

CREDIT RISK, INSURANCE AND BANKING: A STUDY OF
MORAL HAZARD AND ASYMMETRIC INFORMATION

by

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Abstract

This dissertation investigates agency problems within risk transfer contracts. We pay particular attention to the consequences of credit risk transfer in the context of banking. The first two chapters provide an introduction and literature review. We then analyze the effect of counterparty risk on financial insurance contracts in the following two chapters, and uncover a new moral hazard problem on the part of the insurer. If the insurer believes it is unlikely that a claim will be made, it is advantageous for them to invest in assets which earn higher returns, but may not be readily available if needed. We find that counterparty risk can create an incentive for the insured to reveal superior information about the risk of their “investment”. In particular, a unique separating equilibrium may exist even in the absence of any signalling device. This constitutes a first example in which the separation of types can be achieved without a costly signalling device. Our research suggests that regulators should be wary of risk being offloaded to other, possibly unstable parties, especially in financial markets such as that of credit derivatives.

The fifth chapter models loan sales and loan insurance (e.g. credit default swaps) as two key instruments of risk transfer within the banking environment. Recent empirical evidence suggests that the asymmetric information problem is as relevant in loan insurance as it is in loan sales. Contrary to previous literature, this paper allows for informational asymmetries in both markets. Our results show that a well capitalized bank will tend to use

loan insurance regardless of loan quality in the presence of moral hazard and relationship banking costs of loan sales. Finally, we show that a poorly capitalized bank may be forced into the loan sales market, even in the presence of possibly significant moral hazard and relationship banking costs that can depress the selling price.

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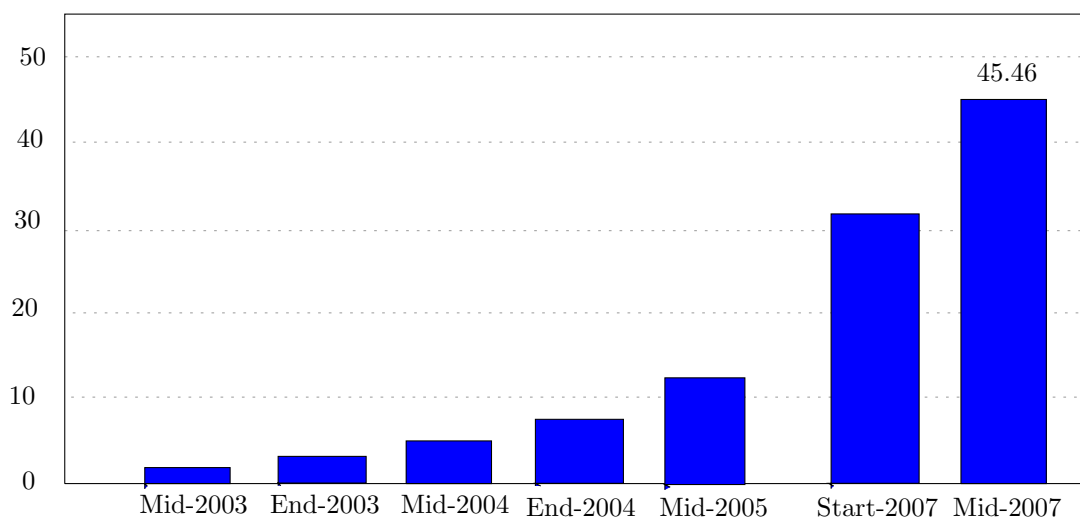
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Chapter 1

General Introduction

The market for risk protection is one of the most important markets available today. The development of new Credit Risk Transfer (CRT) techniques has fundamentally changed the banking system. Whereas banks were once confined to a simple borrow-short-and-lend-long strategy, they can now disperse credit risk through credit derivatives¹ to better implement risk management policies. Figure 1.1 shows the growth rate in credit derivatives since 2003. It is easy to see the rapid expansion that these financial markets have experienced. Table 1.1, which is based on a survey of some of the largest financial institutions in the world, reports the average weekly trading volume for various derivative instruments. It is noteworthy that credit derivatives have overtaken “plain vanilla” equity derivatives in options activity for these banks.

¹A credit derivative, and specifically a *credit default swap* is an instrument of credit risk transfer whereby an insurer agrees to cover the losses of the insured that take place if pre-defined events happen to an underlying borrower. (In many cases, this event is the default of the underlying bond. However, some contracts include things like re-structuring and ratings downgrades as triggering events.) In exchange for this protection, the insured agrees to pay an ongoing premium at fixed intervals for the life of the contract. Note that the values in figure 1.1 are likely subject to some double counting.



Source: ISDA 2005,2007

Figure 1.1: Notional Value of Credit Derivatives (in Trillions of Dollars)

Instrument	2003	2004	2005
Currency Options	427	559	905
Equity Derivatives - Vanilla	291	153	153
Credit Derivatives	79	103	206

Source - ISDA 2005 Operations Benchmark Survey

Table 1.1: Average weekly trading volume (per institution) of three common types of derivatives

Recently, problems with credit risk transfer instruments have surfaced in what is being called the ‘credit crises’ of 2007/2008. Prior to the problems, the ability of banks to easily shed loan risk allowed the creation of what some refer to as a ‘shadow banking system’ (see Milne (2008) and Brunnermeier (2008) for a more detailed discussion). This *new* banking system was conducted with off-balance-sheet vehicles through structured products and asset backed securities (ABS). These instruments allowed a substantial amount of credit risk to exist outside of the regulator’s reach. With credit risk easy to shed, the

demand for additional risk rose. This demand was met by extending loans to parties which otherwise may not have had access to credit. On the surface this seems like a positive development: credit can now be extended to those who otherwise would not qualify. It is now known that the relative lack of transparency of the structured products made the pricing of these instruments very difficult. As the market for these products dried up, counterparties to these transactions found themselves unable to value the instruments through traditional models. This then brings the issue of counterparty risk into the forefront: if a bank links itself through risk transfer to another party, they are then exposed to the risk that that party may fail as well. Therefore, it is important to understand how counterparties may behave when they engage in credit risk transfer. This will be the focus of the first theory chapter of this dissertation (chapter 3), however, chapter 2 first provides a literature review.

Chapter 3 analyzes the effect of counterparty risk on financial insurance contracts using the case of credit risk transfer in banking. The traditional moral hazard problem is one in which an insured party has perverse incentives because some or all of the risk is no longer internalized. We uncover a new moral hazard problem on the other side of the market. This problem arises because the insurer can take actions outside of the control of the insured party. If the insurer believes it is unlikely that a claim will be made, it is advantageous for them to invest in assets which earn higher returns, but may not be readily available if needed. This may increase the counterparty risk that an insured party is exposed to.

We find an interesting consequence of the new moral hazard problem: it can create an incentive for the insured to reveal superior information about the risk of their “investment”. In particular, a unique separating equilibrium may exist, even in the absence of any signalling device.

Chapter 4 extends the model from chapter 3 in three important ways. First, we allow

there to be multiple insured parties. We show that all the results from chapter 3 follow through when each insured party is small, provided that there is aggregate risk that they share in common. Second, we allow there to be multiple insurers, thereby permitting diversification of the insured parties risk. We obtain the somewhat surprising result that counterparty risk need not decrease as the number of insurers with which an insured party contracts with increases. We show that the less risk each insurer takes on, the more severe the new moral hazard problem can become. Third, we incorporate the traditional moral hazard problem into the model. If we use the example of a bank insuring itself on one of its loans, the literature typically assumes that a bank possesses a proprietary monitoring technology (due to a relationship with the underlying borrower). It is straightforward to see that if the bank is fully insured, it may not have the incentive to monitor the loan and, consequently, the probability of default could rise. In this extension we show that the new moral hazard introduced may increase the desire of the insured to monitor.

Chapter 5 analyzes a bank's choice of risk transfer instruments. Specially, we uncover factors which may influence a bank in its choice between loan insurance (credit default swaps) and loan sales. This chapter extends the work of Duffee and Zhou (2001) and relaxes two key assumptions: (1) we allow for informational asymmetries in the loan insurance market; and (2), we relax the maturity mismatch assumption. Our results show that a well capitalized bank will tend to use loan insurance regardless of loan quality in the presence of moral hazard costs of loan sales. Furthermore, we show that a poorly capitalized bank may be forced into the loan sales market, even in the presence of possibly significant moral hazard and relationship banking costs that can depress the selling price.

The remainder of the thesis proceeds as follows: Chapter 2 presents a literature review. Chapter 3 develops a model of counterparty risk. Chapter 4 extends the model presented in

chapter 3 to allow for multiple insured parties, multiple insurers and moral hazard on the part of the insured. Chapter 5 constructs a model of both loan insurance and loan sales. The final chapter concludes.

Chapter 2

Literature Review

This dissertation contributes to two streams of literature: that of credit risk transfer and credit derivatives and that of insurance economics. The literature on credit risk transfer is small, but is growing. Gorton and Pennachhi (1995) provide an early and fundamental discussion of the moral hazard that can arise in CRT. With loan sales being the only instrument available in the model, they show how a bank can overcome the moral hazard problem by continuing to hold a fraction of the loan, and offering explicit guarantees on loan performance. In this setting, the incentive of the bank to continue monitoring the firm remains after the loan is sold. Duffee and Zhou (2001) extend the work of Gorton and Pennachhi (1995) to analyze the consequences of introducing credit derivatives as an instrument of risk transfer. Chapter 5 builds on Duffee and Zhou (2001), but departs from it in two important ways. Recent empirical evidence by Acharya et al. (2007) suggests that banks are acting on their privileged information in credit default swaps (loan insurance) markets. In other words, private information does exist in these markets. Consequently, we allow for informational asymmetries in the loan insurance market. The second assumption we relax is the assumption that insurance is written only on the first period of a two period

loan. The Basel Committee on Banking Supervision (2005) found that supervisors penalize banks if there is a maturity mismatch. Contrary to Duffee and Zhou (2001), we analyze the case in which the insurance is written on the entire length of the loan.

In recent work, Parlour and Plantin (2007) analyze credit risk transfer through the bank-borrower relationship. Specifically, they use loan sales as the only instrument of CRT and propose an asymmetric information problem due to a bank that has a private stochastic discount shock. They analyze the case when a liquid CRT market can arise, and the socially inefficient outcome that may result. In contrast to their paper, our work in chapter 5 focuses on the co-existence of both sales and insurance, which is not discussed in Parlour and Plantin (2007). As such, we are not interested in the existence of CRT as in their paper. Rather, we are interested in the effect that multiple instruments can have on CRT. Wagner and Marsh (2005) and Allen and Carletti (2006) model CRT in terms of loan sales to outside the banking sector. Wagner and Marsh (2005) study the social impact of CRT analyzing cases where CRT itself may not be efficient. They argue that setting regulatory standards that reflect the different social costs of instability in the bank and insurance sector would be welfare improving. Allen and Carletti (2006) show how a default by an insurance company can cascade into the banking sector causing a contagion effect when these two parties are linked through credit derivatives. Our work in chapters 3 and 4 differs from these papers because they do not consider the agency problems of insurance contracts. As a result, they do not discuss the consequences that instability can have on the contracting environment, and how this affects the behavior of the parties involved. These papers differ from our work in chapter 5 in that they do not address the choice of CRT instrument as we do. Instead, they assume sales is the only instrument possible.

We contribute to the literature on insurance economics by raising the issue of counterparty risk which has received little attention. Henriët and Michel-Kerjan (2006) recognize that insurance contracts need not fit the traditional setup in which the insurer is the principal and the insured, the agent. The authors relax this assumption and allow the roles to change. Their paper however does not consider the possibility of counterparty risk as our work in chapters 3 and 4 does, as they assume that neither party can fail. Plantin and Rochet (2007) raise the issue of prudential regulation of insurance companies. They give recommendations for countries to better regulate these parties. This work does not consider the insurance contract itself under counterparty risk as is done here. Consequently, the authors do not analyze the effects of counterparty risk on the informational problems. Instead, they conjecture an agency problem arising from a corporate governance standpoint. In chapters 3 and 4, we analyze an agency problem driven entirely by the investment incentives of the insurer.

Chapter 3

Counterparty Risk in Financial

Contracts: Should the Insured Worry

About the Insurer?

In this chapter, we develop an agency model to analyze an insurer's optimal investment decision when failure is a possibility. We demonstrate that an insurer's investment choice may be inefficient by showing that a moral hazard problem exists on this side of the market. This insurer moral hazard problem does have an upside however, as we show that it can alleviate the adverse selection problem on the part of the insured. To analyze the insurance contract, we will use the market for credit risk transfer as our motivation; however, we can think of this chapter as developing a general insurance model

The credit crisis of 2007/2008 has given considerable media attention to counterparty risk.¹ The bond insurers and, specifically, the monoline insurers² have experienced nothing

¹For a review on the causes and symptoms of the credit crises see Greenlaw et al. (2008) and Rajan (2008).

²Monoline insurers guarantee the timely repayment of bond principal and interest when an issuer defaults.

short of a crisis as they try to pay claims to insured parties. The question then remains: what are the incentives of those parties who insure credit risk? This chapter will attempt to address this question in a general insurance framework. Accordingly, we will not focus specifically on bond insurers. The framework can be easily adapted however to include some of the features that are unique to the bond insurance environment.

The development of credit risk transfer markets in itself may be a positive development; however, two features make these markets potentially different (and dangerous) when compared to traditional insurance markets. First, the potential for unstable counterparties. In other words, potentially large credit risks are being ceded to parties such as hedge funds which may or may not be in a better position to handle them.³ The second feature which is unique to this market is the large size of the contracts.⁴ It would seem prudent then to ask the question of how stable is, and what are the incentives of the insurer? This entails a study of counterparty risk. In what is to follow, we define counterparty risk as the risk that when a claim is made, the insurer is unable to fulfil its obligations.

This chapter arrives at two novel results. The first is that there can exist a moral hazard on the part of the insurer. We call this the *moral hazard result*. This moral hazard arises because the insurer may choose an excessively risky portfolio. The intuition behind this result is as follows. There are two key states of the world that enter into the insurer's decision problem: the first in which a claim is not made, and the second in which it is. We assume that the insurer can default in both of these states if it receives an unlucky draw. However, it can invest and influence the chances that it fails. This investment choice comes with a tradeoff: what reduces the probability of failure the most in the state in which a

³Fitch (2006) reports that banks are the largest insured party in this market. On the insurer side, banks and hedge funds are the largest, followed by insurance companies and other financial guarantors. It should be noted that the author's of the Fitch report suspect that banks are the largest insurers, followed by hedge funds; however, they add that the data is poor and that other research reports do not support this.

⁴The two typical credit default swap contract denominations are \$5 and \$10 million.

claim is not made, makes it more likely that the insurer will fail in the state in which it is. For example, if the insurer believes that the contract is relatively safe, it may be optimal to put capital into less liquid assets to reap higher returns, and lower the chance of failure in the state in which a claim is not made. However, assets which yield these higher returns can also be more costly to liquidate, and therefore make it more difficult to free up capital if a claim is made. The moral hazard arises because the premium is not made conditional on an observed outcome, rather it is paid upfront.⁵ Therefore, there is no way to influence the insurer's investment decision by imposing penalties. We show that the resulting equilibrium is inefficient.

The second result deals with the adverse selection problem that may be present because of the superior information that the insured has about the underlying claim. Akerlof (1970) describes the dangers of informational asymmetries in insurance markets. In his seminal paper, it is shown how the market for good risks may break down, and one is left with insurance only being issued on the most risky of assets, or in Akerlof's terminology, lemons. The incentive that underlies this result is that the insured only wishes to obtain the lowest insurance premium. This incentive will still be present in our model; however, we uncover an opposing incentive.

In our work we show that the safer the underlying claim is perceived to be, the more severe the moral hazard problem is. Consequently, conditional on a claim being made, counterparty risk is higher for insured assets perceived by the insurer as safer. We show that truthful revelation can be optimal for the insured with a poor quality asset. In this case, the insurer will have incentives more in line with the insured, and consequently the insured is subjected to less counterparty risk. We show that this new effect, which we

⁵To have a conditional premium would require a higher payment from the insured to the insurer when the insurer is able to pay than when it is not. This goes against the nature of an insurance contract: when a claim is made, the insured party does not want to pay the insurer.

call the *counterparty risk effect* allows a unique separating equilibrium to be possible. This result is new in that separation can occur in the absence of a costly signalling device. After Akerlof's (1970) model showed that no separating equilibrium can exist, the literature developed the concept of signalling devices with such famous examples as education in Spence's job market signalling paper. These papers allowed the high (safe) type agents to separate themselves by performing a task which is "cheaper" for them than for the low (risky) type agents. In this chapter, we achieve separation by the balance between the insured's desire for the lowest insurance premium, and the desire to be exposed to the least counterparty risk. One can think of this result as adding to the *cheap talk* literature by showing an insurance problem in which costless communication can bring about separation of types.⁶ We call this the *separating equilibrium result*.

The chapter proceeds as follows: Section 3.1 outlines the model and solves the insurer's problem. Section 3.2 determines the equilibria that can be sustained when asymmetric information is present. Furthermore, this section shows a moral hazard problem on the part of the insurer by determining that an inefficient investment choice is made. Section 3.3 concludes. Many of the longer proofs are relegated to appendix A.

3.1 The Model Setup

The model is in three dates indexed $t = 0, 1, 2$. There are two main agent types, an insured party, whom we will call a bank, and multiple risk insurers, whom we will call Insuring Financial Institutions (IFIs). As well, there is an underlying borrower who has a loan with the bank. We model this party simply as a return structure. The size of the loan is normalized to 1 for simplicity. We motivate the need for insurance through an exogenous

⁶For a review of the cheap talk literature, see Farrell and Rabin (1996).

parameter (to be explained below) which makes the bank averse to risk. We assume there is no discounting; however, adding this feature will not affect our qualitative results.

3.1.1 The Bank

The bank is characterized by the need to shed credit (loan) risk. We use the example of a bank that faces capital regulation and must reduce its risk, or else could face a cost (which we denote by $Z \geq 0$). It is this cost that makes the bank averse to holding the risk and so finds it advantageous to shed it through insurance. This situation can be thought of as arising from an endogenous reaction to a shock to the bank's portfolio; however for simplicity, we will not model this here. There are two types of loans that a bank can insure, a safe type (S) and a risky type (R). A bank is endowed with one or the other with equal probability for simplicity. We assume that the return on either loan is $R_B > 1$ if it succeeds which happens with probability p_S (p_R) if it is safe (risky), where $1 > p_S > p_R > 0$. We assume that the return of a failed loan is zero for simplicity. The loan type is private knowledge to the bank and reflects the unique relationship between them and the underlying borrower. We assume that the loan can be costlessly monitored, so that there is no moral hazard problem in the bank-borrower relationship. In chapter 4, we relax this assumption and show that introducing costly monitoring does not change the qualitative results of the model. Note that there is nothing in the analysis to follow that requires this to be a single loan. When we interpret this as a single loan, the insurance contracts to be introduced in section 3.1.3 will resemble that of a *credit default swap*. In the case that this is a return on many loans, the insurance contract will closely resemble that of a *portfolio default swap* or *basket default swap*.⁷

⁷A portfolio or basket default swap is a contract written on more than one loan. There are many different configurations of these types of contracts. For example, a first-to-default contract says that a claim can be

The regulator requires the bank to insure a fixed, and equal proportion of either loan. For simplicity, the bank must insure a proportion γ of its loan, regardless of its type.⁸ As will be shown in section 3.2, we are able to obtain a separating equilibrium without any signalling device. In standard models of insurance contracts, a costly signaling device can be the amount of insurance that the safe and risky type take on. The safe type is able to signal that it is safe by taking on less insurance (e.g., a higher deductible). In this chapter, we shut down this mechanism for obtaining a separating equilibrium so that we can better understand the mechanism that counterparty risk creates. We impose the exogenous cost Z on the bank if the loan defaults and it is not insured for the appropriate amount, or if it is insured for the appropriate amount, but the counterparty is not able to fulfil a claim.⁹ This two part linearity of the payoff function is what imposes the same contract size on both bank types.¹⁰

In what follows, we only model the payoff to this loan for the bank; however, it can be viewed as only a portion of its total portfolio. For simplicity, we assume that the bank cannot fail. Allowing the bank to fail will not affect our qualitative results since it will not affect the insurance contract to be introduced in section 3.1.3. We now turn to the

⁸The assumption of a fixed amount of insurance regardless of type is not crucial. We can think of γ being solved for by the bank's own internal risk management. Therefore, we could have a differing γ depending on loan quality. What is important in this case is that the IFI is not able to perfectly infer the probability of default from γ . This assumption is justified when the counterparty does not know the exact reason the bank is insuring. To know so would require them to know everything about the bank's operations, which should be excluded as a possibility. In this enriched case, γ can be stochastic for each loan type reflecting different (private) financial situations for the bank. This topic has been addressed in the new Basel II accord which allows banks to use their own internal risk management systems in some cases to calculate needed capital holdings. One reason for this change is because of the superior information banks are thought to have on their own assets; regulators have acknowledged that the bank itself may be in the best position to evaluate their own risk.

⁹It is not crucial that Z be the same for both the situation in which the bank does not purchase insurance, and when it does but a claim cannot be fulfilled by the counterparty. We could posit different values for each situation; however, this will not affect our qualitative results.

¹⁰A smooth concave payoff function can be employed instead of a two part linear payoff function. This, however, will only distract from the separating mechanism that this chapter uncovers below.

modelling of the IFI.

3.1.2 The Insuring Financial Institution

Without the sale of the insurance contract, we assume that the IFI has a payoff function of the form:

$$\Pi_{IFI}^{No\ Insurance} = \int_0^{\bar{R}_f} \theta f(\theta) d\theta + \int_{\underline{R}_f}^0 (\theta - G) f(\theta) d\theta, \quad (3.1)$$

where $f(\theta)$ is assumed for simplicity to be a uniform probability density function (with corresponding distribution $F(\theta)$) representing the random valuation of the IFI's portfolio,¹¹ and G is a bankruptcy cost. One interpretation of G is lost goodwill, but any reason for which the IFI would not like to go bankrupt will suffice.¹² Note that bankruptcy occurs when the portfolio draw is in the set $[\underline{R}_f, 0]$, where it is assumed $\underline{R}_f < 0$.¹³ It is assumed that the IFI receives this payoff at time $t = 2$, so that at time $t = 1$, the random variable θ represents the portfolio value if it could be costlessly liquidated at that time. However, the IFI's portfolio is assumed to be composed of both liquid and illiquid assets. In practice, we observe financial institutions holding both liquid (e.g., t-bills, money market deposits) and illiquid (e.g., loans, some exotic options, some newer structured finance products) investments on their books.¹⁴ Because of this, if the IFI wishes to liquidate some of its

¹¹The uniform assumption can be relaxed to a general distribution, provided that it satisfies some conditions. For example, there must be mass in a region above and below zero.

¹²Note that in the case of monoline insurance, we can think of bankruptcy as a ratings downgrade. The monoline business is based on having a better credit rating than the client in a process called *wrapping*. Without a good rating, it would not be profitable for firms to insure themselves with a monoline.

¹³The fact that failure of the IFI corresponds to negative draws is not crucial. We could have f with mass only on positive draws, and define a cutoff value that is strictly greater than zero to be interpreted as IFI default.

¹⁴If another bank acts as the IFI, it is obvious that many illiquid assets are on its the books. However, this is also the very nature of many insurance companies and hedge funds businesses. In the case of insurance companies as the IFI, substantial portions of their portfolios may be in assets which cannot be liquidated easily (see Plantin and Rochet (2007)). In the case of hedge funds as the IFI, many of them specialize in trading in illiquid markets (see Brunnermeier and Pederson (2005) for example).

portfolio at time $t = 1$, it will be subject to a liquidity cost which we discuss below in section 3.1.3. Since the IFI's payoff before taking on the insurance contract will not play a role in our results, we set $\Pi_{IFI}^{No\ Insurance} = 0$ for simplicity.

3.1.3 The Insurance Contract

We now introduce the means by which the bank is insured by the IFI. Because of the possible cost Z , at time $t = 0$ the bank requests an insurance contract in the amount of γ for one period of protection. Therefore, the insurance coverage is from $t = 0$ to $t = 1$. To begin, we assume that the bank contracts with one IFI who is in Bertrand competition.¹⁵ The IFI forms a belief b about the probability that the bank loan will default. In section 3.2 we will show how b is formed endogenously as an equilibrium condition of the model. In exchange for this protection, the IFI receives an insurance premium $P\gamma$, where P is the per unit price of coverage. The IFI chooses a proportion β of this premium to put in a liquid asset that, for simplicity, has a rate of return normalized to one in both $t = 1$ and $t = 2$, but can be accessed at either time period. The remaining proportion $1 - \beta$ is put in an illiquid asset with an exogenously given rate of return of $R_I > 1$ which pays out at time $t = 2$.¹⁶ This asset can be thought of as a two period project that cannot be terminated early. It is this property that makes it illiquid. As will be shown below, the payoff to the IFI is linear in β in the state in which a claim is not made and therefore a redefinition of the return would allow us to capture uncertainty in the illiquid asset to make it risky as well as

¹⁵This assumption is relaxed in section 4.3 which is not intended for publication. In the extension, we allow the bank to spread the contract among multiple IFIs.

