsgnEleNbr=DsgnEleNbr+Abs(CompInfo(CompID,3)-CompInfo(CompID,2))+1+Abs(CompInfo(CompID,5)-CompInfo(CompID,4))+1;
    else
        DsgnEleNbr=DsgnEleNbr+Abs(CompInfo(CompID,3)-CompInfo(CompID,2))+1;
    end
end

NonDsgnEleNbr=MaxEleID-DsgnEleNbr;

DsgnEleInfo=Zeros(DsgnEleNbr,2); //Designable Element info: ElementID, PropertyID (Max number of row could be changed)
if NonDsgnEleNbr>0 then
    NonDsgnEleInfo=Zeros(NonDsgnEleNbr,2); //Non-Designable Element info: ElementID, PropertyID (Max number of row could be changed)
else
    NonDsgnEleInfo=Zeros(DsgnEleNbr,2);
end

for i=1, CompDesignNo do
    CompID=CompDesignID[i];
    EleStrtA=CompInfo(CompID,2); EleStrtB=CompInfo(CompID,4);
    EleEndA=CompInfo(CompID,3); EleEndB=CompInfo(CompID,5);
    // To judge if current element is designable
    if EleEndA>0 then // To Check if the number of element of this group in the current component is bigger than 1
        for i=EleStrtA,EleEndA do
            // Code to rewrite the element
        end
    end
end
nbr=nbr+1;
DsgnEleInfo(nbr,1)=i;
end

else // If the number of element in this group is equal to 1
nbr=nbr+1;
DsgnEleInfo(nbr,1)=EleStrtA;
end

if EleEndB>0 then // To Check if the number of element of this group in the current
component is bigger than 1
for i=EleStrtB,EleEndB do
nbr=nbr+1;
DsgnEleInfo(nbr,1)=i;
end
elseif EleStrtB>0 then // If
EleNode=Zeros(MaxEleID,4); // To record the node ID for each element. Assume it is First-order SHELL element here.
//EleCenter=Zeros(MaxEleID,3); // To record the coordinates of the center of each node

//----- Define .fem File location and name, and to remake the TOP.fem file //-----
// FEAfile=Filepath+FEAname  // Name and location of original .fem file
global TOPl; // TOPl
TOPfile=StrSubs(FEAfile, ".fem","TOP.fem");  // New .fem file for topology optimization
GRIDfile=Filepath+"Grid.txt"  // Element file
ELEfile=Filepath+"Element.txt"  // Element file
PROPfile=Filepath+"Property.txt"  // Material file
MATfile=Filepath+"Material.fem"  // Material file

fidFEA=Open(FEAfile,'r+');  // Open the original .fem file
fidTOP=Open(TOPfile,'a+');  // Open the topology .fem file
fidGRID=Open(GRIDfile,'a+');  // Open the Grid .fem file
fidELE=Open(ELEfile,'a+');  // Open the Element .fem file
fidPROP=Open(PROPfile,'a+');  // Open the Property .fem file
fidMAT=Open(MATfile,'a+');  // Open the material .fem file
Create a new file for analysis

```c
// Read the original file line by line
while tline!=nil do
  // Truncate the first string and assign it to 'tok'
  tok, rem=StrTok(tline);
  if !StrCmp(tok,'GRID') && !StrCmp(tok,'CTRIA3') && !StrCmp(tok,'CQUAD4') && !StrCmp(tok,'PSHELL')
    fidTOP::write(tline,"n"); // Copy the lines to **TOP.fem file except detecting lines with those exceptions
  end
  row=row+1; // Record the line number for the original .fem file

// To get the coordinates of each node
if StrCmp(tok,'GRID') then
  if GRIDPointer==0 then
    GRIDPointer=1;
    insert="INCLUDE Grid.txt";
    fidTOP::write('n'n'n'n'n
'n
'n
'n
'n
'n','$$
'n',insert,"n");
  end

  fidGRID::write(tline,"n");
  gridnr=gridnr+1; // To count the grid number in the original .fem file

  // To store each node's coordinate in CNode Matrix:///
  //+++ Get Node ID
  NodeID=StrSubrange(tline,9,16)*1;
  NodeTable(NodeID)=tline;
  Cx=StrSubrange(tline,25,32); // Coordinates of each node. String format.
  Cy=StrSubrange(tline,33,40);
  Cz=StrSubrange(tline,41,48);
  if (StrFind(Cx,"-",2)~=nil) then
    Cx=StrSubrange(Cx,1,2)+StrSubs(StrSubrange(Cx,3,8),'e-','e-');
  elseif (StrFind(Cx,"+",:)~=nil) then
    Cx=StrSubs(Cx,"+","e+");
  end
  if (StrFind(Cy,"-",2)~=nil) then
    Cy=StrSubrange(Cy,1,2)+StrSubs(StrSubrange(Cy,3,8),'e-','e-');
  elseif (StrFind(Cy,"+",:)~=nil) then
    Cy=StrSubs(Cy,"+","e+");
  end
  if (StrFind(Cz,"-",2)~=nil) then
    Cz=StrSubrange(Cz,1,2)+StrSubs(StrSubrange(Cz,3,8),'e-','e-');
end
```
elseif (StrFind(Cz, "+") == nil) then
  Cz = StrSubs(Cz, "+", "e+" );
end

