# Testing Local Realism with Bell's Inequality

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## Abstract

In this experiment, we constructed a single-channel Bell test to demonstrate the non-local nature of quantum mechanics. The apparatus was used to calculate Bell's inequality(S) for entangled particles and show the violation of local realism. The experiment used a 405nm laser shun through a Beta-Barium Borate(BBO) crystal to create entangled photon pairs in a process called parametric down-conversion. Using interferometry techniques, the entangled photons were directed into two collimators both connected to a single-photon coincidence counter through fibre optic cables to detect entangled photon coincidences. The final part included the placement of linear polarizers in front of the collimators to take data for the calculation of S. The predicted result was that Bell's inequality would be violated thus confirming the non-locality feature of entangled particles. Contrary to predictions, we found that Bell's inequality was not violated for the measurement we took. This result is attributed to insufficient alignment as the non-locality in the quantum realm is a well-observed phenomenon.

## 1. Introduction

The abstract nature of this experiment requires an understanding of some well-known physical properties. First, a description of the experiment will help to familiarize with the general process of testing Bell's inequality. We started by aligning a 405nm laser path towards a specialized BBO crystal such that two continuous streams of entangled photons exit the crystal at an angle of 3 degrees from the initial light path. Next, the collimators were placed at the predicted location of the two entangled photon streams. For clarification, the entangled pairs of photons are within the spectrum of infrared light and therefore cannot be seen with the naked eye. Once we confirmed coincidence counts on the computer, linear polarizers set at non-equal angles were place in front of either collimator. The last step was to change the combination of polarizing angled and calculate Bell's inequality for each, although we were not able to complete this final portion. With enough angle combinations, an accurate conclusion of local realism could be made.

The specialized crystal used in this experiment is known as a Beta-Barium Borate(BBO) crystal. This is how we easily and reliably created photon pairs so that we could conduct our experiment. The BBO crystal uses the process of parametric down-conversion to create entangled photons with half the frequency of the original photon entering the crystal. The original incident photon is known as the pump photon and the resulting pairs are called signal and idler photons[1]. It is important to understand only a small proportion of the incident photons undergo this process and become entangled. It is estimated that only about 1 in $10^{10}$ are successfully entangled[2]. This means when the light is shun through the BBO crystal, nearly all of the photons enter and exit in the same direction without any change in frequency.

The focus of this experiment is on the properties of entangled particles and their implications for classical or relativistic styles of physical intuition. A pair of entangled particles, such as the ones in this experiment, can be described under a single state. Where the total state is a linear combination of the of separate states describing the two particles. Say a measurement of the total state is made and it collapses into a single state dependent on the measurement made. If our measurement is that particle one is spin ½ up, and the state after the collapse describes particle two as spin ½ down, then we can conclude without time delay that we will find the second particle in the spin ½ down configuration. Since the collapse of the state is instantaneous in theory, it violates the age-old concept of local realism. Essentially, entanglement is a special property of quantum mechanics which can, in theory, exceed the speed limit of information exchange, or light speed. The argument posed by Bell is that the spin properties of either particle in our previous example were always there. This is known as the hidden variable hypothesis, which is the basis of Bell's inequality[3].

Further, this experiment's goal is to explicitly test the concept of local realism through particle entanglement. First proposed by Albert Einstein who described the properties of entanglement as "spooky action at a distance" [3]. From the perspective of Einstein and the framework of physics he created, no information can travel faster than the speed of light and therefore the information exchange from one entangled particle to the other is constricted by this same limitation. Although, as described above, entanglement and therefore quantum mechanics refutes this idea and abides by a separate set of rules as compared to previous frameworks in physics. Thus, our experiment's purpose is to show through a violation of Bell's Inequality that Quantum mechanics is non-local[4].

The term Bell's inequality actually references numerous different types, with the first derivation coming from John Bell in 1964. Over time, the amount of assumptions for the Inequality were improved and succeeding versions of the original 1964 paper were created. For this experiment we chose the third version from the Clauser and Horne 1974 paper[5]. It was concluded by our group that this Inequality was more simple to understand and calculate but still has the advantage that it improves on John Bell's original inequality.