¹⁶We can think of these as two assets that are in the IFI's portfolio; however, we assume that the amount is small so that the illiquid asset and the original portfolio are uncorrelated. Adding correlation would only complicate the analysis and would not change the qualitative results.

illiquid. Therefore there is no loss of generality assuming this return is certain.¹⁷ The key difference between these two assets is that the liquid asset is accessible at $t = 1$ when the underlying loan may default, whereas the illiquid asset is only available at $t = 2$.¹⁸

For the remaining capital needed (net of the premium put in the liquid asset) if a claim is made, we assume that the IFI can liquidate its portfolio. Recall that the IFI's initial portfolio (with return governed by F) contains assets of possibly varying degrees of liquidity. To capture this, we assume that the IFI has a liquidation cost represented by the invertible function $C(\cdot)$ with $C'(\cdot) > 0$, $C''(\cdot) \geq 0$, and $C(0) = 0$. The weak convexity of $C(\cdot)$ implies that the IFI will choose to liquidate the least costly assets first, but as more capital is required, it will be forced to liquidate illiquid assets at potentially fire sale prices.¹⁹ $C(\cdot)$ takes as its argument the amount of capital needed from the portfolio, and returns a number that represents the actual amount that must be liquidated to achieve that amount of capital. This implies that $C(x) \geq x \forall x \geq 0$ so that $C'(x) \geq 1$. For example, if there is no cost of liquidation and if x is required to be accessed from the portfolio, the IFI can liquidate x to satisfy its capital needs. However, because liquidation may be costly in this model, the IFI must liquidate $y \geq x$ so that after the liquidation function $C(\cdot)$ shrinks the value of the capital, the IFI is left with x . If $C(\cdot)$ is linear, our problem becomes a linear program, and as will soon become apparent, this yields an extreme case of moral hazard.

At time $t = 1$, the IFI learns a valuation of its portfolio; however, the return is not realized until $t = 2$. This could be relaxed so that the IFI receives a fuzzy signal about the return, however, this would yield no further insight into the problem. Also at $t = 1$, a claim

¹⁷As well, the choice between the liquid and illiquid assets is not crucial. The choice can be between a risky and riskless asset (both liquid) and the qualitative results of the chapter will still hold.

¹⁸This can be relaxed to allow some recovery value of the illiquid asset, but the qualitative results of the model would remain the same.

¹⁹There is a growing literature on trading in illiquid markets and fire sales. See for example Subramanian and Jarrow (2001), and Brunnermeier and Pedersen (2005).

is made if the underlying borrower defaults. If a claim is made, the IFI can liquidate its portfolio to fulfil its obligation of γ . If the contract cannot be fulfilled, the IFI defaults. We assume for simplicity that if the IFI defaults, the bank receives nothing.²⁰ At time $t = 2$, the IFI and bank's return are realized. This setup implies that the uncertainty in the model is resolved at time $t = 1$; however, a costly liquidation problem remains from $t = 1$ to $t = 2$. Figure 5.1 summarizes the timing of the model.

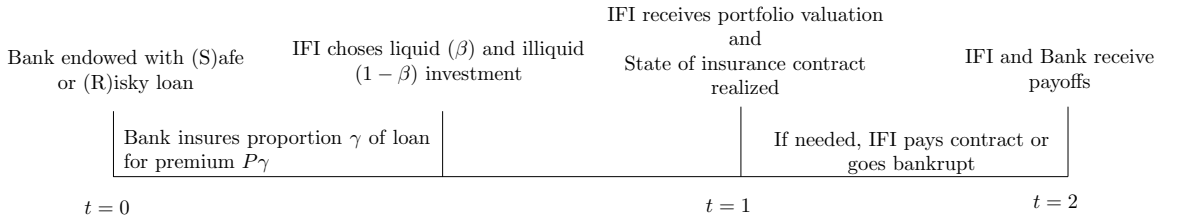


Figure 3.1: Timing of the Model

The expected payoff of the IFI can be written as follows.

$$\begin{aligned}
\Pi_{IFI} = & (1 - b) \left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{\bar{R}_f} \theta f(\theta) d\theta + \int_{\underline{R}_f}^{-P\gamma(\beta+(1-\beta)R_I)} (\theta - G) f(\theta) d\theta \right] \\
& + (b) \left[\int_{C(\gamma-\beta P\gamma)}^{\bar{R}_f} (\theta - C(\gamma - \beta P\gamma) - \beta P\gamma) f(\theta) d\theta \right] \\
& + (b) \left[\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} (\theta - G) f(\theta) d\theta \right] \\
& + P\gamma(\beta + (1 - \beta)R_I)
\end{aligned} \tag{3.2}$$

The first term is the expected payoff when a claim is not made, which happens with probability $1 - b$ given the IFI's beliefs. The $-P\gamma(\beta + (1 - \beta)R_I)$ term in the integrand represents the benefit that engaging in these contracts can have: it reduces the probability of portfolio default when a claim is not made. We assume that \underline{R}_f is sufficiently negative

²⁰In reality, the insuring institution would typically pay the full protection value, but would receive the bond of the underlying borrower in return, which may still have a recovery value. Inserting this recovery value into the model only complicates the mathematics without changing the qualitative results.

so that $P\gamma(\beta + (1 - \beta)R_I) < |\underline{R}_f|$. Since P and β are both bounded from above,²¹ it follows that this inequality is satisfied for a finite \underline{R}_f . This assumption ensures that the IFI cannot completely eliminate its probability of default in this state. Recall that before the IFI engaged in the insurance contract, it would be forced into insolvency when the portfolio draw was less than zero. However, if a claim is not made, it can receive a portfolio draw that is less than zero and still remain solvent (so long as the IFI's draw is greater than $-P\gamma(\beta + (1 - \beta)R_I)$).

The second term is the expected payoff when a claim is made, which happens with probability b given by the IFI's beliefs. The term $C(\gamma - \beta P\gamma)$ represents the cost to the IFI of accessing the needed capital to pay a claim. Notice that the loans placed in the illiquid asset are not available if a claim is made. Furthermore, the probability of default for the IFI increases in this case. To see this, notice that before engaging in the insurance contract, the IFI defaults if its portfolio draw is $\tilde{\theta} \in [\underline{R}_f, 0]$. After the insurance contract is sold, default occurs if the draw is $\tilde{\theta} \in [\underline{R}_f, C(\gamma - \beta P\gamma) > 0]$. To ensure that the IFI prefers to pay the insurance contract when solvent, we assume $G \geq C(\gamma - \beta P\gamma) + \beta P\gamma$.²² Intuitively, if this condition were not to hold, the IFI would rather declare bankruptcy than fulfil the claim, regardless of its portfolio draw. The final term in (3.2) ($P\gamma(\beta + (1 - \beta)R_I)$) is the payoff of the insurance premium given how it was invested.

As stated previously, counterparty risk is defined as the risk that the IFI defaults, conditional on a claim being made. Therefore, counterparty risk is represented in the model by

$$\int_{\underline{R}_f}^{C(\gamma - \beta P\gamma)} f(\theta) d\theta.$$

²¹This is true for β by construction and will be proven for P in Lemma 2.

²²We can think of G as a justification of the assumption that the IFI does not pay a recovery value to the bank if it fails.

3.1.4 IFI Behavior

We now characterize the optimal investment choice of the IFI and the resulting market clearing price. We begin by looking at the IFI's optimal investment decision. The following lemma characterizes the optimal behavior conditional on a belief (b) and a price (P). The IFI is shown to invest more in the liquid asset if it believes a claim is more likely to be made. Let β_S^* (β_R^*) be the optimal choice of the IFI given it believes that the loan is safe (risky).

Lemma 1 *The optimal investment in the liquid asset (β^*) is weakly increasing in the belief of the probability of a claim (b). Consequently, $\beta_R^* \geq \beta_S^*$.*

Proof. *See appendix.*

It follows that the relationship in this proposition is strict when β^* attains an interior solution. Note that the implicit expression for β^* is given by (A.3) found in the proof to this lemma. It is easy to see that the optimal investment is conditional on a price P . We define P^* as the market clearing price. To characterize it, we use the assumption that the IFI must earn zero profit from engaging in the insurance contract.²³ The following lemma yields both existence and uniqueness of the market clearing price P^* .

Lemma 2 *The market clearing price is unique and in the open set $(0, 1)$.*

Proof. *See appendix.*

²³Lemma 4 shows that this assumption can be relaxed to allow more market power to the IFI without affecting our results.

We now analyze the properties of the market clearing price P^* . The following lemma shows that as the IFI's belief about the probability a claim increases, so too must the premium increase to compensate them for the additional risk. Let P_S^* (P_R^*) be the market clearing price given the IFI believes that the loan is safe (risky).

Lemma 3 *The market clearing price P^* is increasing in the belief of the probability of a claim (b). Consequently, $P_R^* > P_S^*$.*

Proof. *See appendix.*

The lemma yields the intuitive result that our pricing function $P(b)$ is increasing in b . The actual price, however, is not the focus of this chapter; it can be computed from the IFI's zero profit condition, $\Pi_{IFI} = 0$ where Π_{IFI} is given by (3.2). We now turn to the issue of bargaining power.

In the preceding analysis, we assumed Bertrand competition among the IFIs. This allowed for a zero profit condition to pin down the market clearing price P^* . The following lemma shows that this is not a crucial assumption. This is done by showing that additional profit by the IFI will have no effect on counterparty risk, unless the underlying loan is 'very' risky.

Lemma 4 *Denote β^* as the IFI's optimal choice given the zero profit price P^* . Consider the IFI being able to make positive profit so that the market clearing price increases. Compared to the zero profit case, counterparty risk remains unchanged when $\beta^* \in [0, 1)$ and decreases if $\beta^* = 1$.*

Proof. *See appendix.*

The intuition behind this result is that if we increase the amount given to the IFI without changing the beliefs, this will have no effect on the marginal benefit of choosing the liquid asset. In its optimization problem, the IFI makes its choice by investing in the liquid asset until the marginal benefit of doing so falls to the level of that of investing in the illiquid asset. Since increasing just the premium will not change the IFI's beliefs (b), this will not change the absolute amount of the premium put in the liquid asset. Instead, all additional capital will be put into the illiquid asset (which will have a higher marginal return at that point). The lemma shows that the only time counterparty risk will decrease is when $\beta^* = 1$, or in other words, when the loan is 'very' risky (recall that Lemma 1 showed that β^* is increasing in b). This case can only be obtained when both before and after the price increase, the underlying loan is so risky that it is never optimal to put any capital in the illiquid asset, so that all additional capital goes into the liquid asset.

3.2 Equilibrium Beliefs

Akerlof (1970) showed how insurance contracts can be plagued by the 'lemons' problem. One underlying incentive in his model that generates this result is that the insured wishes only to minimize the premium paid. It is for this reason that high risk agents would wish to conceal their type. Subsequent literature showed how the presence of a signalling device can allow a separating equilibrium to exist. What is new in our work is that no signalling device is needed to justify the existence of a separating equilibrium. We call the act of concealing one's type for the benefit of a lower insurance premium the *premium effect*. In this section we show that this effect may be subdued in the presence of counterparty risk. This is accomplished by demonstrating another effect that works against the premium effect that we call the *counterparty risk effect*. The intuition of this new effect is that if high

risk (risky) agents attempt to be revealed as low risk (safe), a lower insurance premium may be obtained, but the following lemma shows that counterparty risk will increase.

Lemma 5 *If b decreases, but the actual probability of a claim does not, counterparty risk rises whenever $\beta^* \in (0, 1]$.*

Proof. *See appendix.*

There are two factors that contribute to this result. First, Lemma 3 showed that as the perceived probability of default decreases, the premium also decreases and therefore leaves less capital available to be invested. Second, Lemma 1 showed that the IFI will put more in the illiquid asset as b decreases. Combining these two factors, the counterparty risk increases. The only case in which the counterparty risk will not rise is when the bank is already investing everything in the illiquid asset, so that as b decreases, everything is still invested in the illiquid asset.

To analyze the resulting equilibria, we employ the concept of a Perfect Bayesian Nash Equilibrium (PBE). Define $i \in \{S, R\}$ to represent the two possible bank types, and define the message $\mathcal{M} \in \{S, R\}$ to represent the report that bank type i sends to the IFI. Let the bank's payoff be $\Pi(i, \mathcal{M})$ representing the profit that a type i bank receives from sending the message \mathcal{M} . Formally, an equilibrium in our model is defined as follows.

Definition 1 *An equilibrium is defined as a portfolio choice β , a price P , and a belief b such that:*

1. *b is consistent with Bayes' rule where possible.*
2. *Choosing P , the IFI earns zero profit with β derived according to the IFI's problem.*

3. *The bank chooses its message so as to maximize its expected profit.*

To proceed we ask: is there a separating equilibrium in which both types are revealed truthfully? The answer without counterparty risk is *no*. The reason is that without counterparty risk, it is costless for the bank with a risky loan to imitate a bank with a safe loan. However, with counterparty risk, it is possible that both types credibly reveal themselves so that separation occurs. To begin, assume that the IFI's beliefs correspond to a separating equilibrium. Therefore, if $\mathcal{M} = S$ ($\mathcal{M} = R$) then $b = 1 - p_S$ ($b = 1 - p_R$). We now write the profit for a bank with a risky loan given a truthful report ($\mathcal{M} = R$).

$$\begin{aligned} \Pi(R, R) = & p_R R_B + \gamma(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{\bar{R}_f} dF(\theta) \\ & - \gamma(1 - p_R) Z \int_{\underline{R}_f}^{C(\gamma - \beta_R^* P_R^* \gamma)} dF(\theta) - \gamma P_R^* \end{aligned} \quad (3.3)$$

The first term represents the expected payoff to the bank when the loan does not default. The second term represents the expected payoff on the insured portion of the loan when the loan defaults and the IFI is able to pay the claim. Notice that the IFI's beliefs are such that the IFI is risky. The third term represents the expected payoff when the loan defaults and the IFI fails and so is unable to fulfil the insurance claim. The final term is the insurance premium that the bank pays to the IFI. We now state the profit of a risky bank who reports that they are safe ($\mathcal{M} = S$).

$$\begin{aligned} \Pi(R, S) = & p_R R_B + \gamma(1 - p_R) \int_{C(\gamma - \beta_S^* P_S^* \gamma)}^{\bar{R}_f} dF(\theta) \\ & - \gamma(1 - p_R) Z \int_{\underline{R}_f}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) - \gamma P_S^* \end{aligned} \quad (3.4)$$

We now find the condition under which a risky bank wishes to truthfully reveal its type.

$$\Pi(R, R) \geq \Pi(R, S) \Rightarrow$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R^* - P_S^*}_{\text{amount extra to be paid in insurance premia}} \quad (3.5)$$

From Lemmas 1 and 3 we know that $C(\gamma - \beta_R^* P_R^* \gamma) < C(\gamma - \beta_S^* P_S^* \gamma)$ and therefore the left hand side represents the counterparty risk that a risky bank saves by reporting truthfully. This is the *counterparty risk effect*. The right hand side represents the savings in insurance premia that the bank would receive by misrepresenting its type. This is the *premium effect*. The inequality (3.5) represents the key condition for the separating equilibrium to exist. In a typical insurance problem without counterparty risk, the left hand side must be equal zero. Consequently, in the absence of counterparty risk, the risky type will always want to misrepresent its type. We now turn to a bank with a safe loan and repeat the same exercise.

$$\Pi(S, S) \geq \Pi(S, R) \Rightarrow$$

$$\underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R^* - P_S^*}_{\text{amount to be saved in insurance premia}} \quad (3.6)$$

Since $C(\gamma - \beta_R^* P_R^* \gamma) < C(\gamma - \beta_S^* P_S^* \gamma)$, the left hand side represents the amount of counterparty risk that the bank will save if it conceals its type. The right hand side represents the amount of insurance premia that the bank will save if it reports truthfully. Therefore, when (3.5) and (3.6) simultaneously hold, this equilibrium exists. For an example of when this can hold, take the case in which the safe loan is "very" safe. In particular, we let $p_S \rightarrow 1$ and obtain the following two expressions.

$$\underbrace{0}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R^* - P_S^*}_{\text{amount to be saved in insurance premia}} \quad (3.7)$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R^* - P_S^*}_{\text{amount extra to be paid in insurance premia}} \quad (3.8)$$

Note here that $P_S \rightarrow 0$ since the probability of default of the safe loan is approaching zero. Inequality (3.7) is satisfied trivially, while (3.8) is satisfied for Z sufficiently large. Recall that Z can be interpreted as the cost of counterparty failure when a claim is made. Therefore separation can be achieved when there is a high enough ‘penalty’ on the bank for taking on counterparty risk. The intuition is that a larger penalty forces the bank to internalize the counterparty risk more. As a result, more information is revealed in the market. This is a sense in which counterparty risk may be beneficial to the market, since it can help alleviate the adverse selection problem caused by asymmetric information. We now state the first major result.

Proposition 1 *In the absence of counterparty risk, no separating equilibrium can exist. When there is counterparty risk, the moral hazard problem allows a unique separating equilibrium to exist in which each type of bank truthfully announces its loan risk. Sufficient conditions for this include that the safe loan is relatively safe and the bankruptcy cost Z is large.*

Proof. *See appendix.*

This proposition shows that a moral hazard problem on the part of the insurer can alleviate a possible adverse selection problem on the part of the insured. The separating equilibrium corresponds to the case in which the *premium effect* dominates for the bank with a safe loan, while the *counterparty risk effect* dominates for the bank with a risky loan. Note that there can be no separating equilibrium (different than the one above) in which the safe type reports that it is risky, and the risky type reports that it is safe.

There are also two pooling equilibria that may exist. The first pooling equilibrium occurs when both the safe and risky bank report that they are safe. In this case, the *premium effect* dominates for both types so that the IFI does not update its prior beliefs. The second pooling equilibrium occurs when both the safe and risky bank report that they are risky. In this case, the *counterparty risk effect* dominates for both types.²⁴ We formalize both of these pooling equilibria in the proof to Proposition 1.

We now remove a key contracting imperfection to highlight the inefficiency in the IFI's investment choice and formally prove the existence of a moral hazard problem.

3.2.1 Contract Inefficiency

In this section, we imagine a planning problem wherein the planner can control the investment decision of the IFI. However, we maintain the IFI's beliefs and zero profit condition. We show that regardless of the beliefs of the IFI, the planner can always do better than is done in equilibrium by increasing the amount of capital put in the liquid asset. Therefore, this section will show that we can get closer to a first best allocation by removing this contracting imperfection, thereby highlighting the moral hazard problem. We denote the solution to the planner's problem described given any belief b as β_b^{pl} , with resulting price P_b^{pl} . The following lemma shows that the equilibrium price given the beliefs b , P_b^* must be weakly less than the planning price P_b^{pl} .

Lemma 6 *There is no price $\tilde{P} < P_b^*$ such that the IFI can earn zero profit. This implies that $P_b^* \leq P_b^{pl}$.*

²⁴In the separating equilibrium case, the beliefs are fully defined by Bayes' rule. In the first pooling equilibrium, any off-the-equilibrium path belief with $b > \frac{1}{2}(2 - p_S - p_r)$ if risky is reported is consistent for the IFI with the Cho-Kreps (1987) intuitive criterion. In the second pooling equilibrium, any off-the-equilibrium path belief with $b < \frac{1}{2}(2 - p_S - p_r)$ if safe is reported is consistent with the Cho-Kreps (1987) intuitive criterion.

Proof. By Definition, $\Pi_{IFI}(\beta_b^*, P_b^*) = 0$ (where Π_{IFI} is defined by (3.2)). Then, if $\tilde{P} < P_b^*$ it must be the case that $\Pi_{IFI}(\beta_b^*, \tilde{P}) < \Pi_{IFI}(\beta_b^*, P_b^*)$. Consider $\tilde{\beta} \in [0, 1]$. Since β_b^* is optimal, then $\Pi_{IFI}(\tilde{\beta}, P_b^*) \leq \Pi_{IFI}(\beta_b^*, P_b^*) = 0$. Since the IFI cannot earn negative profit, it follows that $\tilde{P} \geq P_b^*$.

■

We now state the second major result. The following proposition shows that the IFI chooses a β^* that is too small as compared to that of the planner's problem β^{pl} for any belief of the IFI. The proposition shows that the insurer moral hazard problem causes the level of counterparty risk in equilibrium to be strictly too high (so long as $\beta^* \in [0, 1)$).

Proposition 2 *Given an equilibrium portfolio decision β^* with $\beta^* < 1$, a social planner would choose $\beta^{pl} > \beta^*$ so that the level of counterparty risk in equilibrium is too high.*

Proof. See appendix.

The intuition behind this result comes from two sources. First, since the social planning problem corresponds to optimizing the bank's payoff while keeping the IFI at zero profit, the bank strictly prefers to have the IFI invest more in the liquid asset. Second, the IFI must be compensated for this *individually* sub-optimal choice of β by an increase in the premium. Since from Lemma 6, P weakly increases (in the proof of Proposition 2, we show that in this case, the increase is strict), counterparty risk falls (i.e. $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$ falls). In other words, the moral hazard problem on the part of the IFI is characterized by an inefficiency in the investment choice. The key restriction on the contracting space that yields this result is that the insurance premium is paid upfront and so the bank cannot condition its payment on an observed outcome. In the competitive equilibrium case, the

bank knows that the IFI will invest too little into the liquid asset, and therefore lowers its payment accordingly (as from Lemma 4, any additional payment beyond what would yield zero profit to the IFI would be put into the illiquid asset and have no effect on counterparty risk).

3.3 Conclusion

In a setting in which insurers can fail, we construct a model to show that a new moral hazard problem can arise in insurance contracts. If the insurer suspects that the contract is safe, it puts capital into less liquid assets, to take advantage of a higher return to those assets. However, the downside of this is that when a claim is made, the insurer is less likely to be able to fulfil the contract. We show that the insurer's investment choice is inefficiently illiquid. The existence of this moral hazard is shown to allow a unique separating equilibrium to exist wherein the insured freely and credibly relays its superior information. In other words, the new moral hazard problem can alleviate the possible adverse selection problem.

Chapter 4

Multiple Insured Parties, Multiple Insurers and Moral Hazard in the Insured's Problem

4.1 Introduction

In this chapter, we relax three assumptions posed in the chapter 3. The moral hazard result shown in chapter 3 holds regardless of the contract size or number of insured parties, however, the separating equilibrium result holds if one or both of the following two conditions are met: first, a contract is sufficiently large to affect the insurer's investment decision, and second, there is aggregate private risk shared among a pool of insured parties. The former case is plausible in some situations (e.g., as discussed above, in some financial markets single contracts can be large), however, the latter likely constitutes a wider range of cases. Therefore, in this chapter we generalize the model to the case of multiple insured parties,

each of which is insignificant to the insurer's investment decision. We consider the case in which the insured parties share a common component of risk (i.e., correlated risk). Martin et al. (2008) serve as an example of the importance of this type of risk in the context of the credit crises. It has become clear that there was correlated information that inside financial institutions had on the key risks being traded (i.e., bundled mortgages).

Next, We extend the model to analyze the case of multiple insurers. We show that the moral hazard problem increases as the size of the contract that each insurer takes on decreases. In a setup in which all the insurers are ex-ante identical (but not necessarily ex-post, i.e. they receive iid portfolio draws), we find that counterparty risk may remain unchanged from the case in which there is only one insurer.

Finally, we enrich the model to include a possible moral hazard problem on the part of the insured. This moral hazard arises by the insured's ability to affect the probability that a claim is made. If we use the example of a bank insuring itself on one of its loans, the literature typically assumes that a bank possesses a proprietary monitoring technology (due to a relationship with the borrower). It is straightforward to see that if the bank is fully insured, it may not have the incentive to monitor the loan and, consequently, the probability of default could rise. This represents the classical moral hazard problem in the insurance literature. This extension shows that the new moral hazard introduced in this chapter may increase the desire of the insured to monitor. This happens because counterparty risk forces the bank to internalize some of the default risk which it otherwise would not. In this section we show that with a redefinition of a payoff distribution function, the addition of this insured moral hazard problem does not affect the results of chapter 3.

We begin by analyzing the case of multiple insured parties.

4.2 Multiple Insured Parties

In this section, we analyze the case of multiple banks and one insurer. We assume there are a measure $M < 1$ of banks. This assumption is meant to approximate the case in which there are many banks, and the size of each individual bank's insurance contract is insignificant for the IFI's investment decision. Using an uncountably large number instead of a finite but large number of banks helps simplify the analysis greatly.¹ Each bank requests an insurance contract of size γ . At time $t = 0$, each bank receives both an aggregate and idiosyncratic shock (both private to the banks) which assigns them a probability of loan default. For simplicity, as in the case when there was only one bank, the return on the loan is R_B if it succeeds and 0 if it does not. We define the idiosyncratic shock by the random variable X and let it be uniformly distributed over $[0, M]$. The CDF can then be written as follows.

$$\Psi(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \frac{x}{M} & \text{if } x \in (0, M) \\ 1 & \text{if } x \geq M \end{cases}$$

Next, denote the aggregate shock as q_A and let it take the following form:

$$q_A = \begin{cases} s & \text{with probability } \frac{1}{2} \\ r & \text{with probability } \frac{1}{2}, \end{cases}$$

where $0 < s < r < 1 - M$. It follows that the probability of default of bank i is $q_i = q_A + X_i$.² We will refer to the aggregate shock as either (s)afe or (r)isky. We write the

¹An advantage to using a finite number of banks is that we could avoid any measurability issue. In the proof to Proposition 3 (which is stated at the end of this section), we detail this issue briefly and discuss how to handle it. Since the results do not depend on the continuous setup, we opt to use it for its simplification of the problem.