CNode(NodeID, 1) = Cx * 1; // Convert string to number
CNode(NodeID, 2) = Cy * 1;
CNode(NodeID, 3) = Cz * 1;

//////// Judgement2: To get the element ID, each element will be assigned a unique property and material
if StrCmp(tok, "CTRIA3") || StrCmp(tok, "CQUAD4") then
  elenr = elenr + 1; // To count the number of element assigned with material property, excluding
  ELEPointer = 0;
  insert = "INCLUDE Elementrewritten.txt";
  fidTOP::write("
  n
  n
  n
  n
  n
  n", insert, "n");
end

tlineELE = tline;

ID = StrSubrange(tline, 9, 16); // ELEMENT ID is a string
ELEPROPID(1, ID * 1) = ID * 1; // Get element ID, which is a number now
ELEPROPID(2, ID * 1) = StrSubrange(tline, 17, 24) * 1; // Get element ORIGINAL Property ID

// To get the nodes ID of one element
N1 = StrSubrange(tline, 25, 32) * 1;
N2 = StrSubrange(tline, 33, 40) * 1;
N3 = StrSubrange(tline, 41, 48) * 1;
// N4 = StrSubrange(tline, 49, 56) * 1;

///// Change 1: CTRIA3 OR CQUAD4
if StrSubrange(tline, 49, 56) == " " then
  N4 = 0;
else
  N4 = StrSubrange(tline, 49, 56) * 1;
end

// To store element node ID
EleNode(ID * 1, 1) = N1 * 1;
EleNode(ID * 1, 2) = N2 * 1;
EleNode(ID * 1, 3) = N3 * 1;
EleNode(ID * 1, 4) = N4 * 1;

// To calculate the center of each element using the average of the coor of each node
EleCenter(ID * 1, 1) = 1 / 4 * (CNode(N1 * 1, 1) + CNode(N2 * 1, 1) + CNode(N3 * 1, 1) + CNode(N4 * 1, 1));
EleCenter(ID * 1, 2) = 1 / 4 * (CNode(N1 * 1, 2) + CNode(N2 * 1, 2) + CNode(N3 * 1, 2) + CNode(N4 * 1, 2));
// EleCenter(ID*1,3)=1/4*(CNode(N1*1,3)+CNode(N2*1,3)+CNode(N3*1,3)+CNode(N4*1,3));

// flag=0;
// for i=1, CompDesignNo do
// CompID=CompDesignID(i);
// EleStrtA=CompInfo(CompID,2); EleStrtB=CompInfo(CompID,4);
// EleEndA=CompInfo(CompID,3); EleEndB=CompInfo(CompID,5);
// ////////// To judge if current element is designable ///////
// if (EleStrtA<=ID*1 && ID*1<=EleEndA)||(EleStrtB<=ID*1 && ID*1<=EleEndB) then
// flag=1;
// nbr=nbr+1;
// DsgnEleInfo(nbr,1)=ID*1;
// DsgnEleInfo(nbr,2)=ID*1; // NEW Property ID same as designable Element ID
// for j=1,8 do
// tlineELE[16+j]=ID[j]; // Assign the Property ID to Element line
// end
// break;
// end
// end
// if flag==0 then
// Nonnbr=Nonnbr+1;
// NonDsgnPropID=MaxEleID+ELEPROPID(2,ID*1); // ELEPROPID(2,ID*1) stores the original property ID
// NonDsgnEleInfo(Nonnbr,1)=ID*1;
// NonDsgnEleInfo(Nonnbr,2)=NonDsgnPropID; // NEW Property ID for non-designable elements. New Property ID=MaxEleID + original Property ID
// NonDsgnPropID=StrFormat("%8s",NonDsgnPropID);
// for j=1,8 do
// tlineELE[16+j]=NonDsgnPropID[j]; // Assign the New Property ID to Element line
// end
// end