## 2. Theory

Our experimental purpose was to show how Bell's inequality(S) is violated by the known properties of quantum entanglement. As previously mentioned, we used the Inequality from the Clauser Horne 1974 paper. But the setup of this experiment is similar to many of the other versions of testing Bell's inequality. Equation 1[5][6] shows the value of Bell's inequality as a function of coincidence counts for different polarization angle combinations. The equation is geared for a single channel Bell test, which is the apparatus used in this experiment. The violation of S occurs when its value exceeds the upper and lower boundaries described in Equation 2[5][6]. The CH74 inequality takes the form

$$S = N(\alpha,\beta) - N(\alpha,\beta') + N(\alpha',\beta) - N(\alpha',\beta') + N(\alpha') - N(\beta) \tag{1}$$

where S is the nominal value of Bell's inequality, with upper and lower bounds given by

$$1 \leq S \leq 0 \tag{2}$$

When S exceeds these limits, we consider it a violation of the inequality and therefore local realism. *N* in Equation 1 represents the coincidence counts for certain polarization angles $\alpha,\ \alpha',\ \beta,\ \beta'$ . Where either polarizer corresponds to a pair $\alpha,\ \alpha'$ or $\beta,\ \beta'$. An important stipulation is that the angle difference between pairs must be the same for both polarizers. By repeating the process for many different combinations of angles and noting the difference between angles for each measurement of S, a graph of data can be made. Therefore, the graph is S as a function of angle separation, denoted $\Delta$ and we can rewrite Equation 1 to show this explicitly.

$$S(\Delta) = N(\alpha,\beta) - N(\alpha,\beta+\Delta) + N(\alpha+\Delta,\beta) - N(\alpha+\Delta,\beta+\Delta) + N(\alpha+\Delta) - N(\beta) \tag{3}$$

Using Equation 3, we can make a graph of $S(\Delta)$ and place horizontal lines at the restricted bounds of Equation 2. This way, the maximal violation of Bell's inequality can be displayed in something analogous to Figure 1.