²Note that in chapter 3 we referred to p as a probability of success, whereas here we refer to q as a probability of failure. We make this notational change because it is more intuitive in this section to have probabilities of failure when we introduce the IFI's beliefs over the measure of defaults. Of course, the simple relationship $p = 1 - q$ holds.

conditional distribution's of bank types as $\mu(q_i : q_i \leq x | q_A = s) = \Psi(x - s)$ and $\mu(q_i : q_i \leq x | q_A = r) = \Psi(x - r)$. It follows that $\Psi(x - s)$ first order stochastically dominates $\Psi(x - r)$ since $\Psi(x - s) \geq \Psi(x - r) \forall x$. Note that this is in contrast to the usual definition of first order stochastic dominance which entails higher draws providing a 'better' outcome. In the case of this model, the opposite is true, since lower draws refer to a lower probability of default; a 'better' outcome.

4.2.1 The IFI's Problem

Because of the asymmetric information problem, the IFI does not know ex-ante whether the aggregate shock was $q_A = s$ or $q_A = r$. However, the IFI does know that the aggregate shock hits all the banks in the same way.³ Therefore, if only a subset of the banks can successfully reveal their types, this reveals the aggregate shock for the rest of them.

If solvent, the IFI must pay γ to each bank whose loan defaults. In Lemma 8 we will show that there can be no separation of types within the idiosyncratic shock. Because of this, it follows that given a fixed realization of the aggregate shock, each bank pays the same premium P .⁴ We assume that the IFI has the same choice as in section 3.1.3, so that it invests β in the liquid storage asset and $(1 - \beta)$ in the illiquid asset with return R_I . To represent the IFI's beliefs we let Y represent the measure of defaults. Furthermore, let $b(y) = \text{prob}(Y \leq y)$ be the beliefs over the measure of defaults defined over $[0, M]$. It follows that $\Psi(x - s) \geq \Psi(x - r) \forall x$ implies $b(y | q_A = s) \geq b(y | q_A = r) \forall y$. In other words, first order stochastic dominance is preserved. Since each bank insures γ , the total size of contracts insured by the IFI is: $\int_0^M \gamma d\Psi(x) = M\gamma$. The IFI's payoff can now be

³We assume this for simplicity. We can relax the assumption that all the banks receive the same aggregate shock and allow them to receive correlated draws from a distribution.

⁴We are not concerned with pinning down the price in this section. As in chapter 3, we can assume that the IFI has market power; however, it is not crucial.

written as follows (denoting ‘MB’ as ‘Multiple Banks’).

$$\begin{aligned}
\Pi_{IFI}^{MB} = & \underbrace{\int_0^{\beta PM\gamma} \int_{-PM\gamma(\beta+(1-\beta)R_I)+y\gamma}^{\bar{R}_f} \theta dF(\theta) db(y)}_{\text{Term 1}} \\
& + \underbrace{\int_0^{\beta PM\gamma} \int_{\underline{R}_f}^{-PM\gamma(\beta+(1-\beta)R_I)+y\gamma} (\theta - G) dF(\theta) db(y)}_{\text{Term 2}} + \\
& \underbrace{\int_{\beta PM\gamma}^M \int_{C(y\gamma-\beta PM\gamma)}^{\bar{R}_f} (\theta - C(y\gamma - \beta PM\gamma) - \beta PM\gamma) dF(\theta) db(y)}_{\text{Term 3}} \\
& + \underbrace{\int_{\beta PM\gamma}^M \int_{\underline{R}_f}^{C(y\gamma-\beta PM\gamma)} (\theta - G) dF(\theta) db(y)}_{\text{Term 4}} \\
& + \underbrace{(\beta + (1 - \beta) R_I) PM\gamma}_{\text{Term 5}} \tag{4.1}
\end{aligned}$$

Terms one and two represent the case in which the IFI puts sufficient capital in the liquid asset so that there is no need to liquidate its portfolio to pay claims. This happens if a sufficiently small measure of banks make claims. Since the IFI receives $PM\gamma$ in insurance premia, it puts $\beta PM\gamma$ into the liquid asset. It follows that if less than $\beta PM\gamma$ is needed to pay claims (i.e. less than βPM banks fail), portfolio liquidation is not necessary. The third and fourth terms represent the case in which the IFI must liquidate its portfolio if a claim is made. This happens if the amount they need to pay in claims is greater than $\beta PM\gamma$. $C(y\gamma - \beta PM\gamma) + \beta PM\gamma$ represents the total cost of claims, where $y\gamma - \beta PM\gamma$ is the total amount of capital the IFI needs to liquidate from its portfolio. The final term represents the direct proceeds from the insurance premium. We make the usual assumption that $G \geq C(y\gamma - \beta PM\gamma) + \beta PM\gamma$ so that the IFI wishes to fulfil the contract when they are solvent. Note that for simplicity, as in chapter 3, we assume that if a claim is made and the IFI defaults, the banks receive nothing from the IFI.

The following lemma both derives the optimal β^* and proves that counterparty risk is less when the IFI believes that the loans are more risky.

Lemma 7 *For a given aggregate shock, there is less counterparty risk when the IFI's beliefs put more weight on the aggregate shock being risky ($q_A = r$) as opposed to it being safe ($q_A = s$).*

Proof. *See appendix.*

The intuition for this result is similar to that of Lemma 5. If the IFI believes that the pool of loans is risky, it is optimal to invest more in the liquid asset. This happens because the expected number of claims is higher in the risky case so that the IFI wishes to prevent costly liquidation by investing more in assets that will be readily available if a claim is made.

We now give the conditions under which the IFI's beliefs ($b(y)$) are formed.

4.2.2 Equilibrium Beliefs

No Aggregate Shock

To analyze how the beliefs of the IFI are formed, consider the case where there is no aggregate shock. Since there is no aggregate uncertainty, the IFI's optimal investment choice remains the same regardless of whether it offers a pooling price or individual separating prices.⁵ It follows that since an individual bank's choice will have no effect on counterparty risk, only the *premium effect* is active. It is for this reason that a separating equilibrium

⁵To see this, note that with no aggregate risk, the IFI knows the average quality of banks and will use that to make its investment decision. Any bank claiming that they received the lowest idiosyncratic shock will not change the IFI's beliefs about the average quality.

in the idiosyncratic shock cannot exist. To see this, assume that each bank reveals its type truthfully. Now consider the bank with the highest probability of default, call it bank M . Since it is paying the highest insurance premium, it can lie about its type without any effect on counterparty risk, and obtain a better premium, and consequently, a better payoff. The following lemma formalizes.

Lemma 8 *There can be no separating equilibrium in which the idiosyncratic shock is revealed.*

We now introduce the aggregate shock and show that separation of aggregate types can occur.

Aggregate and Idiosyncratic Shock

Each individual bank now receives both an aggregate and an idiosyncratic shock. We can think of this procedure as putting the banks in one of two intervals, either $[s, s + M]$ or $[r, r + M]$. We know that if one bank is able to successfully reveal its aggregate shock, then the aggregate shock is revealed for all other banks. The following proposition shows that a unique separating equilibrium can exist in this setting.

Proposition 3 *There exists a parameter range in which a unique separating equilibrium in the aggregate shock can be supported.*

Proof. *See appendix.*

This insight follows from an individual bank's ability to affect the IFI's investment choice (through the IFI's beliefs). If a bank could only reveal its own shock, its premium would be insignificant to the IFI's investment decision. However, since by successfully

revealing itself, a bank also reveals the other banks, an individual's problem can have a significant effect on the IFI's investment choice. The parameter range that can support this equilibrium is similar to the case in which there was only one bank. Conditions that can support this equilibrium as unique are: Z sufficiently high, and the safe aggregate shock sufficiently low.

We now analyze the case in which the bank can contract with multiple IFIs.

4.3 Multiple IFIs

We now relax the assumption that a bank can only insure with one IFI by allowing a bank to insure with multiple IFIs. Let us consider the case with only one bank. We assume that the bank insures with a finite (and exogenous) number, N , IFIs in the market. For tractability, we assume that the N IFIs all have a portfolio that takes an IID draw from the distribution F with corresponding density f . Therefore, this gives the bank a chance to reduce how much each counterparty holds (as compared to the case in which there was only one counterparty). To contrast with the case in which $N = 1$, we assume that the penalty incurred (Z) is now linearly proportional to the number of IFIs that fail (e.g. if 1 out of 2 IFIs fail, the bank faces a cost of $\frac{Z}{2}$). The following lemma shows that in the environment described, it is equivalent to view the bank interacting with only one (modified) representative IFI. More specifically, if the bank insures with all N of the IFIs, the expected profit is derived given that there can be up to N failures. The result says that there is an equivalent problem in which there is one representative IFI, however, that IFI solves its investment problem as though it was only insuring $\frac{\gamma}{N}$ of the loan. The result comes from two key features of the setup: First, we study an extreme case in which each IFI takes a IID draw from the same distribution. Second, the bank has a linear payoff function.

Lemma 9 *Aggregation* - Denote a as the probability of failure that each of the N counterparties have. Also, denote $\Pi_{bk}(i, N - i)$ as the (expected) profit of the bank when i counterparties fail and $N - i$ counterparties succeed. Given this setup, the expected profit of the bank can be written in a simple form as follows:

$$\sum_{n=0}^N \frac{N!}{n!(N-n)!} a^n (1-a)^{N-n} \Pi_{bk}(n, N-n) = a\Pi_{bk}(1, 0) + (1-a)\Pi_{bk}(0, 1). \quad (4.2)$$

Proof. See Appendix B.

The proof proceeds by rearranging the expected profit of the bank to apply the binomial theorem to show that it collapses down to that as if only one IFI was providing the insurance. However, each IFI is now responsible for $\frac{\gamma}{N}$; a reduction in liability.

Casual intuition should tell us that when the number of IFIs increases, the counterparty risk should decrease. In Lemma 10, we show that when the optimal choice of each IFI is a corner solution, this intuition holds true. However, when an interior solution is achieved, we obtain the unexpected result that counterparty risk remains unchanged. The reason for this counterintuitive result is that when $N > 1$, each IFI behaves differently than when $N = 1$. For what is to follow, we denote the optimal β when $N = 1$ ($N > 1$) as β_1^* (β_N^*). Similarly, we denote the market clearing price per unit of protection that the bank must pay to each IFI when $N > 1$ as P_N^* . The following proposition shows that the smaller the size of the contract that an IFI engages in, the riskier it will behave, and consequently counterparty risk will remain unchanged (provided an interior solution for β_N^* is attained). What is happening is that the IFI has less obligation so that the state of the world in which a claim is made will see it liquidating less of its portfolio. Therefore, the IFI will have an incentive to put more into the illiquid asset to take advantage of its higher return.

Proposition 4 *Given an interior solution for β_N^* :*

1. *The optimal proportion of the illiquid asset bought is decreasing in the size of insurance contracts per IFI.*
2. *Counterparty risk that the bank is subjected to does not depend on N .*

Proof. See Appendix B.

This proposition shows that as the amount of the insurance contract that each IFI takes on decreases, the IFI reduces the percentage of the premium put in the liquid asset. This reduction is by the exact amount so that the counterparty risk remains unchanged. This yields the counterintuitive result that even though each IFI is insuring less of the loan, the counterparty risk that the bank must endure does not decrease.

We now analyze the consequences of a corner solution in the IFI's problem. If the IFI is investing as risky as it can ($\beta_N^* = 0$), counterparty risk decreases from the case in which $N = 1$ whenever the boundary constraint is strictly binding.⁶ What is happening is that as N becomes large, the savings in the counterparty risk comes entirely from the reduction in liability of each IFI.

The final case in which counterparty risk decreases when $N > 1$ compared to when $N = 1$ is when $\beta_N^* = 1$. What is happening is that the IFI is so cautious that even after its liability decreases, it still invests as safe as possible. One case this may apply is when the insurance contract is being written on a "very" risky loan (b close to one). The following lemma formalizes these two cases.

Lemma 10 *If $\beta_N^* = 0$ or $\beta_N^* = 1$, counterparty risk decreases in N .*

⁶This implies there is one technical case in which the counterparty risk would remain constant. It arises when the $\beta_N^* = 0$ but with the lower bound constraint not binding.

Proof. Plug $\beta_N^* = 0$ and $\beta_N^* = 1$ into the counterparty risk term $C(\gamma - \gamma P\beta)$ and notice that this is an increasing function of γ . This in turns implies that counterparty risk decreases with respect to a decrease in γ . ■

We now revert to the case of one IFI and one bank and turn our attention to the bank-borrower relationship.

4.4 Moral Hazard in the Bank-Borrower Relationship

We now relax the assumption that monitoring of the borrower is costless for the bank, thereby introducing the traditional moral hazard problem into our framework. For simplicity, we do away with the asymmetric information problem, or alternatively, assume that the needed parameterization underlying Proposition 1 is satisfied.⁷ This will allow us to focus on only one bank loan type. Define M as the amount monitored that takes a value in the compact interval $[0, \bar{M}]$. We introduce a cost of monitoring function for a loan: $c(M)$ with $c'(\cdot) > 0$, $c''(\cdot) > 0$ and $c(0) = 0$. For simplicity, we rule out corner solutions by assuming $c(\cdot)$ satisfies the Inada conditions: $c'(0) = 0$ and $c'(\bar{M}) = +\infty$.

We enrich the bank's loan to a continuous return conditional on the amount monitored (M). The expected return on the bank's loan is given by:

$$E(R; M) = \int_0^{\bar{R}} \psi h(\psi; M) d\psi - c(M), \quad (4.3)$$

where ψ is the total return from the loan and h is a density function with corresponding distribution H . We define the upper bound of the support as $\bar{R} > 1$. To include the possibility default of the bank loan, we assume that the loan fails if the realized value, $\tilde{\psi} \in [0, 1]$.

⁷The analysis of this section carries through if we allow for asymmetric information; there will be expressions for each loan type in both the pooling and separating equilibria yielding the same qualitative results.

Next, we assume that $H(\psi; M)$ satisfies the usual Monotone Likelihood Ratio Property (MLRP) so that $\frac{\partial}{\partial \psi} \left(\frac{h_M(\psi; M)}{h(\psi; M)} \right) > 0$. Finally, we make the standard assumption that the distribution satisfies the convexity-of-distribution function (CDFC) assumption (as in Hart and Holmström, 1987).⁸ This assumption implies that for any $\lambda \in [0, 1]$, and for any M, M' :

$$h(\psi; \lambda M + (1 - \lambda)M') \leq \lambda h(\psi; M) + (1 - \lambda)h(\psi; M'). \quad (4.4)$$

The MLRP and CDFC assumptions together intuitively say that increasing the monitoring, increases, at a decreasing rate, the probability that the return will be above some level ψ .⁹ We begin by analyzing the case in which the bank cannot insure itself to avoid the penalty (Z) if the loan fails.

No Insurance

When the bank does not use insurance, the optimal amount of monitoring is the incentive feasible level as follows:

$$\begin{aligned} & \int_1^{\bar{R}} \psi dH(\psi; M) + (1 - \gamma) \int_0^1 dH(\psi; M)\psi + \gamma \int_0^1 (\psi - Z) dH(\psi; M) - c(M) \\ & \geq \int_1^{\bar{R}} \psi dH(\psi; M') + (1 - \gamma) \int_0^1 dH(\psi; M') + \gamma \int_0^1 (\psi - Z) dH(\psi; M') - c(M') \\ & \qquad \qquad \qquad \forall M' \neq M. \end{aligned} \quad (4.5)$$

⁸This assumption is used in the first order approach to principal agent problems when the monitoring space is continuous. It allows us to write the infinite number of incentive constraints in one equation. We do not wish to weigh in on the debate that began in the late 1970's as to the validity of the first order approach. For those who find the CDIC assumption unpalatable, Jewitt (1988) theorem 1 shows how it can be relaxed with additional assumptions on the utility function. The alternative approach to the continuous case is to discretize the monitoring space so that there is a finite number of incentive constraints. The qualitative results of this section follow through with such a procedure, provided there are greater than 2 levels of monitoring (for reasons which will soon become apparent). We use the continuous setup for convenience.

⁹See Laffont and Martimort (2002) for a review of the first order approach to principal agent problems.

Hart and Holmstrom (1987) showed that given MLRP and CDIC, these constraints can be re-written as:

$$c'(M) = \int_0^{\bar{R}} \psi dH_M(\psi; M) + \gamma \int_0^1 Z dH_M(\psi; M), \quad (4.6)$$

where H_M is the partial with respect to M . Note that the MLRP assumption implies the weaker condition of First Order Stochastic Dominance (FOSD) which in turn implies that: $\int_0^{\bar{R}} \psi dH_M(\psi; M) > 0$. The left hand side represents the marginal cost of increasing monitoring and is given by the marginal increase in the cost of monitoring itself. The right hand side represents its marginal benefit and is comprised of both the increase in the expected value of the loan, and the reduced probability of loan default that monitoring brings.

We now analyze the case in which the bank can perfectly insure (i.e. no counterparty risk) themselves to avoid the possible cost of Z .

Insurance, No Counterparty Risk

When the bank uses insurance with no counterparty risk, the optimal amount of monitoring is as follows.

$$\begin{aligned} & \int_1^{\bar{R}} \psi dH_M(\psi; M) + (1 - \gamma) \int_0^1 \psi dH_M(\psi; M) \\ & + \gamma \int_0^1 (1 + \psi) dH_M(\psi; M) + \gamma P_M^{NCR} - c'(M) = 0, \end{aligned} \quad (4.7)$$

where P_M^{NCR} represents the marginal price with no counterparty risk. Note that since FOSD implies $\int_0^1 dH_M(\psi; M) < 0$ and Lemma 3 implies $\frac{\partial P}{\partial b} > 0$, it follows that:

$$P_M = \frac{\partial P}{\partial M} = \frac{\partial b}{\partial M} \frac{\partial P}{\partial b} = \int_0^1 dH_M(\psi; M) \frac{\partial P}{\partial b} < 0. \quad (4.8)$$

Finally, since FOSD implies both $\int_0^1 dH_M(\psi; M) < 0$ and $\int_1^{\bar{R}} dH_M(\psi; M) > 0$, we

can rewrite (4.7) in a more intuitive form:

$$c'(M) + \gamma \int_0^1 dH_M(\psi; M) = \int_0^{\bar{R}} \psi dH_M(\psi; M) + \gamma P_M^{NCR}. \quad (4.9)$$

The left hand side represents the marginal cost of monitoring. For an increase in monitoring, the bank incurs the monitoring cost itself, plus a decrease in expected payout from the claim (because claims are made less often with more monitoring). The benefits to monitoring are the increase in the expected return of the loan, plus the reduced insurance premium the bank will enjoy by reducing the probability that a claim will be made.

Comparing (4.6) and (4.9) we see that insurance reduces the incentive to monitor when the following holds:

$$Z > \frac{P_M^{NCR} - \int_0^1 dH_M(\psi; M)}{\int_0^1 dH_M(\psi; M)}. \quad (4.10)$$

In other words, when default of the loan without protection is sufficiently costly, the firm will choose to monitor more when it is not insured. Note that the sign of $P_M^{NCR} - \int_0^1 dH_M(\psi; M)$ is ambiguous and depends on the underlying parameters of the model. When $P_M^{NCR} \leq \int_0^1 dH_M(\psi; M)$, the bank will always monitor more when it is not insured (for any $Z > 0$).

We continue by adding counterparty risk to the insurance contract and show that the moral hazard problem may be less severe than in the current case.

Insurance with Counterparty Risk - Double Moral Hazard

When the bank uses insurance with counterparty risk, a double moral hazard problem is present: both the monitoring by the bank, and investment decision by the IFI occur simultaneously. Therefore, there is an optimization problem for both the bank and the IFI. We

now write the first order condition for the bank taking β^* as given.

$$\int_0^{\bar{R}} \psi dH_M(\psi; M) + \gamma \int_0^1 dH_M(\psi; M) \int_{C(\gamma-\beta^*P^*\gamma)}^{\bar{R}_f} dF(\theta) - Z\gamma \int_0^1 dH_M(\psi; M) \int_{\underline{R}_f}^{C(\gamma-\beta^*P^*\gamma)} dF(\theta) - c'(M) - \gamma P_M^{CR} = 0 \quad (4.11)$$

Where P_M^{CR} is the marginal price with counterparty risk. Because $\int_0^1 dH_M(\psi; M) < 0$, $P_M^{CR} < 0$ and $\int_0^{\bar{R}} dH_M(\psi; M) > 0$, we can rewrite (4.11).

$$\underbrace{c'(M) + \gamma \int_0^1 dH_M(\psi; M) \int_{C(\gamma-\beta^*P^*\gamma)}^{\bar{R}_f} dF(\theta)}_{\text{Marginal Cost of Monitoring}} = \underbrace{\int_0^{\bar{R}} \psi dH_M(\psi; M) + Z\gamma \int_0^1 dH_M(\psi; M) \int_{\underline{R}_f}^{C(\gamma-\beta^*P^*\gamma)} dF(\theta) + \gamma P_M^{CR}}_{\text{Marginal Benefit of Monitoring}} \quad (4.12)$$

Altering Lemma 1 to include an optimal choice of monitoring by the bank, we obtain β^* for a given M^* .

$$\begin{cases} \beta^* = 0 \\ -(1 - b(M^*))(R_I - 1)G + b(M^*)[C'(\gamma - \beta^*P\gamma)(G - C(\gamma - \beta^*P\gamma) - \beta^*P\gamma) \\ + (\bar{R}_f - C(\gamma - \beta^*P\gamma))(C'(\gamma - \beta^*P\gamma) - 1)] = (R_I - 1)(\bar{R}_f - \underline{R}_f) \\ \beta^* = 1 \end{cases}$$

Where the first case holds if $b(M^*) \leq b^*$, the second if $b(M^*) \in (b^*, b^{**})$ and the third if $b(M^*) \geq b^{**}$. Note that:

$$b^* = \frac{(R_I - 1)(G + \bar{R}_f - \underline{R}_f)}{G(R_I - 1) + C'(\gamma)(G - C(\gamma)) - (\bar{R}_f - C(\gamma))(C'(\gamma) - 1)},$$

$$\text{and } b^{**} = \frac{(R_I - 1)(G + \bar{R}_f - \underline{R}_f)}{G(R_I - 1) + C'(\gamma - P\gamma)(G - C(\gamma - P\gamma) - P\gamma) - (\bar{R}_f - C(\gamma - P\gamma))(C'(\gamma - P\gamma) - 1)}.$$

Comparing (4.12) and (4.9) we derive a condition under which the bank monitors strictly more when counterparty risk is present.

$$Z > \frac{-\int_0^1 dH_M(\psi; M) \left(1 - \int_{C(\gamma-\beta^*P^*\gamma)}^{\bar{R}_f} dF(\theta)\right) + P_M^{NCR} - P_M^{CR}}{\int_{\underline{R}_f}^{C(\gamma-\beta^*P^*\gamma)} dF(\theta)} \quad (4.13)$$

We conclude that if Z is sufficiently high, the traditional moral hazard problem will be less severe. The intuition is that the parameter Z ties the bank to the loan. If the loan and the IFI default, the bank is not protected and is subject to the cost Z . Therefore, the higher is Z , the more vigorously the bank will monitor the loan.

One of the key elements that emerges from this section is that when we introduce the classical moral hazard into the model, we need only modify the return structure of the bank loan to include a monitoring amount. In other words, the IFI simply adjusts its belief of the probability of a claim given the amount of monitoring the bank will engage in.

4.5 Conclusion

In this chapter we show that the model of chapter 3 can be extended in a number of interesting ways. First, we allowed for multiple insured parties thereby relaxing the assumption that the contract be large enough to affect the insurer's investment decision. We show that our moral hazard problem still exists, and can obtain the separating equilibrium result when there is private aggregate risk.

Second, we extend the model to analyze the case of multiple insurers. We show that the moral hazard problem increases as the size of the contract that each insurer takes on decreases and find that counterparty risk may remain unchanged from the case in which there is only one insurer.

Finally, we enrich the model to include a possible moral hazard problem on the part of the insured. We show that the new moral hazard introduced in this chapter may increase the desire of the insured to monitor. This happens because counterparty risk forces the bank to internalize some of the default risk which it otherwise would not. We also conclude in this section that our results from chapter 3 still hold with the introduction of a moral hazard on the part of the insured.

Chapter 5

Credit Risk Transfer: To Sell or To Insure

5.1 Introduction

In this chapter we analyze credit risk transfer within the banking environment. In an agency model with asymmetric information in banks assets, we find conditions under which a bank would use loan insurance versus loan sales.