fidELE::write(tlineELE,"n"); // This ' + ' line is copied down.

end

// Since there could be RBE2 or RBE3 elements in this model, nbr+Nonnbr!=MaxEleID //

EID=0; PID=0; NEWPROPID=0;
MID=0; PMATID=0;

// Judgement3: To rewrite the PROPERTY part /////
if StrCmp(tok,'PSHELL')&&! PSHELLPointer==0 then

PSHELLPointer=1;
insert="INCLUDE Property.txt";

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fidTOP::write("\n\n\n\n\n","$
$');

///// Change the NEW Property ID to the same elementID for designable elements /////
for p=1, DsgnEleNbr do
  EID=DsgnEleInfo(p,1);            // To get designable Element ID
  PID=ELEPROPID(2,EID);            // To get the Original Property ID
  tlinePROP=PropTable(PID)         // To call PropTable to get the needy original property line
  NEWPROPID=StrFormat("%8d",EID)   // To assign the Property ID same as current designable Element ID
  for j=1,8 do
    tlinePROP[8+j]=NEWPROPID[j];    // NEW Property ID
    tlinePROP[16+j]=NEWPROPID[j];   // New Material ID
  end
  fidPROP::write(tlinePROP,"\n");
end

///// Change the NEWLY SCALED Property ID for NON-designable elements /////
for q=1, PropNo do
  PID=PropInfo(q,1);             // Original Property ID
  tlinePROP=PropTable(PID);           // To call PropTable to get the needy original property line
  MID=PropInfo(q,2);             // Original MAT ID
  PMATID=MaxEleID+MID*1;              // Newly scaled MAT ID
  PMATID=StrFormat("%8d",PMATID);
  NEWPROPID=MaxEleID+PID*1;      // To assign the SCALED Property ID stored in NonDsgnEleInfo()
  NEWPROPID=StrFormat("%8d",NEWPROPID);
  for j=1,8 do
    tlinePROP[8+j]=NEWPROPID[j];    // NEW Property ID
    tlinePROP[16+j]=PMATID[j];      // nEW Material ID
  end
  fidTOP::write(tlinePROP,"\n");
end
end

MMATID=0;
///// Judgement4: To rewrite the MAT1 part /////
if StrCmp(tok,'MAT1')&& MAT1Pointer==0 then
  MAT1Pointer=1;
  insert="INCLUDE Material.fem"
  fidTOP::write("\n\n\n\n\n","$
$');

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DsgnEleInfo(:,1)=Sort(DsgnEleInfo(:,1));
DsgnEleInfo(:,2)=Sort(DsgnEleInfo(:,2));

for t=1, DsgnEleNbr do
    MMATID=DsgnEleInfo(t,1); // Get NEW MAT ID for deginable elements. MMAID= ID of
designable element
    MMATID=StrFormat("%8d",MMATID);
    tlineMAT=MatTable(DsgnMATID); // MAT line for designable elements
    for j=1,8 do
        tlineMAT[8+j]=MMATID[j];
    end
    fidMAT::write(tlineMAT, 
                  "n");      // Write into Material.fem file
end

for t=1, MatNo do // ATTENTION: MatNo
    tlineMAT=MatTable(t);
    MMATID=MatInfo(t);  // Get original MAT ID
    MMATID=MaxEleID+MMATID;
    MMATID=StrFormat("%8d",MMATID);
    for j=1,8 do
        tlineMAT[8+j]=MMATID[j];
    end
    fidTOP::write(tlineMAT, "n"); //ATTENTION: Write into TOP.fem file
end

tline=fidFEA::read();
end
Close(fidFEA);
Close(fidTOP);
Close(fidGRID);
Close(fidELE);
Close(fidPROP);
Close(fidMAT);