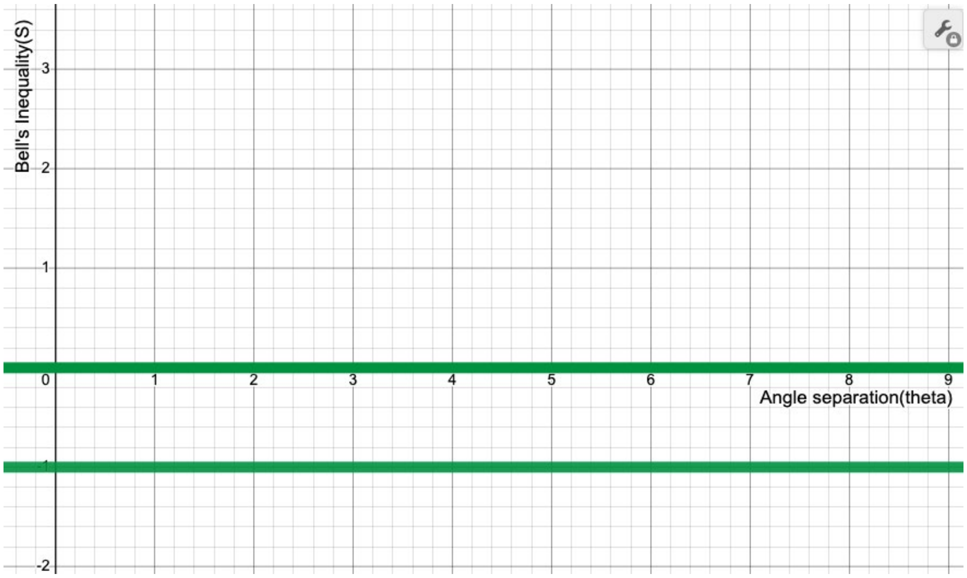
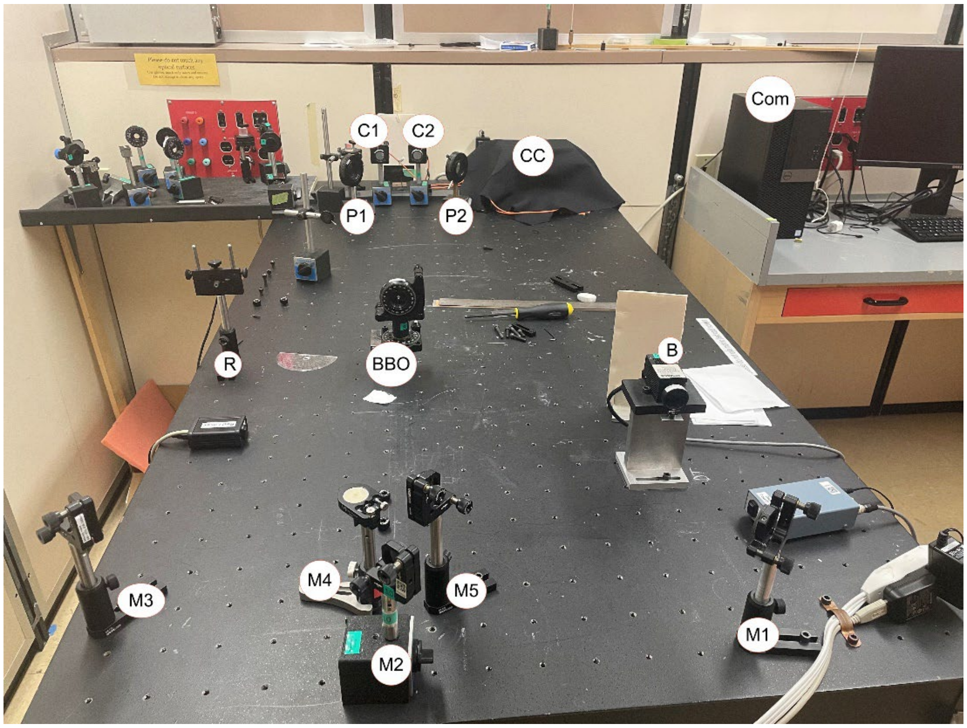


*Figure 1: The graph of Bell's inequality as a function of the separation of angles*$(\Delta)$ *where the area in between the green lines is in agreement with hidden variables and values over or under are considered a display of non-locality.*