Duffee and Zhou (2001) gave us our first insight into how credit derivatives and loan sales can coexist.¹ The authors show how credit derivatives can help alleviate the “lemons”

¹A loan sale trades in the same way as the sale of any other type of asset: When a loan is sold, the future income stream as well as all default risk is taken off the sellers books (note that we are not considering a situation where the bank can make a contractual guarantee about the loan’s outcome, namely, we consider only loan sales without recourse). Alternatively, in a loan insurance contract, the risk buyer agrees to cover the losses that take place if pre-defined events happen to the underlying firm. (In many cases, this event is the default of the underlying loan. However, some contracts also include things like re-structuring as a triggering event). In exchange for this protection, the risk shedder agrees to pay an ongoing premium. Therefore, the credit risk of the underlying loan is transferred from the risk-shedder’s books, but the ownership of the loan still remains with its originator. The instrument we refer to as loan insurance in this chapter most closely resembles a credit default swap contract. As of mid-2005, single-name credit default swaps accounted for two-thirds of all gross sold credit derivative positions (Fitch 2005).

problem that plagues the loan sales market and that it is possible that the introduction of credit derivatives could shut down the loan sales market. This chapter builds on Duffee and Zhou (2001), but departs from it in two important ways. First, an assumption that is pivotal to their lemons result is that loan insurance is used when no informational asymmetries exist between the bank and the potential insurer. Recent empirical evidence by Acharya et al. (2007) suggests that banks are acting on their privileged information in credit default swaps (loan insurance) markets. In their analysis, they find significant information is revealed within these derivatives markets. This information revelation is a tell-tale sign that banks are trading with asymmetric information that can give rise to adverse selection. The first contribution of this chapter is to extend the Duffee and Zhou framework by allowing for informational asymmetries in the credit default swap market. Second, Duffee and Zhou (2001) assume that loan insurance is written on the first period of a two period loan. This assumption is restrictive too, because it implies a maturity mismatch.² The Basel Committee on Banking Supervision (2005) found that supervisors penalize banks in terms of regulatory capital if there is a maturity mismatch. There are even cases where this practice would yield no regulatory capital relief at all. The new Basel II agreement formalizes what most supervisors are currently doing by only allowing maturity mismatches in some cases, but reducing the regulatory capital benefit of the hedge in those instances (BIS 2005). Therefore, we analyze the consequences for the credit derivatives (and sales) market when the insurance contract has the same maturity as the underlying loan.³ The predictions of our model are significantly different than Duffee and Zhou (2001) and will be discussed below.

²A maturity mismatch introduces an example of an additional risk referred to as basis risk.

³Although the maturity of the contract is the same, the loan still belongs to the bank. The no maturity mismatch rule we introduce simply allows the bank to trade all of the credit risk away, instead of just a timed portion of it.

In this chapter we look at *how* credit risk is disseminated in the banking sector. This sets our work apart from the work of others,⁴ who assume a structure of credit risk transfer, and analyze the consequences for issues such as market liquidity and financial stability. We begin by putting structure on the asymmetric information problem so that we can price our instruments.⁵ First, through the unique relationship with their borrower, the bank may learn that a loan is of poor quality. Second, there may be another investment available, which, when combined with the original loan, may create a risk level that is unpalatable for the bank. We show how both loan sales and credit derivatives can be used to achieve optimal levels of investment, while minimizing undue banking risk. We seek to differentiate the two products within the banking environment by concisely determining under what conditions one is advantageous to the other, and when each can be sustained in an equilibrium setting. In an agency model, where the institution that takes on the banks risk does not know its quality, we find that in equilibrium, no separation of (quality) types can occur. We find that two pooling equilibria can exist: one insurance and one sales. Determining when each pooling equilibrium is unique, we find that well capitalized banks will wish to exclusively use loan insurance. Alternatively, banks who must utilize costly capital may need to turn to the loan sales market, even when there are relationship banking⁶ and moral hazard concerns that can depress the selling price. By introducing these features of loan sales and loan insurance, our results differ significantly from those of Duffee and Zhou (2001). Here, loan insurance is in direct competition with loan sales, and the result is that it may or may not be optimal to use, depending on the new factors we introduce. The fact that uniqueness of the two possible pooling equilibria can be determined by the relative severity of costly

⁴Excluding Duffee and Zhou (2001).

⁵This was not an issue in the Duffee and Zhou (2001) framework since the absence of asymmetric information in the credit derivatives market yielded fully informed pricing.

⁶Relationship banking refers to the unique bank-borrower relationship that is established through the course of a loan or loans.

capital to moral hazard and relationship banking costs constitutes the new predictions of our model, and the main contribution of this chapter.

The intuition of our results is as follows. When the bank needs to reduce the risk in its portfolio, it can decide whether to use loan sales or loan insurance. The risk buyer then prices these contracts given the available information. If the perceived probability of default from the risk buyers' perspective is the same for both instruments, then a bank with a good loan would prefer to use insurance, since it need only secure their initial investment, and the return remains solely the bank's. However, banks with bad loans have no incentive to truthfully reveal the quality of their loan by using sales. Therefore, only a pooling insurance, or pooling sales equilibrium can exist. We are left with identifying which pooling equilibrium will prevail. To do this, we extend the previous analysis to the more realistic case in which capital is costly, and where moral hazard and relationship banking issues are present in the bank-firm relationship. In this case, the results get more complicated, but the intuition is clear. In loan sales, the bank will have little incentive to continue to monitor the loan after a sale and could also lose some of the relationship it has built with the underlying firm. In loan insurance, ties are maintained to the underlying loan so the incentive to continue monitoring is greater. Furthermore, the firm need not know that an insurance contract was signed, so the relationship is affected little. It is easy to see that if the relationship banking or moral hazard problems are severe, then the bank will have an incentive to use insurance regardless of the loan quality. However, since insurance requires an upfront premium from the bank, whereas sales does not, costly capital works in the opposite direction. If capital is particularly costly for the bank, it may be optimal for the bank to use sales, regardless of loan quality.

The chapter proceeds as follows. Section 5.2 outlines the model. Section 5.3 analyzes

the model in the absence of CRT. Section 5.4 analyzes CRT in the base case with no externalities. Section 5.5 extends the previous section on CRT to cases in which there is moral hazard, relationship banking and costly capital present. Finally, in Section 5.6 we conclude. The appendix can be found in Section C where the proofs to a number of propositions are contained.

5.2 The Model Setup

The model shares the following features with Duffee and Zhou (2001): There are three dates, indexed as $t = 0, 1, 2$. There are three types of agents: a bank, a risk-taking counter parties (which we will refer to as risk buyers) behaving competitively and a firm (or entrepreneur) requiring capital for a project. The risk buying counter-party is risk neutral, while the bank, although maximizing a linear profit function will display risk aversion through an exogenous “regulation” parameter B to be explained below. The firm will be modelled simply as a production technology that can generate a fixed return or fail.

At time $t = 0$, the firm (entrepreneur) requests L_0 units of capital that yields a rate of return to the bank of $R_0 > 1$ if the firm’s project succeeds at time $t = 2$. The bank then chooses $L \leq L_0$; we will discuss this choice further in section 5.3. The project is worth nothing if terminated at the interim period, $t = 1$. There are two types of projects: high quality and low quality. We assume there are half of each type of project in the economy, and a bank is assigned randomly to a project at time $t = 0$. Define p_h (p_l) as the probability that a high (low) quality project defaults (and returns zero), with $1 > p_l > p_h$. We assume that the bank privately learns the quality of the project at time $t = 1$. Without loss of generality, we normalize the risk free rate in the economy to zero. We also assume that the projects have positive net present value (NPV) so that it makes sense that the bank would

take on such a loan. The bank is endowed with sufficient costless capital to undergo all desired investments. We will depart from the Duffee and Zhou (2001) setup and analyze the case of costly capital in section 5.5.2.

We add a new feature that Duffee and Zhou (2001) did not pursue by putting structure on the adverse selection problem. This departure is needed so that the prices can differentiate the two instruments to be introduced below. With no adverse selection in credit derivatives in Duffee and Zhou (2001), this structure was not needed. Equivalent to Parlour and Plantin (2006), we add a new investment opportunity that becomes available to the bank with probability q at time $t = 1$ that is private information to the bank. This investment has a return $R_1 > 1$ at time $t = 2$ if it succeeds but returns nothing with probability p_N . L_1 is required to be invested to pursue this new project.⁷ This investment represents the dynamic nature of banking. The bank does not know what new opportunities will arise in the future when a loan is issued now. The fact that the risk buyer cannot observe whether this new investment was available is not crucial to our results. We discuss this further in section 5.4.

The Basel Committee on Banking Supervision (2005) cites two main reasons for the use of CRT by banks: The first is to free up credit to take on new business, while the second is to reduce risk due to capital requirements. Both of these points are captured by the two reasons a bank uses CRT in our model: Either they learn there is a new investment opportunity, and they need to shed loan risk to pursue it (to be described in greater detail below), or they are exploiting private information about the quality of the loan.

There is ample evidence that maintaining capital reserves is an important factor in banks' decisions to engage in CRT. Pennacchi (1988) provides the argument that a prime

⁷Note that the model could easily be adapted to fit other reasons for which a bank may wish to reduce risk. For example, Parlour and Plantin (2006) use a private stochastic bank discount factor.

incentive for loan sales is to boost a bank's capital ratio. Dahiya et al. (2003) find empirically that most banks that engage in CRT fall into the bottom quartile when ranked against all banks by tier 1 capital.⁸ Cebenoyan and Strahan (2002) find more supporting evidence of the capital motivation of CRT by directly showing that banks that sell their loans have less capital. To capture this capital consideration in a reduced form, we assume that the bank suffers a cost of $B > 0$ if its losses exceed some level, \hat{L} . We will address the interesting case in which $L_0 < \hat{L}$, $L_1 < \hat{L}$, but $L_0 + L_1 > \hat{L}$ (so a default of both loans causes this cost to be incurred). B is a loss that is unique to the banking environment. Because of the nature of their business, falling below certain levels of capital can be more costly for a bank than other types of institutions. We can interpret B in a couple ways. Consider B being associated with an event that causes fragility or even default of a bank.⁹ As well, B could represent simply a regulatory penalty for a bank falling below a pre-determined level of capital.

Turning to the process of credit risk transfer, a risk buyer can insure the bank against losses in its original loan, or purchase that loan outright. Fitch (2005) reports that more than half of the credit derivatives traded remain in the banking system, with the next highest going to the insurance system, and the third highest to hedge funds. Because of this observation, we simply model the risk buyer as any risk-neutral party, and give no characteristics that distinguish it as any one of the three key players on the risk buying side. The risk buyer does not learn the quality of the firm (while the bank does), and does not learn whether the new investment is available to the bank, but knows they appear with probability

⁸Tier 1 capital refers to financial capital considered to be the most reliable and liquid, equity being the most prevalent example.

⁹We could also think of a the banks excess risk taking is instability of the system as a whole. One could argue that the social cost to instability or failure in the banking sector is higher than it is with other types of financial institutions, because banks deal with the public, as in Wagner and Marsh (2005). Therefore, we could also think of B as a means for the bank to internalize the externality they are causing on the financial system.

q (alternatively, the risk buyer sees the new investment, but is not able to determine if it is profitable for the bank; rather, they have a prior belief that with probability q , it is profitable). Note that the firm only enters this model through a project that needs funding. We will assume that the quality of the firm is only deduced through the bank-firm relationship; therefore, whether the firm knows its type or not is irrelevant. We present the timing of the model in the following figure.

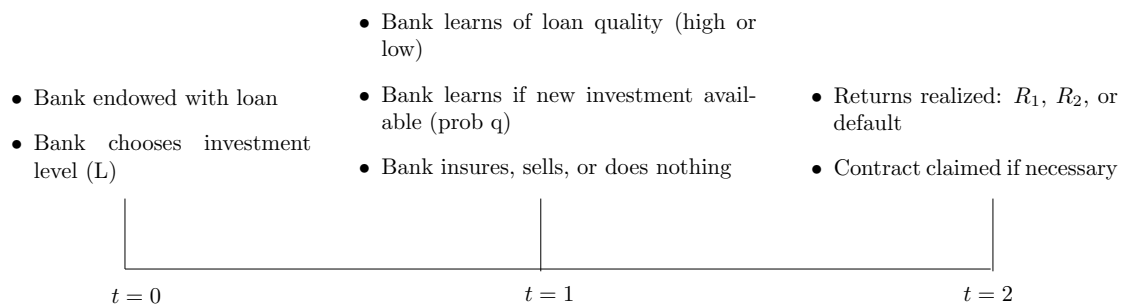


Figure 5.1: Timing of the model

5.3 No CRT Available

We start with the benchmark case in which there are no CRT markets available. The following lemma allows us to pinpoint what initial investment levels the bank will wish to pursue. Specifically, the firm requests an investment of size L_0 and the bank chooses an investment level $L \leq L_0$. For simplicity, we will assume that if the initial investment is less than L_0 , the project still proceeds with a return of $R_0 > 1$.¹⁰ We will call the case in which $L = L_0$ *full (initial) investment*. Accordingly, $L < L_0$ will be called *under-investment*. We begin with the following lemma that gives the optimal investment strategy at time $t = 1$.

¹⁰Our assumption of constant returns to scale is not restrictive. We could modify this to be decreasing returns to scale so that the marginal return may go up when the project is underfunded. However, total profit, and therefore, total payment to the bank will go down. This is all that is required to get the results in the chapter. Of course, assuming increasing returns to scale would reinforce our results even more.

Lemma 11 *Suppose the new investment is available at $t = 1$. The bank's optimal investment strategy at $t = 1$ is characterized as follows: (i) if full initial investment is pursued at $t = 0$, then the bank will pursue the new investment when $B \leq \frac{R_1 L_1 (1-p_N)}{p_N p_h}$ ($B \leq \frac{R_1 L_1 (1-p_N)}{p_N p_l}$) when the loan is revealed as the high (low) type. (ii) If there is initial under-investment, the bank will always pursue the new investment.*

Proof. See appendix.

Interpreting the condition from this lemma is relatively straightforward. The numerator represents the expected value of the new investment, conditional on it being available. We see that the lower the probability of default of the new investment, the higher B can be and still maintain incentive to pursue it (for a fixed R_1 and L_1). We also see that the inequality is decreasing in the probability of default of the high or low quality original loan. Alternatively, we can rearrange the condition to $p_j \leq \frac{R_1 L_1 (1-p_N)}{p_N B}$, $j = \{h, l\}$ and the interpretation is simply that the new investment is pursued if and only if the old one is sufficiently safe. We now turn to time $t = 0$ and find the optimal investment strategy.

Lemma 12

In equilibrium, the bank chooses L_0 or $\hat{L} - L_1$ at $t = 0$.¹¹

1. *If $B \leq \frac{R_1 L_1 (1-p_N)}{p_N p_l}$, the bank will set $L = L_0$ when $L_0 - (\hat{L} - L_1) \geq \frac{q p_N B (p_h + p_l)}{R_0 [(1-p_h) + (1-p_l)]}$, and $L = \hat{L} - L_1$ otherwise.*
2. *If $B > \frac{R_1 L_1 (1-p_N)}{p_N p_h}$, the bank will set $L = L_0$ when $L_0 - (\hat{L} - L_1) \geq \frac{2q(1-p_N)L_1 R_1}{R_0 [(1-p_h) + (1-p_l)]}$, and $L = \hat{L} - L_1$ otherwise.*

¹¹*In this second case, L is simply the level of initial investment such that, when combined with the capital needed for the new investment, its maximum losses cannot exceed \hat{L} .*

3. If $B \in \left(\frac{R_1 L_1 (1-p_N)}{p_N p_l}, \frac{R_1 L_1 (1-p_N)}{p_N p_h} \right]$, the bank will set $L = L_0$ when $L_0 - (\hat{L} - L_1) \geq \frac{q(1-p_N)L_1 R_1 - B p_N p_l}{R_0[(1-p_h) + (1-p_l)]}$, and $L = \hat{L} - L_1$ otherwise.

Proof. See appendix.

The first part of lemma 12 follows from the discontinuity in the payoff function over investment choices.¹² To interpret the second part of the lemma, we can re-write the condition in a more intuitive way:

$$R_0(L_0 - (\hat{L} - L_1))\left[\frac{1}{2}(1 - p_h) + \frac{1}{2}(1 - p_l)\right] \geq B q p_N (p_h + p_l)$$

The left hand side is the return that the bank would forgo if it reduced its initial investment from L_0 to $\hat{L} - L_1$. The right hand side is the expected amount the bank would lose if it pursued full-investment. If $L_0 - (\hat{L} - L_1)$ is small, this means that the initial investment need not be reduced by much to avoid the cost of B . In this case, as long as B is not too big, the bank will find it advantageous to under-invest. Alternatively, when the left hand side is large, this means that the bank must reduce its investment by a lot to avoid the cost of B . In this case, unless B is very small, the bank will wish to invest fully. The third and fourth parts of the proposition can be re-arranged and interpreted in a similar fashion.

We now look at the base case where CRT is available to the bank. We do this so that we can enrich the model in section 5.5 to obtain our main results.

5.4 CRT Available

We now consider the case in which the bank pursues the initial investment fully, and can decide whether to pursue the new investment. With the availability of credit risk transfer

¹²We can generalize this by making B a decreasing function of L_0 . However, this would yield no further intuition about our problem, thus we use the simplest setup possible.

markets, the risk buyer must price the loan sale or insurance premium, given the available information. The bank will wish to engage in credit risk transfer (CRT) if either it learns that the loan is of low quality, or the new investment becomes available. This can be assured by an assumption on B that will be derived and discussed in section 5.4.1. Given the available information, the risk buyer can deduce the probability that a loan is of high quality (h) or low quality (l):

$$Prob(h|CRT) = \frac{q}{q+1}$$

The risk buyer can now form a belief about the probability that the loan will default:

$$Prob(Default|CRT) = \frac{pl + qp_h}{q+1}$$

We allow the bank to insure its initial investment, or sell the loan outright. Because of their zero profit condition (competitiveness assumption), the risk buyer must be indifferent between insuring and not insuring, as well as selling and not selling. We assume that the bank insures its initial investment L ,¹³ and therefore, the risk buyer will demand a premium of $L_0(\frac{pl+qp_h}{q+1})$. As well, the risk buyer would be willing to buy this loan for $R_0L_0(1 - \frac{pl+qp_h}{q+1})$. The latter is simply the expected payoff of the loan from the risk buyer's perspective.

If we relax the assumption that the risk buyer cannot observe the bank engaging in the new investment, we see that the adverse selection problem will still be present so long as the new investment is available. In this case, the risk buyer will use its prior beliefs to determine the probability of default. In the case where the new investment is not available, the risk

¹³This assumption parallels that of Duffee and Zhou (2001) where the bank insures only its initial investment. If we allow the bank to insure less of their loan, i.e. partial insurance, this will not have any effects on the qualitative results of the chapter, so long as the adverse selection problem is maintained. In other words, if the high type can reveal itself by insuring less of the loan, and still be protected from the cost B , the adverse selection problem would be solved. Since we have highlighted evidence that adverse selection is present in these markets, this chapter will focus only on this case.

buyer will know that the loan must be bad. However, in this chapter, we are interested in the consequences for the insurance and sales market when adverse selection is present, so we rule out this revealing case by having the new investment be private information to the bank. Alternatively, we could assume that the new investment is public information but it is always available and all the results to be discussed would follow through.

5.4.1 Incentives to engage in CRT

In the previous section we outlined how the risk buyer would price a loan sale or insurance premium under the information structure given. We now outline restrictions on the parameter space that will see the bank using CRT in the correct states. We assume that the bank invests the entire amount requested (L_0) in the initial investment. In Proposition 5 we will confirm that this level of investment will prevail in the presence of a CRT market. We begin by analyzing the incentives to insure, and then repeat the exercise for sales.

Incentives to Buy Insurance

We start by verifying that the bank will wish to purchase loan insurance in the appropriate states. We denote the state where a high (low) quality loan is realized as H (L), and the state where the new investment opportunity is realized (not realized) as NEW (NONEW). Therefore $\{H,NEW\}$ represents a high quality firm and a new investment available.

It is easy to show that if the incentives are such that the bank insures in the high state, then this implies they will insure in the low state. We therefore need to check two states: $\{H,NEW\}$ to make sure they wish to insure, and $\{H,NONEW\}$ to make sure they do not wish to insure. Let us analyze $\{H,NONEW\}$ first. Let π_{NI} denote the bank's payoff from no insurance in the high state, π_I denote the payoff from insurance in the high state, and P_I

denote the price per unit of the insurance contract. We can consider the price P_I to be the insurance premium.

$$\begin{aligned}\pi_{NI} &= R_0(1 - p_h)L_0 \\ \pi_I &= R_0(1 - p_h)L_0 - P_I L_0 + p_h L_0\end{aligned}$$

From the above, for $\pi_{NI} \geq \pi_I$, $P_I \geq p_h$ must hold. We will refer to this condition as (I-Bound). We will assume that this condition holds by putting it as a restriction in the optimal contracting problem to be set out in section 5.4.3.

We now analyze {H,NEW} to see under what condition they will use loan insurance.

$$\begin{aligned}\pi_{NI} &= R_0(1 - p_h)L_0 + (1 - p_N)L_1 R_1 - p_h p_N B \\ \pi_I &= R_0(1 - p_h)L_0 - (P_I)L_0 + p_h L_0 + (1 - p_N)L_1 R_1\end{aligned}$$

For $\pi_I \geq \pi_{NI}$ the following condition must hold:

$$B \geq \frac{L_0(P_I - p_h)}{p_h p_N} \quad (5.1)$$

Since $P_I \geq p_h$, the R.H.S of (5.1) is positive, and therefore we place this restriction on B .

Incentives to Sell the Loan

We can conduct a similar exercise for loan sales. We begin by analyzing {H,NONEW}. Let π_{NS} denote the payoff from no loan sales, and P_S be the price per unit of a sales contract with a net return on the underlying loan of 1.¹⁴

$$\begin{aligned}\pi_{NS} &= R_0(1 - p_h)L_0 \\ \pi_S &= R_0(P_S)L_0\end{aligned}$$

¹⁴ P_S is defined in this way to ease the comparison of loan sales to loan insurance. Therefore, the total price of the loan sale contract is $R_0 L_0 P_S$.

For $\pi_{NS} \geq \pi_S$, $P_S \leq 1 - p_h$ must hold. As in the insurance case, this condition will be a constraint in the optimal contracting problem. We will refer to this condition as (S-Bound). We now analyze {H,NEW} to see under what condition they will use loan sales.

$$\begin{aligned}\pi_{NS} &= R_0(1 - p_h)L_0 + (1 - p_N)L_1R_1 - p_Np_hB \\ \pi_S &= R_0(P_S)L_0 + (1 - p_N)L_1R_1\end{aligned}$$

For $\pi_S \geq \pi_{NS}$ the following must hold:

$$B \geq \frac{R_0L_0(1 - p_h - P_S)}{p_hp_N} \quad (5.2)$$

Therefore, (5.2) is the parametrization that we make for loan sales. If we consider each market in isolation, we get $P_I = Prob(Defaul\!t|insurance) = Prob(Defaul\!t|sales) = 1 - P_S$. Therefore, one can see that the only difference between (5.1) and (5.2) is a factor of R_0 . The reason for this difference is intuitive when we look at how the two contracts are priced. Whereas the insurance premium is independent of the return on the initial investment, the sales contract involves an entitlement to the return in the future, and must depend on R_0 . If R_0 is very high, B must also be high so that in {H,NEW} the bank still has an incentive to use CRT. Therefore, we make the following assumption so that CRT can arise.

Assumption 1 *To permit the existence of both CRT markets,*

$$B \geq \max\left\{\frac{L_0(P_I - p_h)}{p_hp_N}, \frac{R_0L_0(1 - p_h - P_S)}{p_hp_N}\right\}.$$

The form of this assumption deserves some explanation. We allow the assumption to depend on our two endogenous variables (P_I and P_S) because we are looking at the consequences of the two CRT markets. In other words, we assume the existence of a CRT market for the equilibrium price in which the risk buyer will earn zero profit. The parameter space can always be chosen to accomplish this.

5.4.2 CRT Available - Either Loan Sales or Loan Insurance (but not both)

Below, we will show the ex-ante expected payoff equivalence of *loan sales* and *loan insurance*. Because of this, we need only check one or the other to investigate under what conditions full investment can arise. The case in which the bank does not use CRT and opts to reduce risk exposure by under-investing is given in the appendix as equation (C.3). We can derive the expected (ex-ante) payoff if the bank uses CRT to reduce credit exposure:

$$\begin{aligned}
E(\pi_I) &= \left(\frac{1}{2}\right)(1-q)\left[(1-p_l)R_0L_0 + p_lL_0 - L_0\left(\frac{p_l + qp_h}{q+1}\right)\right] \\
&+ \left(\frac{1}{2}\right)(q)\left[(1-p_l)R_0L_0 + p_lL_0 - L_0\left(\frac{p_l + qp_h}{q+1}\right) + (1-p_N)L_1R_1\right] \\
&+ \left(\frac{1}{2}\right)(q)\left[(1-p_h)R_0L_0 + p_hL_0 - L_0\left(\frac{p_l + qp_h}{q+1}\right) + (1-p_N)L_1R_1\right] \\
&+ \left(\frac{1}{2}\right)(1-q)\left[(1-p_h)R_0L_0\right] \\
&= \frac{1}{2}R_0L_0(1-p_h) + \frac{1}{2}R_0L_0(1-p_l) + q[(1-p_N)L_1R_1] \tag{5.3}
\end{aligned}$$

Equation (5.3) shows us that the expected payoff to the bank is simply the expected return from the initial loan ($\frac{1}{2}R_0L_0(1-p_h) + \frac{1}{2}R_0L_0(1-p_l)$) plus the expected return from the new investment ($q[(1-p_N)L_1R_1]$). Similarly, denoting expected profit from loan sales by $E(\pi_S)$ we derive the following expression:

$$\begin{aligned}
E(\pi_S) &= \left(\frac{1}{2}\right)(1-q)\left[R_0L_0\left(1 - \frac{p_l + qp_h}{q+1}\right)\right] \\
&+ \left(\frac{1}{2}\right)(q)\left[L_0R_0\left(1 - \frac{p_l + qp_h}{q+1}\right) + (1-p_N)L_1R_1\right] \\
&+ \left(\frac{1}{2}\right)(q)\left[L_0R_0\left(1 - \frac{p_l + qp_h}{q+1}\right) + (1-p_N)L_1R_1\right] \\
&+ \left(\frac{1}{2}\right)(1-q)\left[(1-p_h)R_0L_0\right] \\
&= \left(\frac{1}{2}\right)R_0L_0(1-p_h) + \left(\frac{1}{2}\right)R_0L_0(1-p_l) + q[(1-p_N)L_1R_1] \tag{5.4}
\end{aligned}$$

The equivalence of (5.3) and (5.4) has been established.