%%%%%%%%%%%%%%%% Initialization for the element density %%%%%%%%%%%%%%%%%%%%%%%%%% 
global density; //
density=Zeros(DsgnEleNbr,1); 
density([1:DsgnEleNbr,1]=STEEL; // *** Initial real density for each element.
global Adjele;
tt1=CpuTime();
Adjele=4;                                  // Max columns for Adjacent solid elements
Adjacent=Zeros(DsgnEleNbr,Adjele);         // There are max 4 unique adjacent elements for CQUAD or CTRIA (excluding corner element)
global ELEPROPID;
CurrentID=0; AdjacentID=0;
TempNO=0;    SetA=Zeros(1,4); SetB=Zeros(1,4); SameELENO=0;
/////////  To  find  temp  adjacent elements of each element (including duplicates)/////////
AdjElefile=Filepath+"attachedElems.txt"
fidtxt=Open(AdjElefile,"r")
// fidtxtcopy=Open(’E:\Temp\Cantilever_3D\AdjacentElems.txt’,’w’)
AdjacentTemp = FReadText(fidtxt,’’,1)
///////// Change 3://///////////////
for e=1,DsgnEleNbr do                        // Current element
    temp=0;
    // To count the adjacement element
    CurrentID=DsgnEleInfo(e,1);
    TempNO=AdjacentTemp(CurrentID,2);     // Temp Adjacent Element Number
    SetB=EleNode(CurrentID,:);              // Element Node set of current element
    if EleNode(CurrentID,4)>0 then
        // To judge if it is a CQUAD element
        for i=1,TempNO do
            AdjacentID=AdjacentTemp(CurrentID,2+i);
            SetA=EleNode(AdjacentID,:);              // Element Node set of Adjacent element
            SameELENO=Sum(Intersect(SetA,SetB)~=0)    // To count the NO. of same element of the two sets. Rule out zeros.
            if SameELENO==2 then
                // 2 nodes on each face of a CQUAD element
                temp=temp+1;                        // Normally, Max value of temp should be 4.
                Adjacent(e,temp)=AdjacentID;       // Store the adjacent elements
            end
        end
    else
        // which means it is a CTETRA element
        // for i=1,TempNO do
AdjacentID=AdjacentTemp(CurrentID,2+i);

SetA=EleNode(AdjacentID,:);  // Element Node set of adjacent element
SetB=EleNode(CurrentID,:);  // Element Node set of current element

SameELENO=Sum(Intersect(SetA,SetB)~=0)  // To count the NO. of same element of the two sets. Rule out zeros.

if SameELENO==3 then  // 3 nodes on each face of a CTETRA element
  temp=temp+1;  // Normally, Max value of temp should be 4, if there is no T-
  Shape feature of the geometry.
  Adjacent(e,temp)=AdjacentID;  // Store the ajacent elements
end

Close(fidtxt)
tt2=CpuTime();

print("Read original Adjacent Eles Time:",tt2-tt1);

// Subroutine: HyperMath_OC /////////
///////// This subroutine is the Optimality Criteria Algorithm for Single Material //////////

function top(DsgnEleInfo,NonDsgnEleInfo,Adjacent,density,volfrac,volfracy,penal)
  global x;  // 'x' is the relative density vector to determine if current element will be assigned with material
  global y;  // 'y' is the relative density vector to determine if current element will be assigned with STEEL
  global xnew;
global ynew;
global density;
global MAXiteration;
global StrainE;
global EST;
global EAL;
global STEEL;
global AL;
global DsgnEleNbr;
global NonDsgnEleNbr;
global MaxEleID;
global AlldensityX;
global AlldensityY;
global loop;
global RBE2NO;

x=Zeros(DsgnEleNbr,1);  // To store the relative density for designable elements
y=Zeros(DsgnEleNbr,1);
xnew=Zeros(DsgnEleNbr,1);
ynew=Zeros(DsgnEleNbr,1);
StrainE=Zeros((MaxEleID-RBE2NO),2);  // To store All the Strain Energy: Total StrainE, Scaled StrainE
x([1:DsgnEleNbr],1)=volfrac;  // Initialize density_1
y([1:DsgnEleNbr],1)=volfracy;  // Initialize density_2