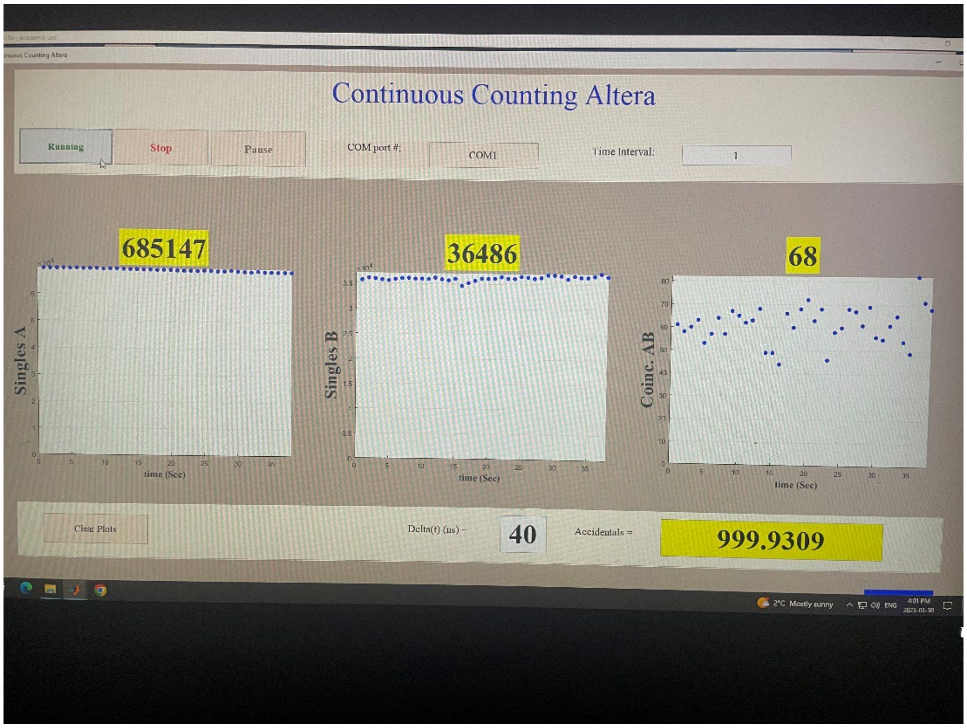
## 3. Apparatus and Procedure

The design project apparatus is a recreation of a single-channel Bell test which at its core is a tool to detect pairs of entangled photons. Our apparatus includes a 405nm laser which has a light path directed straight into a BBO crystal. The BBO crystal can be adjusted rotationally or tilted vertically and horizontally for alignment purposes. Most of the laser light goes through the crystal unphased onto a beam block while a small number of photons are entangled and exit at 3 degrees. A separate red laser is used to align the position of the collimators so that the entangled photons enter the two collimators positioned behind the crystal. These collimators are connected to a single photon coincidence counter via fiber optic cable and transport the incident photons. The coincidence counter is connected to a computer and a coincidence count program was provided through Queen's University on the computer to record counts. The placement of two linear polarizing films in front of each collimator corresponding to either $\alpha$, $\alpha'$ or $\beta$, $\beta'$ complete the apparatus. Figure 2 shows the final setup of our apparatus. And Figure 3 shows the coincidence counter program on the computer.



| M1, M2, M3, M4, M5 | Mirrors |
|---|---|
| B | Blue 405nm laser |
| R | Red Laser |
| C1, C2 | Collimators |
| P1, P2 | Linear Polarizing Films |
| CC | Single Photon Coincidence Counter |
| Com | Computer with Coincidence count program |

*Figure 2: A photon of our single-channel bell test apparatus where the orientation of the photon is directed parallel to the path of the 405nm laser. The associated legend with the components is in the table beneath.*



COM port #: COM1

Time Interval: 1

Delta(t) (ns) = 40

Accidentals = 999.9309

*Figure 3: A photo of the Coincidence counting program on the computer, where the leftmost graph is the total photon counts in Detector A, the middle is the counts for Detector B and the rightmost graph is the coincidence counts between either. The Graph shows a dynamic value of each every second, although the latency of measurement is 40ns.*

The procedure for this setup may seem straightforward, which it is. Although, getting it to work properly proved difficult, as the signal and idler photons are half the frequency of the pump photon (405nm) and therefore they fall in the infrared spectrum which cannot be seen with the naked eye. The first step of our apparatus setup is aligning the blue laser so it shines in a direction exactly parallel to the table. Tiny holes in the table can be seen in Figure 2 much like an electrical breadboard. These holes, with the aid of an iris helped us direct the laser properly towards the other side of the breadboard. Since the final goal is to get the light into the collimator and the collimators are at a fixed, proper height, we use them as a vertical adjustment tool to align the blue laser so it shines at a height equal to the collimator. This eliminates one degree of freedom so that when we were making final adjustments to the setup, the vertical alignment required minimal adjustment. An interferometry technique learned earlier in the course was used by adjusting the height of the laser through an iris at a point close to source and then repeated at increasing distances to ensure the laser is not shining with at vertical angle. So, after making sure the pump photons were directed properly in both the vertical and horizontal directions, measurements for the placement of the crystal were made. We started by measuring 1 meter from a point we wanted to “catch” the entangled photons in either collimator to the eventual position of the BBO crystal. We then marked two more points where we would eventually place the collimators, which was done using simple geometry since we knew the trajectory of 3 degrees and the distance of the collimators to the crystal position. An Iris was then placed where the BBO crystal would be and adjusted so that the blue laser travels directly through it. Furthermore, a semi-circle with a radius of 1 meter made of wood, constructed for a previous year's experiment was placed and secured where the collimators would be positioned. The advantage to this tool is that during the final alignments we can move the collimators along the semi-circle and they constantly face a direction perpendicular to the BBO crystal.