We are now equipped to show the main proposition of this section. The following proposition analyzes the use of CRT and its effect on investment choice.

Proposition 5 *CRT is ex-ante more profitable than without and yields the efficient level of initial and new investment.*

Proof. *See appendix.*

From lemmas 5 and 12, it is straight-forward to see that the ex-ante expected loss due to B can be eliminated by either investing in CRT in both states of the world, or under-investing.

5.4.3 Both markets available - Equilibrium

The equilibrium concept we apply is that of a Bayesian Nash equilibrium (BNE). Given Assumption 1, there are only two equilibria that can prevail when both CRT markets are open at the same time. The first equilibrium is where all types use loan insurance, while the second is where all types use loan sales. We first verify that a separating equilibrium cannot exist, and then proceed to see which pooling equilibria can be sustained.

Non-existence of Separating Equilibria

From proposition 5, we know that full initial investment dominates under-investment. For what follows, we will assume that the conditions of lemma 11 are satisfied so that the bank prefers to invest in the new investment when it is available. Checking both $\{L, \text{NONEW}\}$ and $\{L, \text{NEW}\}$ is redundant. First, the participation constraint on $\{H, \text{NEW}\}$ is a stronger

condition than either of the participation constraints for the low types. As well, the new investment will yield the same additional payoff in both sales and insurance, so the one incentive constraint is redundant.¹⁵ Finally, assumption 1 guarantees that the bank will not wish to use CRT in the state {H,NONEW}. Thus, we need only check {L,NONEW} and {H,NEW}.

Consider first a separating equilibrium where a bank with a high (low) quality loan chooses to insure (sell). Given the information structure of this separating equilibrium, we know that $Prob(Default|insurance) = p_h$ and $Prob(Default|sales) = p_l$. We can show that this equilibrium cannot exist simply by looking at the incentive constraint on the low type (IC1 - which says that the low type wishes to use sales over insurance) as well the two zero profit conditions. The optimal prices (P_I, P_S) must satisfy the following:

$$R_0L_0(P_S) \geq (1 - p_l)R_0L_0 - L_0P_I + p_lL_0 \quad (\text{IC1})$$

$$L_0(P_I - p_h) = 0 \quad (\text{zero-}\pi_I)$$

$$R_0L_0[(1 - p_l) - P_S] = 0 \quad (\text{zero-}\pi_S)$$

By (zero- π_I) and (zero- π_S), we can see that the only candidate prices are $P_I = p_h$ and $P_S = 1 - p_l$. We can quickly verify that under these prices, (IC1) cannot hold:

$$(1 - p_l)R_0L_0 \geq (1 - p_l)R_0L_0 - L_0p_h + L_0p_l \\ \Rightarrow p_h \geq p_l$$

¹⁵To see this, consider the participation constraints in the state {L,NEW}: $(1 - p_l)R_0L_0 - L_0P_I + p_lL_0 + (1 - p_N)L_1R_1 \geq (1 - p_l)R_0L_0 + (1 - p_N)L_1R_1 - p_l p_N B$ which simplifies to: $p_lL_0 - l_0P_I \leq p_lL_0 + p_l p_N B$. Now consider the participation constraint on {H,NEW}: $(1 - p_h)R_0L_0 - L_0P_I + p_hL_0 + (1 - p_N)L_1R_1 \geq R_0L_0(1 - p_h) + (1 - p_N)L_1R_1 - p_N p_h B$ which simplifies to: $p_hL_0 - l_0P_I \leq p_hL_0 + p_h p_N B$. Since $p_h < p_l$ it is obvious that if the participation constraint on {H,NEW} binds, the participation constraint on {L,NEW} will automatically bind. To see the redundancy in the incentive constraints, consider the incentive constraint where {L,NEW} wishes to use sales over insurance: $R_0L_0(P_S) + (1 - p_N)L_1R_1 \geq (1 - p_l)R_0L_0 - L_0P_I + p_lL_0 + (1 - p_N)L_1R_1$. However, this simplifies to: $R_0L_0(P_S) \geq (1 - p_l)R_0L_0 - L_0P_I + p_lL_0$ which is the incentive constraint on {L,NONEW}. This reasoning also applies if we are looking at the low type using insurance over sales. Therefore, only one of the incentive constraints on the low type is needed.

Since $p_l > p_h$, (IC1) is violated. Therefore, this separating equilibrium cannot be supported. We can also verify that the separating equilibrium where banks with high quality loans sell, and banks with low quality loans insure cannot be supported. The proof is very similar to the above and is omitted. The separating equilibria above are ruled out because the risk buyer is forced to earn zero profit in each market. Allowing them to subsidize one market by over-charging in the other will give rise to one separating equilibrium: a bank with high type loans will use insurance, and low type loans, sales. We pursue this case after we have finished analyzing the case in which the risk buyer must earn zero profit in each market. Any separating equilibrium where a type does not engage in CRT is ruled out by assumption 1.

Pooling Equilibrium with Insurance

We proceed by showing there are two possible pooling equilibria. Consider first the case where banks that have either high or low quality loans both choose to insure. Given the information structure in this pooling equilibrium, we know that $Prob(Default|insurance) = \frac{p_l + qp_h}{q+1}$. In this case we will have two participation constraints (I-PC1, I-PC2 - ensuring the bank with either the low or high type loans wishes to engage in CRT) and two incentive constraints (I-IC1, I-IC2 - ensuring that the bank with either the low or high type loans wish

to use insurance over sales).¹⁶ We can characterize the optimal prices (P_I, P_S) as follows:

$$(1 - p_l)R_0L_0 - L_0P_I + p_lL_0 \geq (1 - p_l)R_0L_0 \quad (\text{I-PC1})$$

$$(1 - p_h)R_0L_0 - L_0P_I + p_hL_0 + (1 - p_N)L_1R_1 \geq R_0L_0(1 - p_h) + (1 - p_N)L_1R_1 - p_Np_hB \quad (\text{I-PC2})$$

$$(1 - p_l)R_0L_0 - L_0P_I + p_lL_0 \geq R_0L_0(P_S) \quad (\text{I-IC1})$$

$$(1 - p_h)R_0L_0 - L_0P_I + p_hL_0 + (1 - p_N)L_1R_1 \geq R_0L_0(P_S) + (1 - p_N)L_1R_1 \quad (\text{I-IC2})$$

$$L_0\left(P_I - \frac{p_l + qp_h}{q + 1}\right) = 0 \quad (\text{zero-}\pi)$$

$$P_I \geq p_h \quad (\text{I-Bound})$$

$$P_S \leq 1 - p_h \quad (\text{S-Bound})$$

From (zero- π), we can see that the only admissible insurance premium is $P_I = \frac{p_l + qp_h}{q + 1}$. At this price, (I-PC1) is satisfied, while (I-PC2) is satisfied by assumption 1.

From (I-IC1), we find:

$$P_S \leq (1 - p_l) + \frac{q(p_l - p_h)}{(q + 1)R_0} \quad (5.5)$$

Next, from (I-IC2), we find:

$$P_S \leq (1 - p_h) - \frac{p_l - p_h}{(q + 1)R_0} \quad (5.6)$$

Since $R_0 > 1$, $q \in (0, 1)$ and $p_l < p_h$, it follows that (5.5) \Rightarrow (5.6). Therefore, (5.5) defines the price range that can be assigned to loan sales to sustain this pooling equilibrium, and represents the off-the-equilibrium path beliefs that the risk-buyer assigns to the sales market that can support this pooling equilibrium.¹⁷ It is easy to see that (I-Bound)

¹⁶Recall that we need not check both the low type with and without the new investment as it is redundant.

¹⁷Note that equilibrium refinements like the Cho-Kreps Intuitive Criterion have no bite in this setting (and in all equilibria to be shown in this chapter). Therefore, we are able to focus on all off-the-equilibrium path beliefs that sustain the pooling equilibrium.

and (S-Bound) are satisfied at the admissible values of P_I and P_S . Henceforth, if this equilibrium exists, we will refer to it as the *insurance equilibrium*.

Pooling Equilibrium with Loan Sales

We continue by shifting our focus to the pooling equilibrium where both high and low types chose loan sales. We know $Prob(No\ Default|CRT) = 1 - Prob(Default|CRT) = 1 - \frac{p_l + qp_h}{q+1}$. The optimal prices (P_I, P_S) must satisfy:

$$P_S R_0 L_0 \geq (1 - p_l) R_0 L_0 \quad (\text{S-PC1})$$

$$P_S R_0 L_0 + (1 - p_N) L_1 R_1 \geq R_0 L_0 (1 - p_h) + (1 - p_N) L_1 R_1 - p_N p_h B \quad (\text{S-PC2})$$

$$P_S R_0 L_0 \geq (1 - p_l) R_0 L_0 - L P_I + p_l L \quad (\text{S-IC1})$$

$$P_S R_0 L_0 + (1 - p_N) L_1 R_1 \geq (1 - p_h) R_0 L_0 - L_0 P_I + p_h L_0 + (1 - p_N) L_1 R_1 \quad (\text{S-IC2})$$

$$1 - \frac{p_l + qp_h}{q+1} - P_S = 0 \quad (\text{zero-}\pi)$$

$$P_I \geq p_h \quad (\text{I-Bound})$$

$$P_S \leq 1 - p_h \quad (\text{S-Bound})$$

From (zero- π), we can see that $P_S = 1 - \frac{p_l + qp_h}{q+1}$. Given this, it is easy to verify that (S-PC1) is satisfied. As well, (S-PC2) is satisfied by assumption 1. Plugging the value for P_S into (S-IC1) yields:

$$P_I \geq p_l - R_0 \left[\frac{q}{q+1} (p_l - p_h) \right] \quad (5.7)$$

Plugging P_S into (S-IC2) yields:

$$P_I \geq p_h + R_0 \left[\frac{q}{q+1} (p_l - p_h) \right] \quad (5.8)$$

Since $R_0 > 1$, it follows that (5.8) \Rightarrow (5.7) and therefore, the insurance premium can take on any value in the range defined by the off-the-equilibrium path beliefs, (5.8). With

the off-the-equilibrium path beliefs being defined by (5.8), we can assign an upper bound as: $P_I \leq p_l$. This restriction must hold because of the zero profit condition of the risk buyer.¹⁸ Thus, $R_0 \leq \frac{q+1}{q}$ must hold for this pooling equilibrium to exist. We can see that if a project has a large return, then the bank will turn to the loan insurance market. It is easy to see that (I-Bound) and (S-Bound) are satisfied at the admissible values of P_I and P_S . Henceforth, if this equilibrium exists, we will refer to it as the *sales equilibrium*.

Duffee and Zhou (2001) conclude that if a sales market exists, and a loan insurance market is introduced, pooling in the sales market may no longer be possible. This in turn would cause the break down of the sales market altogether. Since Duffee and Zhou (2001) have insurance being written on a portion of the loan with no adverse selection, they show that the sales market can break down because of the flexibility of loan insurance. Our model produces a similar result without the flexibility in insurance, but from an entirely different channel. Since the sales market can exist in isolation with $R_0 > \frac{q+1}{q}$, if insurance is introduced under these circumstances, pooling in the sales market would not be possible. This would cause the sales market itself to break down.

All other equilibria can be ruled out in essentially the same way as was done above so the proofs are omitted.

We are now ready to analyze an enriched version of the model so that we can derive our main results.

¹⁸The reason for this is that if the bank is charged more than p_l for insurance, they would necessarily make positive profit since p_l is the highest probability the loan can default with.

5.5 Moral Hazard, Relationship Banking, and Costly Capital

One important point about having multiple equilibria is that there is no way of telling which will occur. This non-uniqueness stems from the fact that we have left key attributes of each instrument unmodelled thus far. Enhancing the model to a more realistic setup will give us insight into the choice of insurance versus sales. First, the relationship between a bank and a borrower can cause a moral hazard problem to develop. Consider a bank that has a special technology to verify that the firm is operating in a manner that is in keeping with the bank's interests. We refer to this as a monitoring technology. When the bank transfers away risk from a loan, it may no longer have the incentive to invest in this monitoring technology. In this chapter, we do not analyze the origins of the moral hazard problem, but rather, we analyze the effect of its presence. For a review of moral hazard in banking, see for instance Gorton and Pennachhi (1995).

The second issue with CRT arises only with loan sales. In a loan sale, the underlying firm and new lender must expend resources to build a new relationship which can devalue the loan. We will refer to this cost as *relationship banking*. In reality, the cost of selling a loan could go farther than just a devaluation of the current loan, as it could hurt future business with the underlying firm.¹⁹ In some loan sale contracts, the underlying firm may even try to prevent a bank from selling their loan by specifying a no-sale stipulation. The costs associated with relationship banking are not generally applicable to loan insurance because the originating bank maintains ownership of the loan and need not inform the underlying firm of their intent to insure. It has been shown empirically that these costs are

¹⁹We will not model this channel here.

present in loan sales. Dahiya et al. (2003) find that when a bank sells a loan, the market reacts negatively to it by devaluing the bank's stock.²⁰ With moral hazard and relationship banking costs, conditions can be set so that the bank strictly prefers loan insurance in all states or vice versa. Moral hazard and relationship banking will be analyzed in Section 5.5.1.

In Section 5.5.2, we relax the assumption that capital comes without cost. Realistically, banks have investors who provide capital, but who expect a rate of return on their investment.²¹ Competitiveness in the banking sector will provide us with different cost structures for sales and insurance. We will see that this addition has the possibility of making loan sales attractive to a bank who must acquire relatively costly capital.

The introduction of these two unique features are two of the contributions this chapter makes to the literature. Duffee and Zhou (2001) does not consider the possible trade-offs between sales and insurance on these grounds. We will see that the addition of these two costs drives the equilibrium choice between sales and insurance, whereas Duffee and Zhou (2001) could make no such direct comparison.²²

5.5.1 Moral Hazard and Relationship Banking Costs

To consider the possibility that there are additional costs to using loan sales, we add an exogenous cost parameter $\alpha \in [0, 1]$. This new parameter represents the degree to which the project is worth less in the hands of the risk buyer due to both the moral hazard problem of the bank and the relationship banking cost incurred by the risk buyer. When α is low, the

²⁰This evidence may also incorporate the adverse selection problem discussed before.

²¹We will not analyze the choice between investor and depositor financing for the bank in this chapter. We simply assume a rate of return that a bank must pay on any capital it uses.

²²Recall that Duffee and Zhou (2001) differentiated sales and insurance through a maturity mismatch, which this chapter contends is no longer a driving feature of these markets.

costs associated with selling are high, and the bank must take a significantly lower price for the loan sale if it wishes to pursue this instrument of CRT. For simplicity, we assume that moral hazard and relationship effects are not present in loan insurance; however, the qualitative results will follow through if we allow for moral hazard in the loan insurance market. However, since the originating bank is still tied to the return of the loan, we would expect moral hazard to be smaller with loan insurance. This argument will be formalized below when we show how moral hazard can be endogenized in the model. If we consider a different setting where the bank maintains no ties to the return on the loan (i.e it insures both the principal and the return), then the moral hazard problem would be the same in insurance as in sales. However, α would still be larger for loan insurance because of the relationship banking cost of loan sales. We can show the new expected profit from loan sales.

$$E(\pi_S) = \frac{1}{2}R_0L_0[1 - p_h(1 - q(1 - \alpha))] + \frac{1}{2}R_0L_0(1 - \alpha p_l) + q[(1 - p_N)L_1R_1]$$

Not surprisingly, the expected profit from loan sales is unambiguously lower than that of loan insurance (as determined in (5.3)) when $\alpha < 1$. This result already gives us the intuition behind what will be the equilibrium outcome. Our main result, Proposition 7 will confirm that the smaller is α , the less likely it is that sales can be sustained in equilibrium.

It is important to recognize that this reduced form representation of moral hazard can be generated endogenously in an extension to the model. In the appendix (section C.0.1) we show that little is lost by assuming that moral hazard imposes an exogenous cost on the price of loan sales.

5.5.2 Costly Bank Capital

In this section, we generalize the previous exercises with the more realistic assumption that bank capital may come at a cost. An important question to ask is what would cause a bank to have different costs of capital? One key factor is how well capitalized a bank is. Investors in the bank will be willing to accept lower rates of return if the bank has sufficient equity to cover potential loss in case of default. If the bank is poorly capitalized, the risk to the investor is greater, and they will charge a greater amount representing the extra risk they must bear. However, we assume that the risk-buyer is not under this type of constraint.²³ Therefore, we assume that the risk buyer is not subjected to this cost of capital. This assumption can be relaxed so that the risk buyer does have a cost, but it is less than that of the bank.

Let there be two investors in the bank: an early investor, and a late investor. The early investor is endowed with unlimited capital at time $t = 0$, but none at times $t = 1, 2$. We represent their preferences as in Allen and Gale (2005) with the following risk-neutral utility function:

$$U(c_0, c_1, c_2) = (R_f - 1)c_0 + c_1 + c_2$$

where $R_f - 1$ represents the rental rate of capital and c_t is the consumption at time t . One of the key insights from the functional form is that investors are indifferent between consumption at $t = 1$ and $t = 2$. Because of this, they will require the same return, $R_f - 1 > 0$, regardless of how long they loan the capital to the bank. The late investor is endowed with unlimited capital at time $t = 1$, but none at time $t = 2$. We represent their utility function as:

$$U(c_0, c_1, c_2) = (R_f - 1)c_1 + c_2$$

²³This assumption is justified given our assumption that the risk buyer is well diversified.

This type of investor simply gives capital at time $t = 1$ and requires a return of $R_f - 1$ at time $t = 2$. Note that we have equalized the outside opportunity cost of each investor type for simplicity.²⁴ The rental rate of capital deserves some explanation. We use a rental rate to be consistent with the base case where the bank owns its own capital ($R_f = 1$). Alternatively, we could modify the base case so that the bank does not own its capital, but need not pay a return on it. The results do not change with either way of treating the capital cost. Therefore, we assume the bank will return the principal after the final date, but for simplicity, and without loss of generality, we normalize the principal to zero.

We now turn to the expected (ex-ante) profit for the bank. Recall we derived the expressions (5.3) and (5.4) earlier without costly capital. We can calculate the new expected costs of this additional feature. Denote the expected additional cost of insurance and sales with costly capital as $E(C_I)$ and $E(C_S)$ respectively.

$$\begin{aligned} E(C_I) &= \frac{1}{2}(1-q)(R_f-1)L_0 + q[(R_f-1)L_0 + (R_f-1)L_0P_I + (R_f-1)L_1] + \\ &\quad \frac{1}{2}(1-q)(R_f-1)L_0 \\ &= (R_f-1)(L_0 + qL_1) + (R_f-1)qL_0P_I \end{aligned}$$

$$\begin{aligned} E(C_S) &= \frac{1}{2}(1-q)(R_f-1)L_0 + q[(R_f-1)L_0 + (R_f-1)(L_1 - R_0L_0(P_S))] + \\ &\quad \frac{1}{2}(1-q)(R_f-1)L_0 \\ &= (R_f-1)(L_0 + qL_1) - (R_f-1)qR_0L_0P_S \end{aligned}$$

We can see that insurance is unambiguously more costly than sales. Therefore, loan sales yields more profit (ex-ante) than loan insurance under costly capital without moral hazard and relationship costs (i.e $\alpha = 1$). The intuition behind this result is that loan

²⁴We have also assumed that the two types of investors cannot trade with each other.

insurance forces the bank to obtain even more costly capital to engage in CRT $((R_f - 1)qL_0P_I)$. At the same time, loan sales allows them to free up capital (and reinvest the payoff from the sale) when they wish to pursue the new investment $(-(R_f - 1)qR_0L_0P_S)$.

We will see that the intuition of the previous analysis is confirmed in Proposition 7, where we show that the higher is R_f , the less likely it is that insurance can be sustained in an equilibrium setting.

5.5.3 Moral Hazard, Relationship Banking, and Costly Capital Together

A similar exercise to that of section 5.4.1 can give us the following assumption that permits the existence of our CRT markets in our generalized framework. The derivation can be found in the appendix (section C.0.2).

Assumption 2 $B \geq \max\left\{\frac{L_0(R_f(P_I) - p_h)}{p_N p_h}, \frac{R_0 L_0((1 - p_h) - \alpha P_S R_f)}{p_N p_h}\right\}$

We begin by ruling out all separating equilibria in this generalized setting.

Proposition 6 *No separating equilibria can exist in the generalized model where $\alpha \in [0, 1)$ and $R_f < 1$.*

Proof. *See appendix.*

Let us now consider the insurance pooling equilibrium where banks with both high and low type loans choose to insure. To differentiate, given the knife-edge case where the incentive compatibility constraints hold with equality, we assume they choose insurance. This will make our incentive compatibility constraints in the sales equilibrium case strict.

It can be shown that the incentive and participation constraints of the state {H,NEW} are implied by those in state {L,NEW} so we drop them. In what follows, we assume that the bank wishes to pursue full investment, as we did in the simpler case of the previous section. The optimal prices (P_I, P_S) are given by:

$$(1 - p_l)R_0L_0 + p_lL_0 - (R_f - 1)L_0 - R_fP_IL_0 \geq (1 - p_l)R_0L_0 - (R_f - 1)L_0 \quad (\text{I-PC1})$$

$$(1 - p_l)R_0L_0 + p_lL_0 + (1 - p_N)L_1R_1 - (R_f - 1)L_0 - R_fP_IL_0 - (R_f - 1)L_1 \geq (1 - p_l)R_0L_0 + (1 - p_N)L_1R_1 - (R_f - 1)L_0 - (R_f - 1)L_1 - p_Np_lB \quad (\text{I-PC2})$$

$$(1 - p_l)R_0L_0 + p_lL_0 - (R_f - 1)L_0 - R_fP_IL_0 \geq \alpha R_0L_0(P_S) - (R_f - 1)L_0 \quad (\text{I-IC1})$$

$$(1 - p_l)R_0L_0 + p_lL_0 + (1 - p_N)L_1R_1 - (R_f - 1)L_0 - R_fP_IL_0 - (R_f - 1)L_1 \geq \alpha R_0L_0(P_S) + (1 - p_N)L_1R_1 - (R_f - 1)L_0 - (R_f - 1)[L_1 - (\alpha R_P S L_0)] \quad (\text{I-IC2})$$

$$L_0\left(P_I - \frac{p_l + qp_h}{q + 1}\right) = 0 \quad (\text{zero-}\pi)$$

$$P_I \geq \frac{p_h}{R_f} \quad (\text{I-Bound})$$

$$\alpha P_S \leq 1 - p_h \quad (\text{S-Bound})$$

On the left hand side of (I-IC1) and (I-IC2), we see that with insurance, the bank holds the investment for two periods, and incurs a cost of $(R_f - 1)L$. As well, they borrow an

additional $L_0 P_I$ for one period to pay for the cost of insuring, and incur a cost of $R_f L_0 P_I$.

On the right hand side of (I-IC1) and (I-IC2), the cost of capital for loan sales deserves some explanation. The bank acquires the capital for the initial loan at a cost of $(R_f - 1)L$. At time $t = 1$, they need not borrow the full amount of capital for the new investment. This is because they can reinvest the proceeds of the loan sale. The cost of the extra capital that is needed for the new investment is $(R_f - 1)[L_1 - \alpha R_0 P_S L_0]$. For simplicity we assume that $L_1 \geq \alpha R_0 P_S L_0$.²⁵ This assumption is innocuous since we will soon see that our characterizing solutions do not depend on L_1 .