dca=Zeros(DsgnEleNbr,1);  // To store the sensitivity of Mat_1
dcb=Zeros(DsgnEleNbr,1);  // To store the sensitivity of Mat_2
AlldensityX=Zeros(DsgnEleNbr,MAXiteration);  // To store the relative density_1 for all the iterations
AlldensityY=Zeros(DsgnEleNbr,MAXiteration);  // To store the relative density_2 for all the iterations
AlldensityZ=Zeros(DsgnEleNbr,MAXiteration);  // To store the relative density_3 for all the iterations, density_3 is not a design variable.
AllSE=Zeros((MaxEleID-RBE2NO),MAXiteration);  // To store the TOTAL strain energy for all the iterations
AllSEC=Zeros((MaxEleID-RBE2NO),MAXiteration);  // To store the SCALED strain energy for all the iterations
loop=0;  // To count the iteration number
scalefactor=0;
changeX=1.;  changeY=1.;  change=1.;

///// Define the output files /////
global Filepath;

Compliancefile=Filepath+"Output_Compliance.fem"
Changefile=Filepath+"Output_Change.fem"
DensityXfile=Filepath+"Output_EleDensityX.txt"
DensityYfile=Filepath+"Output_EleDensityY.txt"  // Export relative density of STEEL
DensityZfile=Filepath+"Output_EleDensityZ.txt"  // Export relative density of Aluminum
SEfile=Filepath+"Output_StrainEnergy_total.txt"
SECfile=Filepath+"Output_StrainEnergy_scaled.txt"

outputC=Open(Compliancefile,'a+');
outputCh=Open(Changefile,'a+');
fidDensityX=Open(DensityXfile,'a+');
 fidDensityY=Open(DensityYfile,'a+');
 fidDensityZ=Open(DensityZfile,'a+');
 fidSE=Open(SEfile,'a+');
 fidSEC=Open(SECfile,'a+');

global TOPfile;
// global OutfileName;
OutfileName=StrSubs(TOPfile,".fem",".h3d");

```
runanalysis="E:\Altair12.0\hwsolvers\scripts\radioss.bat" +TOPfile+" -dir" // Pay attention to the space
```

global Codepath;
// ReadSEfile=Codepath+"ReadSE_Advance.hml" // The file path of the ReadSE.html subroutine
ReadSEfile=Codepath+"ReadSE_2D.html"
DensityRewritten=Codepath+"UpdateDensity.html" // The file path of the UpdateDensity.html subroutine
global MATfile;
Objchange=1; // Percentage of change of the Compliance of the two consecutive iterations
cold=0; // Compliance

```
while (change>0.02 && loop<MAXiteration && Objchange>0.00001) do // && Objchange>0.002
    // while (change>0.02 && loop<MAXiteration) do
    ///////////////////////////////////////////////////////////////////////////////////////////
    ///// To call RADIOSS to run the analysis /////
        System(runanalysis)
    ///////////////////////////////////////////////////////////////////////////////////////////
```

```
///// To call ReadSE.html /////
Include(ReadSEfile)
StrainE=ReadSE(OutfileName);  // To call the ReadSE() function
```

```
loop=loop+1;
xold=x;
yold=y;
c=0;
```

```
AlldensityX(:,loop)=x;  // To store the density by column
AlldensityY(:,loop)=y;
```

```
///// To calculate the Compliance for all CTRIA and CQUAD elements (excluding RBE2 or RBE3)
for ele=1,(MaxEleID-RBE2NO) do // *** Will be rewritten excluding RBE2 or RBE3 elements
    // c=c+x(ele)^penal*StrainE(ele); // StrainE=1/2*compliance
    c=c+StrainE(ele,1); // Strain Energy for the entire structure, Young's Modulus is the interpolated E
end
```

```
DsgnElecID=0;
for ele=1,DsgnEleNbr do
    // dc(ele)=penal*x(ele)/(penal-1)*StrainE(ele);
    DsgnElecID=DsgnEleInfo(ele,1); // To obtain designable elementID, which is in ascending order
dca(ele)=penal*2*StrainE(DsgnElecID,1)/x(ele); // To calculate the sensitivity for designable element
```

```
if dca(ele)>0 then
    dca(ele)=0;
```