For the next part of the procedure, we setup the red laser in a similar fashion to the blue one by eliminating the vertical degree of freedom and started with the mirror M5. The mirror directed the red light through the Iris onto the points where we marked the path of entangled photons, or the position of collimator C1 in Figure 2. Using a plump bob, we were able to move the mirror until the red light shined directed through the iris onto the string of the plump bob where it was split in half perfectly. We repeated this process for the flip mirror M4 towards collimator C2. We then placed the collimators in the approximated geometric positions. The next interferometry technique we used is to connect either collimator to one another with a fibre optic cable. This way if we shine the red laser into the one collimator, if it is entering properly it will go through the cable and exit the other collimator. We did this for each collimator and adjusted them so that when the red light entered one, exited the other and ended up shining back onto the middle of the iris. This was final process of initial alignment.

The final portion of the procedure we replaced the Iris with BBO crystal and adjusted the individual paths of the blue and red lasers such that they both entered the crystal at the exact same point. After this, we repeated the processes of making sure the red light shined into collimator, exited the other back into the middle of the BBO. We were now ready to test the apparatus by connecting the collimators to the coincidence counter and turning on the MATLAB coincidence counter program provided by the Queen's University. We slowly adjusted both the collimators and BBO crystal to maximize the amount of coincidence counts. Once we were satisfied with the amount of counts, polarizers were placed in front of either collimator and data was taken.

## 4. Data

Our experiment required many points of data for many different intervals of time. The thought process of our group was that if we take enough combinations of polarization angles that some calculation of S would exceed the allowed limits. This way we could graph S for many different angle combinations and eventually pinpoint a maximum violation for Bell's inequality like we proposed in Figure 1. As mentioned in previous undergraduate labs, the coincidence counter requires a short period of time to "warm-up" to optimize the collection of data. Thus, we followed the same principal as this lab from Andrew Wakileh et el, and chose to take a cumulation of data from a period of 60-120 seconds[7]. The program on the computer has a helpful feature where it appends all the data onto three separate excel files. One file is the total photon counts in detector A, another for detector B and one more for the coincidence counts between A and B. To be specific, the file has two columns, the first column represents the time and the second shows the value of counts. So we would note the last row number before taking each measurement so that the time interval of data collection could be identified for each calculation. Next, the data from 60-120 seconds was identified and totaled using excels summation function. For example, if the data for $N(\alpha,\ \beta)$ was identified on the excel sheet from rows 1540 to 1680, we would take a summation of rows (1540+60) to (1540+120) for all three categories of data. By repeating this process for each polarization angle combination, a value of S was calculated. So we began by angling the polarizers at two angles $\alpha$ and $\beta$ , turning the laser and coincidence counter program on, then taking 120 seconds or more of raw data. I repeated this process for all of the required angle combinations in Equation 1. Unfortunately, due to our group failing to align the apparatus properly until the last week, only a single value of S could be calculated. Table 1 is inspired by the previously reference lab group from Queens and shows the data used in the calculation of our S value[7].

| **Measurement** | **Polarizer A** | **Polarizer B** | **Counts A** | **Counts B** | **Coincidences** | **Accidentals** |
|---|---|---|---|---|---|---|
| $N(\alpha,\ \beta)$ | 0° | 45° | 20588540 | 265977 | 237 | 219043 |
| $N(\alpha,\ \beta')$ | 0° | 90° | 20587420 | 74029 | 48 | 60962 |
| $N(\alpha',\ \beta)$ | 45° | 45° | 26340300 | 289803 | 310 | 305339 |
| $N(\alpha',\ \beta')$ | 45° | 90° | 26240010 | 83658 | 112 | 87807 |
| $N(\alpha')$ | 45° | None | 26338190 | 1994899 | 2489 | 2101681 |
| $N(\beta)$ | None | 45° | 39634290 | 1458725 | 2681 | 2312621 |

*Table 1: the measurements of Counts in detector A, B and the coincidence counts between the two detectors for each combination of linear polarization films in front of A and B. this data represents the summations of all data points between 60 and 120 seconds in the associated excel files.*

If we completed the project earlier, we would have taken many measurements resulting in an equal number of tables, one for each calculation of S.