Given (zero- π), we know that $P_I = \frac{p_l + q p_h}{q+1}$. (I-PC1) is satisfied when $R_f \leq \frac{p_l}{P_I}$, while (I-PC2) is implied by assumption 2. P_S is given by the off-the-equilibrium path beliefs of the risk buyer and can be defined given α and R_f . We obtain the following parameterizations for (I-IC1) and (I-IC2):

$$\alpha \leq \frac{R_0(1 - p_l) - R_f P_I + p_l}{R_0 P_S} \quad (\text{I-IC1a})$$

$$\alpha \leq \frac{R_0(1 - p_l) - R_f P_I + p_l}{R_0 P_S R_f} \quad (\text{I-IC2a})$$

Since (I-IC1a) and (I-IC2a) differ by a fraction $\frac{1}{R_f}$, it follows that (I-IC2) \Rightarrow (I-IC1). Therefore, (I-IC2) is binding, while (I-IC1) is slack. We can use a standard approach to find out when this equilibrium cannot exist. We let the off-the-equilibrium path beliefs be $P_S = 1 - p_l$. Therefore, if the equilibrium cannot exist under this condition, it cannot exist for any valid off-the-equilibrium path belief. Substituting $P_S = 1 - p_l$ into (I-IC2) yields this range:

$$\alpha > \frac{R_0(1 - p_l) - R_f P_I + p_l}{R_0(1 - p_l) R_f}$$

We continue by analyzing when the equilibrium can exist.

²⁵This assumption is equivalent to the assumption that the bank has a storage technology with a return of 1.

Lemma 13 *The insurance equilibrium exists whenever one of the following two conditions is met*

1. $\alpha < \frac{P_I(1-p_I)}{P_S p_I}$ and $1 \leq R_f \leq \frac{p_I}{P_I}$
2. $\alpha > \frac{P_I(1-p_I)}{P_S p_I}$ and $1 \leq R_f \leq \frac{R_0(1-p_I)+p_I}{\alpha R_0 P_S + P_I}$

Proof. *See appendix.*

The results of this Lemma are relatively straight-forward. The first condition says that if α is small, then the insurance equilibrium will exist when R_f (low cost of capital) is sufficiently small. The second condition says if α is larger, we will require an even smaller value of R_f than what was required in the first condition.

We can conduct a similar exercise for the sales equilibrium. The participation and incentive constraints of state {L,NEW} are implied by those of state {H,NEW} and are

dropped. The following setup will yield the optimal prices (P_I, P_S) :

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 \geq (1 - p_l)R_0 L_0 - (R_f - 1)L_0 \quad (\text{S-PC1})$$

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)[L_1 - (\alpha R P_S L_0)] \geq \quad (\text{S-PC2})$$

$$(1 - p_h)R_0 L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)L_0 - (R_f - 1)L_1 - p_N p_h B$$

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 > (1 - p_l)R_0 L_0 + p_l L_0 - (R_f - 1)L_0 - R_f P_I L_0 \quad (\text{S-IC1})$$

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)[L_1 - (\alpha R P_S L_0)] > \quad (\text{S-IC2})$$

$$(1 - p_h)R_0 L_0 + p_h L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)L_0 - R_f P_I L_0 - (R_f - 1)L_1$$

$$R_0 L_0 \left[\alpha \left(1 - \frac{p_l + q p_h}{q + 1} \right) - \alpha P_S \right] = 0 \quad (\text{zero-}\pi)$$

$$P_I \geq \frac{p_h}{R_f} \quad (\text{I-Bound})$$

$$\alpha P_S \leq 1 - p_h \quad (\text{S-Bound})$$

From (zero- π) we know that $P_S = 1 - \frac{p_l + q p_h}{q + 1}$. (S-PC1) holds when $\alpha \geq \frac{1 - p_l}{P_S}$, while (S-PC2) holds by assumption 2. P_I is given by the off-the-equilibrium path beliefs of the risk buyer and can be defined given α and R_f . We can find a parametrization in terms of α for

(S-IC1) and (S-IC2):

$$\alpha > \frac{R_0(1 - p_l) + p_l - R_f P_I}{R_0 P_S} \quad (\text{S-IC1b})$$

$$\alpha > \frac{R_0(1 - p_h) + p_h - R_f P_I}{R_f R_0 P_S} \quad (\text{S-IC2b})$$

Using the same method as in the insurance case, we can determine when the sales equilibrium cannot exist. By substituting $P_I = p_l$ as the off-the-equilibrium path belief into (S-IC1b) and (S-IC2b), we can obtain the range for which sales cannot exist (if either one of the following two conditions are met):

$$\alpha \leq \frac{R_0(1 - p_l) + p_l - R_f p_l}{R_0 P_S} \quad (5.9)$$

$$\alpha \leq \frac{R_0(1 - p_h) + p_h - R_f p_l}{R_f R_0 P_S} \quad (5.10)$$

The following Lemma gives the formal conditions for when the sales equilibrium exists.

Lemma 14 *The sales equilibrium exists whenever one of the following three conditions is met*

1. $\alpha \geq \frac{1-p_l}{P_S}$ and $R_f > \max\left\{\frac{R_0(1-p_h)+p_h}{(1-p_l)R_0+P_I}, \frac{p_l}{P_I}\right\}$
2. $\frac{R_0(1-p_h)+p_h}{P_I+R_0P_S} < R_f \leq \frac{R_0(1-p_h)+p_h}{(1-p_l)R_0+P_I}$ and $\alpha > \frac{R_0(1-p_h)+p_h-R_f P_I}{R_f R_0 P_S} \geq \frac{R_0(1-p_l)+p_l-R_f P_I}{R_0 P_S}$
3. $\frac{R_0(1-p_l)+p_l-R_0 P_S}{P_I} < R_f \leq \frac{p_l}{P_I}$ and $\alpha > \frac{R_0(1-p_l)+p_l-R_f P_I}{R_0 P_S} \geq \frac{R_0(1-p_h)+p_h-R_f P_I}{R_f R_0 P_S}$

Proof. See appendix.

The first condition says that so long as α and R_f are sufficiently high, then this equilibrium can exist. The second two conditions simply say that if we force α to be even higher than the first condition, then we can sustain this equilibrium for lower costs of capital

(smaller values of R_f).²⁶ When we combine this Lemma with that of Lemma 13, Proposition 7 will show that the bank will tend to rely on loan insurance when α is low or R_f is close to one. For example, it could be that banks are well capitalized and/or the moral hazard or relationship banking concerns are troublesome in the loan sales market. When α is low and R_f is high, both markets may not exist. By assumption 2, we can rule out the case where a bank with one type of loan wishes not to participate. We can fix this idea by defining a particular off-the-equilibrium path belief in each of these two equilibria. In the insurance case, we set $P_S = 1 - \frac{pl+qp_h}{q+1}$, and for the sales case we set $P_I = \frac{pl+qp_h}{q+1}$. The following proposition shows under this off-the-equilibrium path belief, the choice between insurance and sales is unique.

Lemma 15 *Under the off-the-equilibrium path beliefs assigned, the equilibrium is uniquely determined by R_f and α as either insurance, sales or neither.*

Proof. *See appendix.*

We can now give our main result of the section. The following Proposition states that when capital is relatively cheap, then the insurance equilibrium can be supported when there is sufficient moral hazard/relationship banking costs. Conversely, when the moral hazard/relationship banking costs are low, the sales equilibrium can be supported for sufficiently high costs of capital. The proof of this Proposition follows easily from Lemmas 13 and 14. We can obtain uniqueness of the equilibrium from Lemma 15.

Proposition 7

²⁶Note that in the second two conditions, one must be careful as the the lower bound on R_f cannot be smaller than 1.

1. *When the cost of capital is low, the bank will use insurance when α is sufficiently small.*
2. *When the costs of moral hazard/relationship banking are low, the bank will use sales when R_f is sufficiently high.*

The reason for this result has been discussed earlier, but will be reiterated for clarity. The bank may choose sales over insurance when the cost of capital is high because insurance requires an upfront payment, whereas sales frees up capital immediately. Conversely, since the moral hazard and relationship banking problems will tend to be worse for loan sales, the bank will use insurance when these costs are high.

The results of this chapter show that by introducing adverse selection into the insurance market, the Duffee and Zhou (2001) framework changes quite a bit. In Duffee and Zhou (2001), the existence of the sales equilibrium versus the insurance equilibrium was driven by the timing of the model, and the specific informational assumption on loan insurance. In this model, by relaxing these two key assumptions (which we discuss why they may be unrealistic), we derive properties of a bank (or loan) which can determine whether sales or insurance would be used.

5.6 Conclusion

We use a model where CRT arises because of two factors: first, a bank can use CRT to dump low quality loans, and second, a bank can use CRT when its total risk exceeds a pre-determined level. We show that in the basic setup with no moral hazard or relationship banking costs, only an insurance or sales pooling equilibrium can exist. To determine the conditions under which either equilibrium can be the unique outcome, we extend the model

to allow for costly capital, moral hazard and relationship banking issues. We find that well capitalized banks will use loan insurance in the presence of moral hazard and relationship costs of loan sales. Finally, we show that if the bank is poorly capitalized, so that capital is very costly, they may be forced into the loan sales market even in some cases where the loan sale price could be significantly depressed.

Chapter 6

Summary and Conclusions

The large increase in the usage of credit risk transfer in the past 15 years has fundamentally changed the banking environment. This dissertation aims to analyze some of the new incentives that these forms of contracts create. Chapters 3 and 4 analyze the issue of counterparty risk. When one party contracts with another, it is important to recognize although risk may appear to be transferred, it may end up back on the books of its originator due to a counterparty failure.

In chapter 3, we analyze a setting in which insurers can fail and construct a model to show that a new moral hazard problem can arise in insurance contracts. The moral hazard problem emerges because the insurer makes an investment choice which is beyond the control of the insured party. We show that the insurer's investment choice is inefficient by demonstrating that a central planner would choose a more liquid investment allocation. The presence of this moral hazard is shown to allow a unique separating equilibrium to exist wherein the insured freely and credibly relays its superior information. In other words, the new moral hazard problem can alleviate the possible adverse selection problem.

In chapter 4, we extend the model of chapter two to allow for multiple insured parties,

multiple insurers and moral hazard on the insured side of the market. In all three cases, the results of the model of chapter 3 are shown to be robust to these changes. In particular, we show that with multiple insurers, we can relax the somewhat restrictive assumption that the contract size is sufficiently large to affect the insurers investment decision. Instead, we show that with aggregate private risk, the separating equilibrium result holds. With multiple insurers, we show that the expected diversification benefit that this should give to the insured party may be offset by an increase in the moral hazard problem. Finally, with moral hazard on the insured side of the market, we show that our new moral hazard may actually curtail the desire for the insured party to behave riskier.

Chapter 5 examines the choice between loan insurance and sales for a bank engaging in credit risk transfer. We find that well capitalized banks will use loan insurance in the presence of moral hazard and relationship costs of loan sales. Finally, we show that if the bank is poorly capitalized, so that capital is very costly, they may be forced into the loan sales market where moral hazard may be more prevalent.

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Appendix A

Appendix for Chapter 2

Proof Lemma 1. Using the assumption that $f(\theta)$ is distributed uniform over the interval $[\underline{R}_f, \bar{R}_f]$, we solve for the optimal choice of β for the IFI, given b and P .

$$\max_{\beta \in [0,1]} \Pi_{IFI}$$

Using Leibniz rule to differentiate the choice variable in the integrands, we obtain the following first order equation:

$$\begin{aligned} 0 = & \frac{bP\gamma}{\bar{R}_f - \underline{R}_f} [C'(\gamma - \beta P\gamma) (G - C(\gamma - \beta P\gamma) - \beta P\gamma) \\ & + (\bar{R}_f - C(\gamma - \beta P\gamma)) (C'(\gamma - \beta P\gamma) - 1)] \\ & + (1 - b) \frac{G}{\bar{R}_f - \underline{R}_f} [-R_I \gamma P + \gamma P] + P\gamma(1 - R_I) \end{aligned} \quad (\text{A.1})$$

Where $G - C(\gamma - \beta P\gamma) - \beta P\gamma \geq 0$ by assumption, and $C'(\gamma - \beta P\gamma) - 1 \geq 0$ since $C(x) \geq x \forall x \geq 0$. To ensure a maximum, we take the second order condition and show the inequality that must hold.

$$\begin{aligned} C''(\gamma - \beta P\gamma) (G - C(\gamma - \beta P\gamma) - \beta P\gamma) + (\bar{R}_f - C(\gamma - \beta P\gamma)) C''(\gamma - \beta P\gamma) \\ \geq 2C'(\gamma - \beta P\gamma) (C'(\gamma - \beta P\gamma) - 1) \end{aligned} \quad (\text{A.2})$$

Note that this holds with equality when $C(x) = x \forall x \geq 0$ so that $C'(x) = 1 \forall x \geq 0$ and $C''(x) = 0 \forall x \geq 0$. Plugging in the boundary conditions for β into the FOC, we now derive the optimal proportion of capital put in the liquid asset as an implicit function.

$$\begin{cases} \beta^* = 0 & \text{if } b \leq b^* \\ -(1-b)(R_I - 1)G + b[C'(\gamma - \beta^*P\gamma)(G - C(\gamma - \beta^*P\gamma) - \beta^*P\gamma) \\ + (\bar{R}_f - C(\gamma - \beta^*P\gamma))(C'(\gamma - \beta^*P\gamma) - 1)] = (R_I - 1)(\bar{R}_f - \underline{R}_f) & \text{if } b \in (b^*, b^{**}) \\ \beta^* = 1 & \text{if } b \geq b^{**} \end{cases} \quad (\text{A.3})$$

$$\text{where } b^* = \frac{(R_I - 1)(G + \bar{R}_f - \underline{R}_f)}{G(R_I - 1) + C'(\gamma)(G - C(\gamma)) - (\bar{R}_f - C(\gamma))(C'(\gamma) - 1)},$$

$$\text{and } b^{**} = \frac{(R_I - 1)(G + \bar{R}_f - \underline{R}_f)}{G(R_I - 1) + C'(\gamma - P\gamma)(G - C(\gamma - P\gamma) - P\gamma) - (\bar{R}_f - C(\gamma - P\gamma))(C'(\gamma - P\gamma) - 1)}.$$

We now show that the optimal proportion of capital put in the liquid asset is increasing in b by finding $\frac{\partial \beta}{\partial b}$ from the FOC.

$$\begin{aligned} 0 = & A + b[(-C'(\gamma - \beta P\gamma))(-\frac{\partial \beta}{\partial b}P\gamma)(C'(\gamma - \beta P\gamma) - 1) \\ & + (\bar{R}_f - C(\gamma - \beta P\gamma))(C''(\gamma - \beta P\gamma))(-\frac{\partial \beta}{\partial b}P\gamma) \\ & + C''(\gamma - \beta P\gamma)(-\frac{\partial \beta}{\partial b}P\gamma)(G - C(\gamma - \beta P\gamma) - \beta P\gamma) \\ & + C'(\gamma - \beta P\gamma)(-C'(\gamma - \beta P\gamma))(-\frac{\partial \beta}{\partial b}P\gamma) \\ & - (-\frac{\partial \beta}{\partial b}P\gamma)] + G(R_I - 1)P\gamma \end{aligned} \quad (\text{A.4})$$

Where we define:

$$\begin{aligned} A = & C'(\gamma - \beta P\gamma)P\gamma(G - C(\gamma - \beta P\gamma) - \beta P\gamma) \\ & + (\bar{R}_f - C(\gamma - \beta P\gamma))P\gamma(C'(\gamma - \beta P\gamma) - 1) \geq 0. \end{aligned} \quad (\text{A.5})$$

Assuming an interior solution and rearranging for $\frac{\partial \beta}{\partial b}$ yields to following.

$$\frac{\partial \beta}{\partial b} = \frac{A_2}{A_3} \quad (\text{A.6})$$

$$> 0 \quad (\text{A.7})$$

Where:

$$\begin{aligned} A_2 = & -C'(\gamma - \beta P\gamma) (G - C(\gamma - \beta P\gamma) - \beta P\gamma) \\ & - (\bar{R}_f - C(\gamma - \beta P\gamma)) (C'(\gamma - \beta P\gamma) - 1) - G(R_I - 1) \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} A_3 = & -C''(\gamma - \beta P\gamma) (G - C(\gamma - \beta P\gamma) - \beta P\gamma) \\ & - (\bar{R}_f - C(\gamma - \beta P\gamma)) C''((\bar{R}_f - C(\gamma - \beta P\gamma))) \\ & + 2C'(\gamma - \beta P\gamma) (C'(\gamma - \beta P\gamma) - 1) \end{aligned} \quad (\text{A.9})$$

The numerator of (A.6) is trivially negative while the denominator is negative because of condition (A.2) imposed by the SOC to achieve a maximum.

■

Proof of Lemma 2.

Step 1: Existence

We prove that there exists a P^* that satisfies the following:

$$\begin{aligned} 0 = & -b \left[\int_{C(\gamma - \beta P^*\gamma)}^{\bar{R}_f} (C(\gamma - \beta P^*\gamma) + \beta P^*\gamma) f(\theta) d\theta \right] - b \left[\int_0^{C(\gamma - \beta P^*\gamma)} G f(\theta) d\theta \right] \\ & (1 - b) \left[\int_{-P^*\gamma(\beta + (1 - \beta)R_I)}^0 G f(\theta) d\theta \right] + P^*\gamma (\beta + (1 - \beta)R_I). \end{aligned} \quad (\text{A.10})$$

Consider $P^* \leq 0$. In this case, the IFI earns negative profits. To see this, notice all terms on the right hand side of (A.10) are weakly negative, with the second and third

terms strict (since $C(\gamma - \beta P^* \gamma) > \beta P^* \gamma$ when $P^* \leq 0$). Therefore, it must be that $\Pi_{IFI}(\beta^*, P^* \leq 0) < 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium.

Next, consider $P^* \geq 1$, and $\beta = 1$ (not necessarily the optimal value). In this case, the first term on the right hand side of (A.10) is strictly positive and the third term is zero. The second plus the fourth term is positive since $P^* \gamma > b \int_0^{\bar{R}_f} P^* \gamma f(\theta) d\theta$. Since β^* can yield no less profit than $\beta = 1$ by definition of it being an optimum, it must be that $\Pi_{IFI}(\beta^*, P^* \geq 0) > 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium. Therefore, if it exists, $P^* \in (0, 1)$.

To show that P^* exists in the interval $(0, 1)$, we differentiate the right hand side of (A.10) to show that profit is strictly increasing in P .

$$\begin{aligned} \frac{\partial \Pi_{IFI}}{\partial P} &= b\beta P [C'(\gamma - \beta P \gamma) (G - C(\gamma - \beta P \gamma) - \beta P \gamma) \\ &\quad + (\bar{R}_f - C(\gamma - \beta P \gamma)) \beta \gamma (C'(\gamma - \beta P \gamma) - 1)] \\ &\quad + (1 - b) [G \gamma (\beta + (1 - \beta) R_I)] + \gamma (\beta + (1 - \beta) R_I) \end{aligned} \quad (\text{A.11})$$

$$> 0 \quad (\text{A.12})$$

Where the inequality follows from the assumption that $G \geq C(\gamma - \beta P \gamma) - \beta P \gamma$ and the assumption that $C(x) \geq x \forall x \geq 0$ (which implies $C'(x) \geq 1$). Therefore, since profit is negative when $P^* \leq 0$ and positive when $P^* \geq 1$, and since profit is a (monotonically) increasing function of P^* , profit must equate to zero within $P^* \in (0, 1)$.

Step 2: Uniqueness

Assume the following holds: $\Pi_{IFI}(\beta^*, P_1^*) = 0$. Since we have already shown that profit is a strictly increasing function of P^* , then if $P_2^* > P_1^*$ ($P_2^* < P_1^*$) this implies

$\Pi_{IFI}(\beta^*, P_2^*) > 0$ ($\Pi_{IFI}(\beta^*, P_2^*) < 0$). Therefore, $\Pi_{IFI}(\beta^*, P_2^*) = 0$ implies $P_1^* = P_2^*$ must hold, so our price is unique.

■

Proof of Lemma 3.

From the envelop theorem, we can ignore the effect that changes in b have on β when we evaluate the payoff at β^* . Plugging $\beta = \beta^*$ into (3.2) and taking the partial derivative with respect to b yields:

$$\left. \frac{\partial \Pi_{IFI}}{\partial b} \right|_{\beta=\beta^*} = -\frac{A_4}{\bar{R}_f - \underline{R}_f} < 0 \quad (\text{A.13})$$

Where:

$$\begin{aligned} A_4 = & (\bar{R}_f - C(\gamma - \beta^* P \gamma)) (C(\gamma - \beta^* P \gamma) + \beta^* P \gamma) + C(\gamma - \beta^* P \gamma) G \\ & + P \gamma G(\beta^* + (1 - \beta^*) R_I) \end{aligned} \quad (\text{A.14})$$

The inequality (A.13) follows because $C(\cdot) > 0$ by assumption. Since the envelop theorem is a local condition and does not hold for large changes in b , it serves as an upper bound on the decrease in profits. It follows that an increase in b must be met with an increase in P otherwise the IFI would earn negative profit and would not participate in the market.

■

Proof of Lemma 4. Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$, we find the effect that a change in P has on $C(\gamma - \beta^* P \gamma)$. Since $C(\cdot)$ is monotonic, we focus on $(\gamma - \beta^* P \gamma)$. It should be immediately apparent that when $\beta^* = 0$, changes in P have no effect. Intuitively, if the IFI is already putting everything into the illiquid asset, any additional capital will also be put into the illiquid asset.

We now take the following partial derivative and show that it equates to zero.

$$\frac{\partial (\gamma - \beta^* P \gamma)}{\partial P} = -\gamma \left(\frac{\partial \beta^*}{\partial P} P + \beta^* \right) \quad (\text{A.15})$$

We find $\frac{\partial \beta^*}{\partial P} \equiv \frac{\partial \beta}{\partial P} \Big|_{\beta=\beta^*}$ (where β^* is defined implicitly in the FOC).

$$\begin{aligned} 0 = & \left[-C'(\gamma - \beta^* P \gamma) \left(-\frac{\partial \beta^*}{\partial P} P \gamma - \beta^* \gamma \right) \right] [C'(\gamma - \beta^* P \gamma) - 1] \\ & + [\bar{R}_f - C(\gamma - \beta^* P \gamma)] \left[C''(\gamma - \beta^* P \gamma) \left(-\frac{\partial \beta^*}{\partial P} P \gamma - \beta^* \gamma \right) \right] \\ & + \left[C''(\gamma - \beta^* P \gamma) \left(-\frac{\partial \beta^*}{\partial P} P \gamma - \beta^* \gamma \right) \right] [G - C(\gamma - \beta^* P \gamma) - \beta^* P \gamma] \\ & + C'(\gamma - \beta^* P \gamma) \left[-C'(\gamma - \beta^* P \gamma) \left(-\frac{\partial \beta^*}{\partial P} P \gamma - \beta^* \gamma \right) - \beta^* \gamma - \frac{\partial \beta^*}{\partial P} P \gamma \right] \end{aligned} \quad (\text{A.16})$$

Rearranging for $\frac{\partial \beta^*}{\partial P}$ yields the following.

$$\begin{aligned} \frac{\partial \beta^*}{\partial P} P \gamma A &= -\beta^* \gamma A \\ \Rightarrow \frac{\partial \beta^*}{\partial P} &= -\frac{\beta^*}{P} \end{aligned} \quad (\text{A.17})$$

Where we define:

$$\begin{aligned} A = & C'''(\gamma - \beta^* P \gamma) (G - C(\gamma - \beta^* P \gamma) - \beta^* P \gamma) \\ & + (\bar{R}_f - C(\gamma - \beta^* P \gamma)) C''(\gamma - \beta^* P \gamma) \\ & - 2C'(\gamma - \beta^* P \gamma) (C'(\gamma - \beta^* P \gamma) - 1). \end{aligned} \quad (\text{A.18})$$

Note that $A < 0$ from the assumption on the SOC (A.2) to ensure a maximum (recall that we are interested in interior solutions so that $A \neq 0$). Substituting (A.17) into (A.15) yields the desired result:

$$\frac{\partial (\gamma - \beta^* P \gamma)}{\partial P} = 0. \quad (\text{A.19})$$

Therefore changes in P have no effect on counterparty risk when β attains an interior solution. The final situation is where $\beta^* = 1$. We obtain:

$$\frac{\partial(\gamma - \gamma P)}{\partial P} = -\gamma < 0. \quad (\text{A.20})$$

In this case, the IFI puts all additional premia in the liquid asset and thus reduces the counterparty risk.

■

Proof of Lemma 5. Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$, we are interested in what happens to $C(\gamma - \beta^* P^* \gamma)$ as b changes.

We first focus on the case in which $\beta^* \in (0, 1)$. We take following partial derivative where we define $\frac{\partial \beta^*}{\partial b} \equiv \frac{\partial \beta}{\partial b} \Big|_{\beta=\beta^*}$ and $\frac{\partial P^*}{\partial b} \equiv \frac{\partial P}{\partial b} \Big|_{P=P^*}$.

$$\frac{\partial(\gamma - \beta^* P^* \gamma)}{\partial b} = -\gamma \left(\frac{\partial \beta^*}{\partial b} P^* + \beta^* \frac{\partial P^*}{\partial b} \right) \quad (\text{A.21})$$

From Lemma 1 we know $\frac{\partial \beta^*}{\partial b} \geq 0$. As well, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Since $\beta^* \in (0, 1)$ and $P^* > 0$ (from Lemma 2), it follows that:

$$\frac{\partial(\gamma - \beta^* P^* \gamma)}{\partial b} < 0 \quad (\text{A.22})$$

Therefore, as b increases, counterparty risk decreases when $\beta \in (0, 1)$. Next, consider the case of $\beta^* = 1$. Again, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Therefore, $\frac{\partial(\gamma - \beta^* P^* \gamma)}{\partial b} < 0$ regardless of whether $\frac{\partial \beta^*}{\partial b} = 0$ or $\frac{\partial \beta^*}{\partial b} > 0$. Thus, counterparty risk decreases when b decreases if $\beta^* = 1$.