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scalefactor=EAL/(y(ele)^penal*EST+(1-y(ele)^penal)*EAL);  // Scaled factor calculation
StrainE(DsgnEleledcID,2)=scalefactor*StrainE(DsgnEleledcID,1);
dcb(ele)=-penal*(2*StrainE(DsgnEleledcID,1)-2*StrainE(DsgnEleledcID,2))/y(ele);  // To calculate the sensitivity for designable element
if dcb(ele)>0 then
dcb(ele)=0;
end

dca,dcb=check(DsgnEleNbr,Adjacent,x,y,dca,dcb);            // Call 'sensitivity filtering' function, dont need bracket '[' ]'
x,y=OC(DsgnEleNbr,x,y,volfrac,volfracy,dca,dcb);           // Call 'Optimality Criteria' function, dont need bracket '[' ]'

AllSE(:,loop)=StrainE(:,1);                     // To store the total strain energy by column
AllSEC(:,loop)=StrainE(:,2);                    // To store the scaled strain energy by column

changeX=Max(Abs(x-xold));                  // HyperMath does not support Max(Max()) structure
changeY=Max(Abs(y-yold));
change=Max(changeX,changeY);
Objchange=abs((cold-c)/c)*100;             // Percentage of Change of Compliance
cold=c;

print("It.: ",loop, "Obj: ", c, "Vol.: ", Sum(x)/DsgnEleNbr, "Vol_1.: ", Sum(x.*y)/DsgnEleNbr, "Vol_2.: ", Sum(x.*(1.-y))/DsgnEleNbr, "ch.", change, "ch_X.", changeX, "ch_Y.", changeY, "Obj_ch.", Objchange)
outputC::write(c,"n")
outputCh::write(change, "n")

// System("E:\Altair12.0\hwssolvers\scripts\radioss.bat" E:\Temp\BBMTest\MATLABFEMTEST\MATFILE.fem")
// ///////////////////////////////////////////////////////////////////////////
// Include("E:\Topology\00Research\SwPckg\HyperMath\Topology\HyperMath_Fem\ReadSE.hml")
// ///////////////////////////////////////////////////////////////////////////

end  // This 'end' is used to end the looping statement 'while'

Time1=CpuTime();
Close(outputC);
Close(outputCh);

FWrite(fidDensityX, AlldensityX); // Store the densities for all the iterations
Close(fidDensityX);

FWrite(fidDensityY, AlldensityY); // Store the densities for all the iterations
Close(fidDensityY);

AlldensityZ=1.-AlldensityY;
FWrite(fidDensityZ, AlldensityZ); // Store the densities for all the iterations
Close(fidDensityZ);

FWrite(fidSE, AllSE); // Store the strain energy for all the iterations
Close(fidSE);

FWrite(fidSEC, AllSEC); // Store the strain energy for all the iterations
Close(fidSEC);

Time2=CpuTime();

// print("OCTime: ",Time1-Time0,"OutputFileWrittingTime:",Time2-Time1)

end  // This 'end' is used to end a function

// Optimality Criteria (OC) Algorithm //////////////////////////////////////////////////
function OC(DsgnEleNbr,x,y,volfrac,volfracy,dca,dcb)  // Don't need [xnew,ynew]=...
l1=0;I2=200000; step=0.2;
t1=0;I2=200000; move=0.2;
global xnew;
global ynew;
xnew=x;
lmid=0.5*(l2+l1);
tmid=0.5*(t2+t1);
// Filepath="E:/Temp/Engine_Cradle/"
// xnewfile=Filepath+"xnew.txt"
// ynewfile=Filepath+"ynew.txt"
// fidxnew=Open(xnewfile,'a+');
// fidynew=Open(ynewfile,'a+');

while Max((l2-l1),(t2-t1))>1e-5 do
    ynew=Max(0.001,Max(y-step,Min(1.,Min(y+step,y.*Sqrt(dcb./(x.*lmid)))))); // To update design_variable_2 (Mat_1)

    // FWrite(fidynew,ynew);                  // Store the densities for all the iterations