## 5. Analysis

Our proposed method of data analysis was to graph the values of S as a function of separation angle. This first requires a calculation of S for our only data point. We can use Equation 1 to calculate a value. But first, a there is a row in Table 1 showing the accidentals. Accidentals are a statistical approximation for coincidence counts which are accidentally counted. This can be calculated using Equation 4 by using the product of Counts in either detector A or B multiplied by the latency of photon detection. For our experiment we use a latency or resolving time of 40 nanoseconds. The actual number of coincidences can be intuitively calculated by subtracting the coincidence counts by the accidentals[8].

$$A = C_A \times C_B \times L \tag{4}$$

Where A is the value of accidentals, $C_A$ and $C_B$ are the counts in detector A and B, respectively and L is the latency time between photon detections. As seen in Table 1, our value of accidentals far exceeds the number of coincidences we got. This is due to misalignment of our apparatus such that there not enough coincidence counts as a proportion of the total photon counts in detectors A and B. If we were able to increase the percentage of coincidence counts per total amount of counts, the effect of accidentals would be greatly reduced. Since our accidentals exceed the coincidences, when we subtract by the accidentals we would get a negative value. Although we cannot get a negative coincidence count and thus makes the actual coincidence counts zero for all angle combinations. This leads us to the calculation of S for a separation angle of 45 degrees or $\frac{\pi}{4}$ radians.

$$S = N(0,45) - N(0,90) + N(45,45) - N(45,90) + N(45) - N(45) \tag{5}$$

$$S = 0 \tag{6}$$

Equation 6 is our final calculation of S, which falls within the boundaries of particle locality and thus does not violate Bell's inequality. The reason for this result can be attributed to two things. First, the number of accidentals skewed the collected data over the chosen time interval and secondly, only a single value of S was calculated to compare with.

## 6. Discussion and Results

This success of this experiment relied primarily on the precise alignment of a laser, specialized BBO crystal and photon detectors. Since, the precision of alignment determines the proportion of detected photon coincidences per total photons detected. Our result did not agree with our theoretical expectations and this can be due to many reasons. The first reason is that we took the data for the experiment after a minimal amount of apparatus adjustments. That is, as soon as we obtained significant coincidence counts per second, data was taken. This was because we did not get the apparatus functioning until the last day before the results were presented. Our lab group would have preferred to spend more time adjusting both the collimator and the BBO crystal to maximize the amount of photon coincidence counts. Moreover, having multiple calculated values of S would have helped reduce unknown confounding errors. It is easy to notice that a single calculation of S does not provide a conclusive result.

Some important techniques were taught to us by professors throughout the semester that I believe will improve the rate of apparatus setup progression if another group ever chooses to conduct a similar experiment. To start, when the lights are turned on it is hard to see the path of the blue laser, especially since we are using safety goggles. A cool trick to help this issue is to use a small piece of white paper to track the path of blue light. The light reflects off the paper very brightly, which helps guide the laser in the proper path during alignment. Another technique we learned was employing the plumb bob to accurately position the lasers in the horizontal direction. A plumb bob is a pointed weight hanging from a string so that the string is oriented perfectly vertical due to gravity. So, when you point the weight onto the desired placing of an apparatus component, the string is also in the exact position of where you want to place that apparatus. Therefore, if you desired to shine a laser into the collimators you can first position the plumb bob where you want to eventually place the collimator, and when you shine the laser towards it you will know it is shining in the right direction if the laser is split exactly in half by the string.

## 7. Conclusion

This experiment attempted to show that the laws of Quantum mechanics do not function in the same manner as classical or relativistic physics. We conclude that although our experiment yielded a result that does not violate Bell's inequality or the concept of local realism, with more calculations and more precise alignment, we would find a violation of particle locality. The experiment provided a great experience for our lab group in terms of designing, constructing, and then conducting our own lab. If the experiment were re-done with further efforts to reduce accidental photon counts, I believe a successful presentation of quantum mechanical non-locality could be shown explicitly. Although disappointing, we learned much about the principles of physics and were able to study the transition from relativistic to quantum mechanical thinking.

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