It is obvious that if $\beta^* = 0$ there will be no change in counterparty risk by noting that $\beta^* P^* \gamma$ will be independent of b .

■

Proof of Proposition 1. We begin by ruling out a separating equilibrium when there is no counterparty risk, regardless of the IFI's choice. This implies $\int_{R_f}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) = 0$. It follows that the left hand side of (3.5) and (3.6) are both zero. Since $P_R^* - P_S^* > 0$, (3.5) and (3.6) cannot be simultaneously satisfied so that this separating equilibrium cannot exist.

We proceed by showing the conditions for which the two pooling equilibria can exist. We begin with the case in which both types wish to be revealed as safe. We define $\beta_{1/2}^*$ and $P_{1/2}^*$ as the equilibrium result from the IFI's problem when the belief of the probability of a claim cannot be updated further: $b = \frac{1}{2}(2 - p_S - p_r)$. Finally, we let β_{OE}^* and P_{OE}^* be the result from the IFI's problem when a bank gives an off the equilibrium path report of R . The following two conditions formalize this case:

$$\begin{aligned} \Pi(S, S) \geq \Pi(S, R) \Rightarrow \\ \underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_{OE}^* P_{OE}^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{OE}^* - P_{1/2}^*}_{\text{amount to be saved in insurance premia}} \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} \Pi(R, S) \geq \Pi(R, R) \Rightarrow \\ \underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_{OE}^* P_{OE}^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{OE}^* - P_{1/2}^*}_{\text{amount to be saved in insurance premia}} \end{aligned} \quad (\text{A.24})$$

The binding condition (A.24) is satisfied for Z sufficiently small. The intuition is that if counterparty risk is not too costly, the bank would wish to obtain lowest insurance premium. In other words, the *premium effect* dominates for both types. It follows that for this equilibrium to exist, $b > \frac{1}{2}(2 - p_S - p_R)$. Next, consider the case in which both types report that they are risky. In this case, we use the notation β_{OE2}^* and P_{OE2}^* to indicate the

off the equilibrium path beliefs if a bank reports that it is safe. The conditions can be characterized as follows:

$$\begin{aligned} \Pi(S, R) \geq \Pi(S, S) \Rightarrow \\ \underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_{OE2}^* P_{OE2}^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_{1/2}^* - P_{OE2}^*}_{\text{amount extra to be paid in insurance premia}} \end{aligned} \quad (\text{A.25})$$

$$\begin{aligned} \Pi(R, S) \geq \Pi(R, R) \Rightarrow \\ \underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_{OE2}^* P_{OE2}^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_{1/2}^* - P_{OE2}^*}_{\text{amount extra to be paid in insurance premia}} \end{aligned} \quad (\text{A.26})$$

The binding condition (A.25) is satisfied for Z sufficiently high. Intuitively, the bank is so averse to counterparty risk, that the *counterparty risk effect* dominates for both types. It follows that for this equilibrium to exist, $b < \frac{1}{2}(2 - p_S - p_R)$.

We now show that the separating equilibrium defined by (3.5) and (3.6) can be unique. Combining (3.5) and (3.6) we obtain the following condition for when the separating equilibrium exists:

$$\frac{P_R - P_S}{(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \leq 1 + Z \leq \frac{P_R^* - P_S^*}{(1 - p_S) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \quad (\text{A.27})$$

Turning to the pooling equilibria, we use extreme off the equilibrium path beliefs to illuminate the result (which is valid for the general belief as well). Let $OE = R$ and $OE2 = S$. The condition under which the pooling equilibrium cannot exist (i.e., when (A.25) and (A.26) are not satisfied) can be written as:

$$\frac{P_R^* - P_{1/2}^*}{(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)} < 1 + Z < \frac{P_{1/2}^* - P_S^*}{(1 - p_S) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \quad (\text{A.28})$$

It follows that if (A.27) and (A.28) are satisfied, the separating equilibrium exists and is unique.¹ To see that these conditions can be simultaneously satisfied, let $p_S \rightarrow 1$ so that the right hand side of both (A.27) and (A.28) are satisfied. It follows that if Z is sufficiently large, the left hand side of these two inequalities can be satisfied yielding a unique separating equilibrium.

■

Proof of Proposition 2. The proof proceeds in 3 steps. Step 1 derives the first order condition for the planning problem. Step 2 assumes the equilibrium solution and derives an expression for $\frac{\partial P}{\partial \beta}$ from the IFI's zero profit condition. Step 3 shows that β^{pl} and P^{pl} must be greater than in the equilibrium case when $\beta^* < 1$. Since we need not specify a belief for this proof, it follows that the result holds regardless if there is separation or pooling of banks.

Step 1

The profit for the bank (bk) can written as follows (note here we leave the bank's loan type as $j \in \{S, R\}$ as the proof is valid for both the safe and risky type).

$$\Pi_{bk} = p_j R_B \gamma + \gamma(1 - p_j) \int_{C(\gamma - \beta P \gamma)}^{\bar{R}_f} dF(\theta) - \gamma(1 - p_j) Z \int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} dF(\theta) - \gamma P$$

In the planners case, P^{pl} is now endogenous and determined by $\Pi_{IFI}(\beta^{pl}, P^{pl}) = 0$ (where Π_{IFI} is defined by (3.2)). Using the uniform assumption on F yields the following first order condition.

$$\frac{\partial P}{\partial \beta} = \gamma C'(\gamma - \beta P \gamma) \left(P + \frac{\partial P}{\partial \beta} \beta \right) (1 - p_j)(1 + Z) \quad (\text{A.29})$$

¹Note that the separating equilibrium is unique since any other possible separating equilibrium would only differ in terms of off the equilibrium path beliefs.

The left hand side represents the marginal cost of increasing β , while the right hand side represents the marginal benefit of doing so.

Step 2

We show that if $\beta^{pl} = \beta^*$, then (A.29) cannot hold. We know from the IFI's problem, the following must hold (see the proof to Lemma 1 for its derivation):

$$\begin{aligned}
0 &= \frac{bP^*\gamma}{\bar{R}_f - \underline{R}_f} [C'(\gamma - \beta^*P^*\gamma) (G - C(\gamma - \beta^*P^*\gamma) - \beta^*P^*\gamma) \\
&\quad + (\bar{R}_f - C(\gamma - \beta^*P^*\gamma)) (C'(\gamma - \beta^*P^*\gamma) - 1)] \\
&\quad + (1-b) \frac{G}{\bar{R}_f - \underline{R}_f} [-R_I\gamma P^* + \gamma P^*] + P^*\gamma(1 - R_I) \tag{A.30}
\end{aligned}$$

We now find an expression for $\left. \frac{\partial P}{\partial \beta} \right|_{\beta=\beta^*, P=P^*}$ by implicitly differentiating the equation $\Pi_{IFI}(\beta^*, P^*) = 0$.

$$\begin{aligned}
0 &= (1-b) \left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^0 Gf(\theta)d\theta \right] - b \left[\int_{C(\gamma-\beta P\gamma)}^{\bar{R}_f} (C(\gamma - \beta P\gamma) + \beta P\gamma) \right] \\
&\quad - b \left[\int_0^{C(\gamma-\beta P\gamma)} Gf(\theta)d\theta \right] + P\gamma(\beta + (1-\beta)R_I) \tag{A.31}
\end{aligned}$$

Implicitly differentiating this equation to find $\frac{\partial P}{\partial \beta}$ yields the following.

$$\begin{aligned}
A \left. \frac{\partial P}{\partial \beta} \right|_{\beta=\beta^*, P=P^*} &= (1-b) \frac{G}{\bar{R}_f - \underline{R}_f} [-R_I\gamma P^* + \gamma P^*] + P^*\gamma(1 - R_I) \\
&\quad + \frac{bP^*\gamma}{\bar{R}_f - \underline{R}_f} [C'(\gamma - \beta^*P^*\gamma) (G - C(\gamma - \beta^*P^*\gamma) - \beta^*P^*\gamma) \\
&\quad + (\bar{R}_f - C(\gamma - \beta^*P^*\gamma)) (C'(\gamma - \beta^*P^*\gamma) - 1)] \tag{A.32}
\end{aligned}$$

Where we define:

$$\begin{aligned}
A &= b\beta^*\gamma[C'(\gamma - \beta^*P^*\gamma)(C(\gamma - \beta^*P^*\gamma) + \beta^*P^*\gamma) \\
&\quad - (\bar{R}_f - C(\gamma - \beta^*P^*\gamma))(C'(\gamma - \beta^*P^*\gamma) - 1) \\
&\quad + C'(\gamma - \beta^*P^*\gamma)G].
\end{aligned} \tag{A.33}$$

It follows that $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} = 0$ since the right hand side of (A.32) is the FOC derived in Lemma 1 and must equate to 0 at the optimum, β^* .

Step 3

Substituting $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} = 0$ into (A.29) yields:

$$0 = \gamma C'(\gamma - \beta^*P^*\gamma)(P^*)(1 - p_j)(1 + Z), \tag{A.34}$$

which cannot hold since $\gamma > 0$, $(1 - p_j) > 0$ and $Z > 0$. Therefore, $\beta^{pl} \neq \beta^*$ and $P^{pl} \neq P^*$. To satisfy (A.29), it must be the case that $\beta^{pl} > \beta^*$, and from Lemma 6 it follows that $P^{pl} \geq P^*$. However, if $\beta^{pl} > \beta^*$, then $P^{pl} > P^*$. It follows that $\int_0^{C(\gamma - \beta^{pl}P^{pl}\gamma)} f(\theta)d\theta < \int_0^{C(\gamma - \beta^*P^*\gamma)} f(\theta)d\theta$, i.e., counterparty risk is strictly smaller in the planners case as compared to the equilibrium case.

It is obvious that if $\beta^* = 1$, it is not possible for the planner to make a more liquid investment choice for the IFI. This is the case in which the IFI is already investing everything in the liquid asset.

■

Appendix B

Appendix for Chapter 3

Proof of Lemma 7. Optimizing Π_{IFI}^{MB} choosing β yields the following first order condition (recall F is assumed to be uniformly distributed):

$$\begin{aligned}
0 &= \frac{1}{\bar{R}_f - \underline{R}_f} \int_0^{\beta^* PM\gamma} (-PM\gamma(1 - R_I)) Gdb(y) \\
&+ \frac{1}{\bar{R}_f - \underline{R}_f} [-PM\gamma(\beta^* + (1 - \beta^*)R_I) + \beta^* PM\gamma - \underline{R}_f] GPM \\
&+ \frac{1}{\bar{R}_f - \underline{R}_f} \int_{\beta^* PM\gamma}^M [-C'(y\gamma - \beta^* PM\gamma)(-PM\gamma) (-C(y\gamma - \beta^* PM\gamma) - \beta^* PM\gamma) \\
&+ (\bar{R}_f - C(y\gamma - \beta^* PM\gamma)) (-C'(y\gamma - \beta^* PM\gamma)(-PM\gamma) - PM\gamma) \\
&+ C'(y\gamma - \beta^* PM\gamma)(-PM\gamma)(-G)]db(y) \\
&- \frac{1}{\bar{R}_f - \underline{R}_f} [(\bar{R}_f - C(0)) (-C(0) - \beta^* PM\gamma) + (C(0) - \underline{R}_f) (-G)] PM \\
&+ (1 - R_I)PM\gamma
\end{aligned} \tag{B.1}$$

Recalling $C(0) = 0$ we simplify the above.

$$\begin{aligned}
0 &= - \int_0^{\beta^* PM} \gamma(R_I - 1)G db(y) - PM\gamma(1 - \beta^*)R_I G \\
&+ \gamma \int_{\beta^* PM}^M [C'(y\gamma - \beta^* PM\gamma) (G - C(y\gamma - \beta^* PM\gamma) - \beta^* PM\gamma) \\
&+ (\bar{R}_f - C(y\gamma - \beta^* PM\gamma)) (C'(y\gamma - \beta^* PM\gamma) - 1)] db(y) \\
&+ \bar{R}_f \beta^* \gamma - \underline{R}_f G - \gamma(R_I - 1)(\bar{R}_f - \underline{R}_f)
\end{aligned} \tag{B.2}$$

The SOC implies that the right hand side of (B.2) is decreasing in β^* so that our problem achieves a maximum. Define two belief distributions $b_1(y)$ and $b_2(y)$ such that $b_1(y) \geq b_2(y) \forall y$. As well, let $(\beta_1^*, b_1(y))$ solve the first order condition (B.1). Intuitively, moving from $b_1(y)$ to $b_2(y)$, mass shifts from the interval $[0, \beta^* PM]$ to $[\beta^* PM, M]$. Formally:

$$\int_0^{\beta^* PM} db_1(y) > \int_0^{\beta^* PM} db_2(y) \tag{B.3}$$

$$\int_{\beta^* PM}^M db_1(y) < \int_{\beta^* PM}^M db_2(y). \tag{B.4}$$

Given (B.3) and (B.4) and since the FOC holds with $(\beta_1^*, b_1(y))$, then with $(\beta_1^*, b_2(y))$, it follows that β_1^* must increase for (B.2) to hold. In other words, the riskier the distribution of loans that the IFI insures, the more that it invests in the liquid asset.

To proceed, we use a similar result to that of Lemma 3. It is straight forward to see that when the belief of defaults is higher (as in the risky case), so must the price of the contracts be higher (this can be shown in the same way that Lemma 3 was proved by showing that the profit function is decreasing in the amount of risk in the loans). Next we find what happens to counterparty risk. What is different about the case of multiple banks is that counterparty risk is defined relative to the number of banks that default: $\int_{\beta PM}^M \int_{\underline{R}_f}^{C(y\gamma - \beta PM\gamma)} dF(\theta) db(y)$. In the case in which the IFI puts more weight on the loans being risky ($q_A = r$), β^* and P^* increase, so that $C(\gamma - \beta P\gamma)$ decreases. Furthermore, since from the point of view of the

banks, the probability of a claim does not change, counterparty risk decreases as compared to the case in which the IFI puts more weight on the loans being safe ($q_A = s$).

■

Proof of Proposition 3. The proof proceeds in 3 steps. Steps 1 and 2 determine when the pooling equilibria cannot exist. In particular, we use beliefs of the IFI for which banks have the greatest incentive to pool. In step 1 we assume that all banks report that they received the aggregate shock $q_A = s$ and find a condition wherein some bank (or measure of banks) that received the aggregate shock $q_A = r$ wish to reveal it truthfully.¹ In the second step we repeat a similar exercise to determine when a risky pooling equilibrium does not exist. Step 3 determines when a unique separating equilibrium can exist. We use beliefs such that the banks have the least incentive to separate. In this step we assume separating beliefs for the IFI and find the condition wherein both bank types do not wish to misrepresent their aggregate type.

Step 1

Consider all banks reporting $q_A = s$, regardless of the aggregate shock. Now consider the incentive of banks that receive the aggregate shock $q_A = r$. Given that all banks report that they are safe, we see if there is any incentive for deviation (i.e., some bank to send the message $q_A = r$). If every bank reports that it received the safe aggregate shock, the IFI does not update its beliefs. However, if some bank or measure of banks²

¹Note that there are other ways of arriving at a pooling equilibrium, for example, some banks of the same type report differently than others. These can arise when the IFI's beliefs are such that no new information is gleaned from the reports. Since these yield the same outcome, we will focus only on the cases described.

²There is a technical issue that would not arise if there was a finite number of banks. In the traditional Riemann sense of measurability using the concept of point-wise convergence almost everywhere, a bank of measure zero cannot change the IFI's beliefs. There are two ways to rectify this. The first way is to employ the Pettis-integral as in Uhlig (1994). The second way is to imagine a small but positive measure of banks deviating.

deviate and send the message $q_A = r$, then the IFI employs off the equilibrium path beliefs (OE) about the aggregate shock. We assume that the deviating bank(s) are believed to have received the highest idiosyncratic shock(s).³ It is easy to show that the bank with the greatest incentive to be revealed as risky is the one with the highest idiosyncratic shock, which we denote as bank M . Denote the probability of default of the loan of this bank as q_M^r . If this bank does not deviate, it pays $P^{\frac{1}{2}}$, alongside all other banks. If this bank does deviate, it pays P_M^{OE} , while the rest of the banks pay P^{OE} , corresponding to the average quality of a risky bank. Next, we denote the optimal investment choice of the IFI in the pooling (deviating) case by $\beta^{1/2}$ (β^{OE}). Finally, we let $D^{1/2}$ (D^{OE}) represent the probability that upon a claim being made in the pooling (deviating) case, the IFI fails and so cannot pay. It follows that $D^{1/2} = \int_{\beta^{1/2} P^{1/2} M}^M \int_{\underline{R}_f}^{C(y\gamma - \beta^{1/2} P^{1/2} M\gamma)} dF(\theta) db(y)$ and $D^{OE} = \int_{\beta^{OE} P^{OE} M}^M \int_{\underline{R}_f}^{C(y\gamma - \beta^{OE} P^{OE} M\gamma)} dF(\theta) db(y)$. The condition under which this pooling equilibrium cannot exist is given as follows.

$$\begin{aligned}
& (1 - q_M^r)R_B + q_M^r\gamma(1 - D^{OE}) - q_M^r\gamma D^{OE}Z - \gamma P_M^{OE} \\
& \geq (1 - q_M^r)R_B + q_M^r\gamma(1 - D^{1/2}) - q_M^r\gamma D^{1/2}Z - \gamma P^{1/2} \\
& \Rightarrow q_M^r (D^{1/2} - D^{OE}) (1 + Z) \geq P_M^{OE} - P^{1/2} \tag{B.5}
\end{aligned}$$

Step 2

Consider all banks reporting $q_A = r$, regardless of the aggregate shock. Now consider the incentive of banks that receive the aggregate shock $q_A = s$. We find the condition under which a bank would like to reveal that it is safe ($q_A = s$). Let the beliefs of the IFI be that if one (or some positive measure) of banks report that they are safe, then there are off the equilibrium path beliefs (OE2) about the aggregate shock. Furthermore, those reporting that they are safe are believed to have received the lowest idiosyncratic shock.

³This belief about the idiosyncratic shock gives the banks the greatest incentive to pool. In this way, we can rule out pooling if, under these conditions, there is still a bank that wishes to deviate.

We know that the bank with the greatest incentive to be revealed as safe is the one with the lowest idiosyncratic shock, call it bank 0. Denote the probability of default for the loan of this bank as q_0^s and the individual price if they reveal themselves as safe as P_M^{OE2} . In this case, let the price paid by there other banks be P^{OE2} and β^{OE2} be the optimal investment choice of the IFI. It follows that counterparty risk in this case can be defined by: $D^{OE2} = \int_{\beta^{OE2} P^{OE2} M}^M \int_{R_f}^{C(y\gamma - \beta^{OE2} P^{OE2} M\gamma)} dF(\theta) db(y)$. The condition under which this pooling equilibrium cannot exist is given as follows.

$$\begin{aligned}
& (1 - q_0^s)R_B + q_0^s\gamma(1 - D^{OE2}) - q_0^s\gamma D^s Z - \gamma P_M^{OE2} \\
& \geq (1 - q_0^s)R_B + q_0^s\gamma(1 - D^{1/2}) - q_0^s\gamma D^{1/2} Z - \gamma P^{1/2} \\
& \Rightarrow q_0^s \left(D^{OE2} - D^{1/2} \right) (1 + Z) \leq P^{1/2} - P_M^{OE2} \tag{B.6}
\end{aligned}$$

Therefore (B.5) and (B.6) are simultaneously satisfied when:

$$\frac{P^{1/2} - P_M^{OE2}}{q_0^s (D^{OE2} - D^{1/2})} \geq 1 + Z \geq \frac{P_M^{OE} - P^{1/2}}{q_M^r (D^{1/2} - D^{OE})}. \tag{B.7}$$

Where $q_0^s < q_M^r$. To see that (B.7) can hold, consider the limit as q_0^s approaches zero with Z sufficiently large.

Step 3

We now find the conditions under which the separating equilibrium exists. Consider the following beliefs of the IFI: if all banks report that they are safe, then they are believed to be safe, if all report risky, then they are believed to be risky. Imagine the aggregate shock was $p_A = s$ and all banks report truthfully. Consider the bank with the highest idiosyncratic shock, bank M (we use this bank because they have the greatest incentive to deviate). If it reports $p_A = s$ then the price it receives is P^s and the counterparty risk it is exposed to is D^s (alongside the other banks). If it deviates and reports $p_A = r$ then the counterparty risk

is D^{OE3} and the price is P_0^{OE3} , where $OE3$ represents the off the equilibrium path beliefs of the IFI. Since we are trying to determine conditions under which a separating equilibrium exists, a deviating bank is believed to have received the best idiosyncratic shock. The condition under which the bank would report truthfully is given by the following.

$$\begin{aligned}
& (1 - q_M^s)R_B + q_M^s\gamma(1 - D^s) - q_M^s\gamma D^s Z - \gamma P^s \\
\geq & (1 - q_M^s)R_B + q_M^s\gamma(1 - D^{OE3}) - q_M^s\gamma D^{OE3} Z - \gamma P_0^{OE3} \\
\Rightarrow & P_0^{OE3} - P^s \geq q_M^s (D^s - D^{OE3}) (1 + Z) \tag{B.8}
\end{aligned}$$

Next, imagine the aggregate shock was $p_A = r$ and all banks report that $p_A = r$. Consider the bank with the lowest idiosyncratic shock, bank 0. If it reports $p_A = r$ then the price it receives is P^r and the counterparty risk it is exposed to is D^r (alongside the other banks). If it deviates and reports $p_A = s$ then the counterparty risk is D^{OE4} and the price is P_0^{OE4} , where $OE4$ are the off the equilibrium path beliefs in this case. The condition under which the bank would reveal truthfully is given by the following.

$$\begin{aligned}
& (1 - q_0^r)R_B + q_0^r\gamma(1 - D^r) - q_0^r\gamma D^r Z - \gamma P^r \\
\geq & (1 - q_0^r)R_B + q_0^r\gamma(1 - D^{OE4}) - q_0^r\gamma D^{OE4} Z - \gamma P_0^{OE4} \\
\Rightarrow & q_0^r (D^{OE4} - D^r) (1 + Z) \geq P^r - P_0^{OE4} \tag{B.9}
\end{aligned}$$

It follows that (B.8) and (B.9) are simultaneously satisfied when:

$$\frac{P_0^{OE3} - P^s}{q_M^s (D^s - D^{OE3})} \geq 1 + Z \geq \frac{P^r - P_0^{OE4}}{q_0^r (D^{OE4} - D^r)} \tag{B.10}$$

To show that there can exist a unique separating equilibrium we first need to establish that if there is a separating equilibrium, it must be the one outlined by (B.10). Given that any

separating equilibrium must have the same prices and same level of counterparty risk,⁴ they will only differ by off the equilibrium path beliefs. Therefore, the separating equilibrium above represents the only possible separating equilibrium. It follows that the separating equilibrium exists and is unique when both (B.7) and (B.10) are satisfied. To see that this is possible, we can define specific off the equilibrium path beliefs. Let $OE = r$, $OE2 = s$, $OE3 = r$ and $OE4 = s$. Now consider q_M^s and q_0^s sufficiently small so that the left hand sides of (B.7) and (B.10) are satisfied. It follows that for Z sufficiently large, the right hand sides of (B.7) and (B.10) can be satisfied.

■

Proof of Lemma 9. Define the payoff for a bank (bk) who contracts with N IFIs. (note We leave the banks loan type as $j \in \{S, R\}$ as the proof is valid for both types.

$$\begin{aligned} \Pi_{bk} = & p_j R_B + \gamma(1 - p_j) \sum_{n=1}^N \left(\text{prob}(n \text{ IFIs do not fail}) \frac{n}{N} \right) \\ & - \gamma(1 - p_j) \sum_{n=1}^N \left(\text{prob}(n \text{ IFIs fail}) \frac{Zn}{N} \right) - \gamma N P_N \end{aligned} \quad (\text{B.11})$$

Where $\text{prob}(n \text{ IFIs fail})$ represents the probability that n IFIs fail, $\text{prob}(n \text{ IFIs do not fail})$ represents the probability that n IFIs do not fail, and $\gamma N P_N$ represents the total premium paid by the bank. For simplicity (and since the IFIs are ex-ante identical) we assume that they each individually receive P_N in exchange for their coverage of $\frac{\gamma}{N}$. We now find $\sum_{n=1}^N \left(\text{prob}(n \text{ IFIs do not fail}) \frac{n}{N} \right)$ which represents the expected payment from the IFIs

⁴To see this, notice that in any separating equilibrium, the safe and risky types are revealed, so that the risk the IFI faces is known. Therefore, given the fixed profit assumption on the IFI, they must charge the same price and make the same investment decision (conditional on a type) regardless of the off the equilibrium path beliefs (provided these beliefs sustain the equilibrium).

when a claim is made. Expanding this term yields the following.

$$= \sum_{n=0}^N \binom{n}{N} \left(\frac{N!}{n!(N-n)!} \right) \left[\left(\int_{C(\frac{\gamma}{N}-\beta P_N \frac{\gamma}{N})}^{\bar{R}_f} dF(\theta) \right)^n \left(1 - \int_{C(\frac{\gamma}{N}-\beta P_N \frac{\gamma}{N})}^{\bar{R}_f} dF(\theta) \right)^{N-n} \right] \quad (\text{B.12})$$

Where β is the result of the IFI's optimization problem detailed in Lemma 1. Next, let

$a = \int_{C(\frac{\gamma}{N}-\beta P_N \frac{\gamma}{N})}^{\bar{R}_f} dF(\theta)$ so that (B.12) simplifies to:

$$= \sum_{n=0}^N \binom{n}{N} \left(\frac{N!}{n!(N-n)!} \right) [a^n (1-a)^{N-n}] \quad (\text{B.13})$$

$$= \left[a^N + \binom{N-1}{N} \left(\frac{N!}{1!(N-1)!} \right) a^{N-1} (1-a) + \dots + a(1-a)^{N-1} \right] \quad (\text{B.14})$$

Where the second equality follows by expanding the summation and reversing the order.