    // print("Sum(xnew.*ynew): ",Sum(xnew.*ynew))
    // print("Sum(xnew.*ynew): ",StrFormat("%8d",Sum(xnew.*ynew)),"volfracy*DsgnEleNbr: ",StrFormat("%8d",volfracy*DsgnEleNbr))
    //
    // print(StrFormat("%8d",Sum(xnew.*ynew)-volfracy*DsgnEleNbr));

    if (Sum(xnew.*ynew)-volfracy*DsgnEleNbr)>0 then
        l1=lmid;
    else
        l2=lmid;
    end

    lmid=0.5*(l1+l2);

    xnew=Max(0.001,Max(x-move,Min(1.,Min(x+move,x.*Sqrt(dca./(y.*lmid+tmid))))));

    // FWrite(fidxnew,xnew);                  // Store the densities for all the iterations

    if (Sum(xnew)-volfrac*DsgnEleNbr)>0 then       // *** Will be modified to calculate the area for each element.
        t1=tmid;                                  // *** Volfrac is fraction of the designable volume
    else
        t2=tmid;
    end

    tmid=0.5*(t1+t2);

    // print("l1: ",l1,"l2: ",l2,"l2-l1",l2-l1);
    // print("t1: ",t1,"t2: ",t2,"t2-t1",t2-t1);

end
// Close(fidynew);
// Close(fidxnew);
return [xnew], [ynew];                                // Fuction value to be returned
end                                        // Use 'end' to close a function definition
function check(DsgnEleNbr, Adjacent, x, y, dca, dcb) // Don't need [dcna, dcnb]=...
    dcnb = Zeros(DsgnEleNbr, 1);
    global Adjele
    for i = 1, DsgnEleNbr do
        b = 0;
        for j = 1, Adjele do // Adjele is the maximal adjacent element No. of current element
            if Adjacent(i, j) > 0 && Adjacent(i, j) <= DsgnEleNbr then // Since not all the elements will have 6 adjacent elements, in adjacency matrix, some spots might be zero.
                AjID = Adjacent(i, j); // Put adjacent element ID in AjID
                // AjID = AjID - 394; // Seek the right order in x()
                dcnb(i) = dcnb(i) + x(AjID) * dca(AjID) * 1;
                // Calculate the sensitivity of adjacent elements.
                dcnb(i) = dcnb(i) + dcb(AjID) * 1;
                b = b + 1;
            end
        end
        dcnb(i) = dcnb(i) / (x(i) * b); // ***Search radius could be treated as '1' for one layer elements
        dcb(i) = dcb(i) / b;
    end
    return [dcna], [dcnb];
end

// Subroutine: ReadSE /////
// This subroutine is to update element density in the Material.fem file /////
t8 = CpuTime();
global MATfile;
MATtempfile = StrSubs(MATfile, "\.fem", "Temp.fem");
fidnew = Open(MATfile, "a+");
fidnew01 = Open(MATtempfile, "w+"); // 'w+' is used to clear all the previous data and write new data
global x;
global y;
global EST;
global EAL; //
global penal;

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tline=fidnew::read()
for i=1,DsgnEleNbr do
    tok,rem=StrTok(tline);
    if StrCmp(tok,'MAT1') then
        tlineYoung=tline;
        YoungUpdate=StrFormat("%8.1e",x(i)^penal*(y(i)^penal*EST+(1-y(i)^penal)*EAL));  // MATID is a number
        DensityUpdate='        ';                      // Density is nil in the material card
        Density(i,1)=x(i);
        YoungUpdate=StrFormat("%8s",YoungUpdate);      // Update the Young's Modulus for each
        DensityUpdate=StrFormat("%8s",DensityUpdate);  // Update the Density for each material card
        //
        for j=1,8 do
            tlineYoung[16+j]=YoungUpdate[j];
            tlineYoung[40+j]=DensityUpdate[j];
        end
        fidnew01::write(tlineYoung,"
        n");
    end
end
// To point to the next line
Close(fidnew01)
Close(fidnew)
/// Switch the contents of file-MaterialTemp.fem with file Material.fem///

fidnew=Open(MATfile,"w+");                  // 'w+' is used to clear all the previous data and write new data
fidnew01=Open(MATtempfile,"a+");            // writing is only allowed at the end of the file if 'a+' is used

tlinenew01=fidnew01::read();
while tlinenew01!=nil do
    tline=tlinenew01;
    fidnew::write(tline,"n");
    tlinenew01=fidnew01::read();
end
Close(fidnew01)
Close(fidnew)
T9=CpuTime();
print("MatFileRewrittenTime:",t9-t8)