We now factor out a and simplify:

$$= a \left[a^{N-1} + \frac{(N-1)!}{1!(N-2)!} a^{N-2} (1-a) + \frac{(N-2)!}{2!(N-3)!} a^{N-3} (1-a)^2 + \dots + (1-a)^{N-1} \right] \quad (\text{B.15})$$

Using a change of variable $M = N - 1$:

$$= a \left[a^M + \frac{M!}{1!(M-1)!} a^{M-1} (1-a) + \frac{(M-1)!}{2!(M-2)!} a^{M-2} (1-a)^2 + \dots + (1-a)^M \right] \quad (\text{B.16})$$

$$= \sum_{m=0}^M \frac{M!}{m!(M-m)!} (a^{M-m} (1-a)^m) \quad (\text{B.17})$$

$$= a [a + (1-a)]^M \quad (\text{B.18})$$

Where the final equality follows from the binomial theorem. Since N is finite, this implies M is finite, and therefore $1^{N-1} = 1$. We obtain:

$$\sum_{n=1}^N \left(\text{prob}(n \text{ IFIs do not fail}) \frac{n}{N} \right) = \int_{C(\frac{\gamma}{N} - \beta P_N \frac{\gamma}{N})}^{\bar{R}_f} dF(\theta). \quad (\text{B.19})$$

By letting $a = \int_{\underline{R}_f}^{C(\frac{\gamma}{N} - \beta P_N \frac{\gamma}{N})} dF(\theta)$, we can repeat the above analysis to show:

$$\sum_{n=1}^N \left(\text{prob}(n \text{ IFIs fail}) \frac{n}{N} \right) = \int_{\underline{R}_f}^{C(\frac{\gamma}{N} - \beta P_N \frac{\gamma}{N})} dF(\theta). \quad (\text{B.20})$$

Substituting (B.19) and (B.20) into (B.11) yields an expression for the expected profit of the bank.

$$\begin{aligned} \Pi_{bk} = & p_j R_B + \gamma(1 - p_j) \int_{C(\frac{\gamma}{N} - \beta P_N \frac{\gamma}{N})}^{\bar{R}_f} dF(\theta) \\ & - \gamma(1 - p_j) \int_{\underline{R}_f}^{C(\frac{\gamma}{N} - \beta P_N \frac{\gamma}{N})} Z dF(\theta) - \gamma N P_N \end{aligned} \quad (\text{B.21})$$

We can see that the expected payoff of the bank is given by the expected payoff as if the bank were dealing with only one IFI, but the IFI was making its investment decision with a contract size of $\frac{\gamma}{N}$.

■

Proof of Proposition 4. The proof proceeds in two steps. In step one we show that as γ decreases, the IFI decreases β_N^* (compared to the case of β_1^*). Step two shows that as a result of the decrease of β_N^* , counterparty risk remains unchanged. This proof will follow closely the proof of Lemma 4.

Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$, we find the effect that changes in γ have on $C(\gamma - \beta^* P \gamma)$. Since $C(\cdot)$ is monotonic, we focus on $\gamma - \beta^* P \gamma$. This proposition focuses only on the case in which β^* achieves an interior solution.

Step 1

We wish to take the following partial derivative and show that it equates to zero.

$$\frac{\partial(\gamma - \beta^* P\gamma)}{\partial\gamma} = 1 - \left(\frac{\partial\beta^*}{\partial\gamma} P\gamma + \beta^* P \right) \quad (\text{B.22})$$

We now find $\frac{\partial\beta^*}{\partial\gamma} \equiv \frac{\partial\beta}{\partial\gamma} \Big|_{\beta=\beta^*}$ (where β^* is defined implicitly in the FOC found in the proof to Lemma 1).

$$\begin{aligned} 0 = & \left[C'(\gamma - \beta^* P\gamma) \left(1 - \beta^* P - \frac{\partial\beta^*}{\partial\gamma} P\gamma \right) \right] [C'(\gamma - \beta^* P\gamma) - 1] \\ & + [\bar{R}_f - C(\gamma - \beta^* P\gamma)] \left[C''(\gamma - \beta^* P\gamma) \left(1 - \beta^* P - \frac{\partial\beta^*}{\partial\gamma} P\gamma \right) \right] \\ & + \left[C'''(\gamma - \beta^* P\gamma) \left(1 - \beta^* P - \frac{\partial\beta^*}{\partial\gamma} P\gamma \right) \right] [G - C(\gamma - \beta^* P\gamma) - \beta^* P\gamma] \\ & + C'(\gamma - \beta^* P\gamma) \left[-C'(\gamma - \beta^* P\gamma) \left(1 - \beta^* P - \frac{\partial\beta^*}{\partial\gamma} P\gamma \right) - \beta^* P - \frac{\partial\beta^*}{\partial\gamma} P\gamma \right] \end{aligned} \quad (\text{B.23})$$

Rearranging for $\frac{\partial\beta^*}{\partial\gamma}$ yields the following.

$$\frac{\partial\beta^*}{\partial\gamma} P\gamma A = (1 - \beta^* P) A \quad (\text{B.24})$$

Where we define:

$$\begin{aligned} A = & C''(\gamma - \beta^* P\gamma) (G - C(\gamma - \beta^* P\gamma) - \beta^* P\gamma) \\ & + (\bar{R}_f - C(\gamma - \beta^* P\gamma)) C''(\gamma - \beta^* P\gamma) \\ & - 2C'(\gamma - \beta^* P\gamma) (C'(\gamma - \beta^* P\gamma) - 1). \end{aligned} \quad (\text{B.25})$$

Note that $A < 0$ from the assumption on the SOC to ensure a maximum (recall that we are interested in interior solutions so that $A \neq 0$). Therefore, $\frac{\partial \beta^*}{\partial \gamma} = \frac{1 - \beta^* P}{P \gamma} > 0$. This implies that as γ decreases (N increases), β_N^* decreases as desired.

Step 2

Substituting (B.24) into (B.22) yields the following.

$$\begin{aligned} \frac{\partial (\gamma - \beta^* P \gamma)}{\partial P} &= 1 - P \gamma \frac{(1 - \beta^* P)}{P \gamma} - \beta^* P \\ &= 0 \end{aligned} \tag{B.26}$$

By Lemma 9, we can view the counterparty risk as the probability that one IFI fails when a claim is made (given its investment choice is solved for with a contract size of $\frac{\gamma}{N}$). Since we have shown that this risk does not change in the case when $N > 1$ from $N = 1$, it follows that counterparty risk remains unchanged.

■

Appendix C

Appendix for Chapter 4

Proof of Lemma 11.

Conditional on the bank pursuing full initial investment, we can consider the two states in which the bank may want to avoid investment in the new project separately: {H,NEW} (the project is of the high type, and the new investment is available) and {L,NEW} (the project is of the low type, and the new investment is available). We begin by looking at {H,NEW} and finding the range of B where the bank will wish to pursue the new investment:

$$\begin{aligned} R_0L_0(1 - p_h) + R_AL_1(1 - p_N) - p_Np_hB &\geq R_0L_0(1 - p_h) \\ \Rightarrow B &\leq \frac{R_AL_1(1 - p_N)}{p_Np_h} \end{aligned} \quad (\text{C.1})$$

We now derive the condition for full new investment in the state {L,NEW}:

$$\begin{aligned} R_0L_0(1 - p_l) + R_AL_1(1 - p_N) - p_Np_lB &\geq R_0L_0(1 - p_l) \\ \Rightarrow B &\leq \frac{R_AL_1(1 - p_N)}{p_Np_l} \end{aligned} \quad (\text{C.2})$$

Because $p_l > p_h$ (C.2) \Rightarrow (C.1), and thus (C.2) is the only parametrization needed to ensure

the new investment is pursued when it is available.

If the bank does not pursue full initial investment, then it must always be optimal to pursue the new investment since it has positive expected return, and the possibility of the loss B has already been eliminated. This concludes the proof. ■

Proof of Lemma 12.

For the first part of the proposition, consider investing $L \in (\hat{L} - L_1, L_0)$. This investment is strictly dominated by $L = L_0$ since the project is of positive net present value (NPV), and by investing $L \in (\hat{L} - L_1, L_0)$, you are still subjected to the possibility of the loss, B . Next, consider the case in which $L < \hat{L} - L_1$. This investment level is strictly dominated by $L = \hat{L} - L_1$ since the project is of positive NPV, and by choosing $L = \hat{L} - L_1$, the possibility of the loss of B is still eliminated. $L > L_0$ is not possible since the firm does not request more than L_0 units from the bank.

From the first part of the proposition, we need only focus on two potential levels of investment to address the second part: $L = \hat{L} - L_1$ and $L = L_0$. First, consider the case in which the bank invests $L = \hat{L} - L_1$ and assume that $B \leq \frac{R_A L_1 (1 - p_N)}{p_N p_l}$:

$$E(\pi_{L=\hat{L}-L_1}) = \frac{1}{2}(1 - p_h)R_0(\hat{L} - L_1) + \frac{1}{2}(1 - p_l)R_0(\hat{L} - L_1) + q(1 - p_N)L_1R_1 \quad (\text{C.3})$$

We now consider the case in which the bank invests the requested $L_0 = L$:

$$E(\pi_{L=L_0}) = \frac{1}{2}(1 - p_h)R_0L_0 + \frac{1}{2}(1 - p_l)R_0L_0 + q(1 - p_N)L_1R_1 - \frac{1}{2}q(p_h + p_l)p_N B \quad (\text{C.4})$$

Comparing (C.3) and (C.4) we derive the condition in which full initial investment takes place:

$$L_0 - (\hat{L} - L_1) \geq \frac{Bqp_N(p_h + p_l)}{R_0[(1 - p_h) + (1 - p_l)]} \quad (\text{C.5})$$

Note that if $L_0 - (\hat{L} - L_1) < \frac{Bqp_N(p_h+p_l)}{R_0[(1-p_h)+(1-p_l)]}$, then the bank under-invests in the initial loan and pursues the new loan. Next, consider the case where $B > \frac{R_AL_1(1-p_N)}{p_Np_l}$. In this case, the bank can either have full investment in the initial loan and does not pursue the new loan, or can under-invest in the initial loan, and fully pursue the new loan. The payoff to under-investing is given in (C.3) while the payoff to full investment are given is:

$$E(\pi_{L=L_0}) = \frac{1}{2}(1-p_h)R_0L_0 + \frac{1}{2}(1-p_l)R_0L_0 \quad (C.6)$$

Comparing (C.3) and (C.6), we obtain the following condition for full investment:

$$L_0 - (\hat{L} - L_1) \geq \frac{2q(1-p_N)L_1R_1}{R_0[(1-p_h)+(1-p_l)]}$$

The final case is where $B \in (\frac{R_AL_1(1-p_N)}{p_Np_l}, \frac{R_AL_1(1-p_N)}{p_Np_l}]$. If the banks invests fully at time $t = 0$, then, conditional on the new investment being available, the bank will invest in it only if it is revealed that the initial loan is of high quality. The payoff to under-investment is given in (C.3), while the payoff to full investment is given by:

$$E(\pi_{L=L_0}) = \frac{1}{2}(1-p_h)R_0L_0 + \frac{1}{2}(1-p_l)R_0L_0 + \frac{1}{2}(q(1-p_N)L_1R_1 - Bp_Np_l) \quad (C.7)$$

Comparing (C.3) and (C.7), we obtain the following condition for full investment:

$$L_0 - (\hat{L} - L_1) \geq \frac{q(1-p_N)L_1R_1 - Bp_Np_l}{R_0[(1-p_h)+(1-p_l)]}$$

■

Proof of Proposition 5.

Comparing (C.3) and (5.3), we can see that the existence of either one of the CRT instruments solves the under-investment problem that can occur if (C.5) is not satisfied.

We give the expected, ex-ante profits of a bank that does not pursue the new investment in $\{H,NEW\}$ (denoted by NEW_H), $\{L,NEW\}$ (denoted by NEW_L) or both (denoted by

NONEW).

$$E(\pi_{L_0=L}^{NONEW}) = \frac{1}{2}(1 - p_h)R_0L_0 + \frac{1}{2}(1 - p_l)R_0L_0 \quad (\text{C.8})$$

$$E(\pi_{L_0=L}^{NEW_H}) = \frac{1}{2}(1 - p_h)R_0L_0 + \frac{1}{2}(1 - p_l)R_0L_0 + \frac{1}{2}q(1 - p_N)L_1R_1 - \frac{1}{2}qp_Np_hB \quad (\text{C.9})$$

$$E(\pi_{L_0=L}^{NEW_L}) = \frac{1}{2}(1 - p_h)R_0L_0 + \frac{1}{2}(1 - p_l)R_0L_0 + \frac{1}{2}q(1 - p_N)L_1R_1 - \frac{1}{2}qp_Np_lB \quad (\text{C.10})$$

Comparing (C.8), (C.9) and (C.10) with (5.3), we see that CRT also ensures that the new investment will be fully pursued. Since this is not the case without CRT, and these projects are of positive net present value, we conclude that CRT induces the optimal investment level. This is because under-investment involves leaving a portion of the project unfunded, and forcing it to proceed on a smaller scale. Comparing the expected profit from full investment under CRT and no CRT, we immediately see that the use of CRT is always more profitable for the bank. ■

Proof of Proposition 6.

First, consider the separating equilibrium where the high types chose sales, and the low types chose insurance. The zero profit condition tells us that $P_S = 1 - p_h$ and $P_I = p_l$. The participation constraint in the state $\{L, NONEW\}$: (PC1) can be written as:

$$(1 - p_l)R_0L_0 - R_f p_l L_0 + p_l L_0 - (R_f - 1)L_0 \geq (1 - p_l)R_0L_0 - (R_f - 1)L_0 \\ \Rightarrow R_f \leq 1$$

Since $R_f > 1$, (PC1) will never be satisfied.

Next, consider the separating equilibrium where the high types chose to insure, and the low types chose to sell. The zero profit condition tells us that $P_S = 1 - p_l$ and $P_I = p_h$.

The participation constraint in the state $\{L, NONEW\}$ (PC1) can be written as:

$$\begin{aligned} \alpha(1 - p_l)R_0L_0 - (R_f - 1)L &\geq (1 - p_l)R_0L_0 - (R_f - 1)L_0 \\ &\Rightarrow \alpha \geq 1 \end{aligned}$$

Since $\alpha < 1$, (PC1) will never be satisfied.

Any equilibria where either type is indifferent between loan sales and loan insurance will yield either one of the two previous cases and can be ruled out. ■

Proof of Lemma 13.

There are two cases that we need to consider since the binding constraint will depend on the parameters of the model.

The first condition is derived assuming that (I-PC1) is the binding constraint. We then put the necessary restriction on (I-IC2) to make (I-PC1) bind.

$$\begin{aligned} \frac{R_0(1 - p_l) + p_l}{\alpha R_0 P_S + P_I} &> \frac{p_l}{P_I} \\ \Rightarrow \alpha &< \frac{P_I(1 - p_l)}{P_S p_l} \end{aligned}$$

The second condition assume (I-IC2) binds. We put the necessary restriction on R_f from (I-PC1) to make sure this is the case.

$$\begin{aligned} \frac{p_l}{P_I} &> \frac{R_0(1 - p_l) + p_l}{\alpha R_0 P_S + P_I} \\ \Rightarrow \alpha &> \frac{P_I(1 - p_l)}{P_S p_l} \end{aligned}$$

■

Proof of Lemma 14.

There are three cases that we need to consider since the binding constraint will depend on the parameters of the model.

The first condition is derived assuming that (S-PC1) is the binding constraint. We find the range of R_f such that the R.H.S of (S-IC1b) and (S-IC2b) are less than $\frac{1-p_l}{P_S}$.

$$\begin{aligned}\frac{R_0(1-p_l) + p_l - R_f P_I}{R_0 P_S} &< \frac{1-p_l}{P_S} \\ \Rightarrow R_f &> \frac{p_l}{P_I}\end{aligned}$$

$$\begin{aligned}\frac{R_0(1-p_h) + p_h - R_f P_I}{R_f R_0 P_S} &< \frac{1-p_l}{P_S} \\ \Rightarrow R_f &> \frac{R_0(1-p_h) + p_h}{(1-p_l)R_0 + P_I}\end{aligned}$$

The second condition assumes that (S-IC2b) binds. The condition on R_f allows the R.H.S of (S-IC2b) to be less than $\frac{1-p_l}{P_S}$. The second condition results because for (S-IC2b) to bind, the R.H.S of (S-IC2b) must be greater than the R.H.S of (S-IC1b).

$$\begin{aligned}\frac{R_0(1-p_h) + p_h - R_f P_I}{R_f R_0 P_S} &\geq \frac{1-p_l}{P_S} \\ \Rightarrow R_f &\leq \frac{R_0(1-p_h) + p_h}{(1-p_l)R_0 + P_I}\end{aligned}$$

To obtain a lower bound on R_f , we need to make sure that the value of R_f is not so low as to require $\alpha > 1$. To this we compute:

$$\begin{aligned}\frac{R_0(1-p_h) + p_h - R_f P_I}{R_f R_0 P_S} &\leq 1 \\ \Rightarrow R_f &\geq \frac{R_0(1-p_h) + p_h}{P_I + R_0 P_S}.\end{aligned}$$

The third condition assumes that (S-IC1b) binds. The condition on R_f allows the R.H.S of (S-IC1b) to be less than $\frac{1-p_l}{P_S}$. The second condition results because for (S-IC1b) to bind,

the R.H.S of (S-IC1b) must be greater than the R.H.S of (S-IC2b).

$$\begin{aligned}\frac{R_0(1-p_l) + p_l - R_f P_I}{R_0 P_S} &\geq \frac{1-p_l}{P_S} \\ \Rightarrow R_f &\leq \frac{p_l}{P_I}\end{aligned}$$

To obtain a lower bound on R_f , we need to make sure that the value of R_f is not so low as to require $\alpha > 1$. To this we compute:

$$\begin{aligned}\frac{R_0(1-p_l) + p_l - R_f P_I}{R_0 P_S} &\leq 1 \\ \Rightarrow R_f &\leq \frac{R_0(1-p_l) + p_l - R_0 P_S}{P_I}.\end{aligned}$$

■

Proof of Lemma 15.

Plugging in $P_S = 1 - P_I$ into (I-IC1a), (S-IC1b) and (S-IC2b). If (S-IC1b) is the binding constraint for the sales equilibrium, then the set $S(\alpha | (I-IC1a) \cap (S-IC1b))$ is empty. This implies that sales and insurance are mutually exclusive. Furthermore, we can see that in this case, either of the two cases must occur. If (S-IC2b) is the binding constraint for the sales equilibrium, so that $\frac{R_0(1-p_l) + p_l - R_f P_I}{R_0 P_S} < \frac{R_0(1-p_h) + p_h - R_f P_I}{R_f R_0 P_S}$, then the set $S\left(\alpha \left| \frac{R_0(1-p_l) - R_f P_I + p_l}{R_0 P_S} < \alpha \leq \frac{R_0(1-p_h) + p_h - R_f P_I}{R_f R_0 P_S} \right.\right)$ is non-empty so that neither the insurance nor sales equilibrium exists. Furthermore, it is easy to see that the insurance and sales equilibrium cannot co-exist in this case. ■

C.0.1 Endogenous Moral Hazard

Consider a bank with access to an unverifiable monitoring technology at time $t = 1$ that has a cost, e . Let us assume that without this monitoring, all low quality loans will fail with

probability 1.¹ Consider the case in which there is no CRT available. To ensure that the bank wishes to monitor, the following condition must hold:

$$e \leq R_0 L_0 (1 - p_l)$$

Next, consider the case of loan insurance. There is a trade-off present with this new monitoring technology. The bank can choose not to monitor, but give up the potential return from the low quality loans.² We can put the following assumption on e to ensure that they wish to continue monitoring in the low state when they insure their loan.

$$\begin{aligned} R_0 L_0 (1 - p_l) + p_l L_0 - e &\geq L_0 \\ \Rightarrow e &\leq (R_0 - 1)(1 - p_l) \end{aligned}$$

Finally, if the bank wishes to use loan sales, it can never credibly commit to monitoring the bad loans for any $e > 0$. Therefore, the price of the loan sale will simply be $R_0 L_0 [1 - Prob(Defaul\!t|sales)] = R_0 L_0 [1 - \frac{1+qp_h}{q+1}]$. We can see immediately that this new loan sales price is smaller than the original price without moral hazard. We therefore use the exogenous variable α to represent the amount that the loan sale price is reduced with moral hazard present.³ Intuitively, if $\alpha < 1$, all else equal, the bank may not wish to use loan sales and the market may not exist. For example, consider the participation constrain in the state $\{L, NONEW\}$ of the sales equilibrium:

$$\alpha P_S R_0 L_0 \geq (1 - p_l) R_0 L_0$$

It follows that if $\alpha < \frac{1-p_l}{P_S}$, the participation constraint can never be satisfied, and therefore they will not sell their loan in this state.

¹The qualitative results follow through if we make the assumption that bank monitoring can transform low quality loans into high quality loans.

²If the bank chooses to not monitor the bad loans we know that $Prob(Defaul\!t) = \frac{1+qp_h}{q+1}$.

³Gorton and Pennachhi (1995) and DeMarzo and Duffie (1999) show that if the bank retains a portion of the loan (usually first-loss), the moral hazard can be lowered. A modern example is that of a Collateralized Loan Obligation (CLO). We will not consider tranching in this paper.

C.0.2 Existence of CRT With Moral Hazard and Costly Capital

We now consider the resulting equilibrium when moral hazard, relationship banking and costly capital are added to the analysis. We begin the analysis by redefining the parameter space of interest. We turn to the state {H,NONEW} first.

$$\begin{aligned}\pi_{NI} &= R_0(1 - p_h)L_0 - (R_f - 1)L_0 \\ \pi_I &= R_0(1 - p_h)L_0 + p_hL_0 - (R_f - 1)L_0 - R_fL_0P_I\end{aligned}$$

It follows that for $\pi_{NI} \geq \pi_I$, the condition $P_I \geq \frac{p_h}{R_f}$ must be added to the optimal contracting problem as (I-Bound). We now analyze {H,NEW} to see under what condition they will use loan insurance.

$$\begin{aligned}\pi_{NI} &= R_0(1 - p_h)L_0 - (R_f - 1)L_0 + (1 - p_N)L_1R_1 - p_Np_hB - (R_f - 1)L_1 \\ \pi_I &= R_0(1 - p_h)L_0 + p_hL_0 + (1 - p_N)L_1R_1 - (R_f - 1)L_0 - R_fP_IL_0 - (R_f - 1)L_1\end{aligned}$$

For $\pi_I \geq \pi_{NI}$ the following must hold:

$$B \geq \frac{L_0(R_fP_I - p_h)}{p_Np_h} \quad (\text{C.11})$$

Therefore, (C.11) gives us the parameter bound on B. To find a similar bound for loan sales, we begin by looking at {H,NONEW}.

$$\begin{aligned}\pi_{NS} &= R_0(1 - p_h)L_0 - (R_f - 1)L_0 \\ \pi_S &= \alpha R_0(P_S)L_0 - (R_f - 1)L_0\end{aligned}$$

For $\pi_{NS} \geq \pi_S$, we will add $\alpha P_S \leq 1 - p_h$ to our optimal contracting problem as (S-Bound).

We now analyze {H,NEW} to see under what condition they will use loan sales:

$$\begin{aligned}\pi_{NS} &= R_0L_0(1 - p_h) - (R_f - 1)L_0 + (1 - p_N)L_1R_1 - (R_f - 1)L_1 - p_hp_NB \\ \pi_S &= \alpha R_0L_0(P_S) + (1 - p_N)L_1R_1 - (R_f - 1)L_0 - (R_f - 1)[L_1 - \alpha R_0P_SL_0]\end{aligned}$$

From above, for $\pi_S \geq \pi_{NS}$, the following must hold:

$$B \geq \frac{R_0 L_0 ((1 - p_h) - \alpha P_S R_f)}{p_N p_h} \quad (\text{C.12})$$

The parametrization that characterizes loan sales is given by (C.12).