Paschen's Law in Different Gases

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Abstract

An experiment was performed to measure the Paschen curves of multiple gasses. Paschen curves are generated using Paschen's equation, $V_b = \frac{B\cdot p\cdot d}{ln(A\cdot p\cdot d)-ln\left(ln\left(1+\frac{1}{\gamma_{se}}\right)\right)}$. A plasma discharge tube was used to induce arcs of plasma in low pressure environments filled with each of the specific gasses. Using this apparatus, we measured the breakdown voltage V_b of each gas, as well as the pressure p of the environment, and the distance d between the two electrodes creating the plasma. The data collected was fit to Paschen's equation in order to find the two unknown parameters A and B in the Paschen equation. Our fit method was tested against other experimental data to verify its effectiveness. The values gathered for A and B lead us to results that showed an incorrect value for the composition of atmospheric air. The results of our experiment indicated that there were too many problems with our apparatus to form any strong conclusions regarding the legitimacy of Paschen's law, and our apparatus would likely need adjustments to make it usable, namely fixing leaks to maintain low pressures, and developing a way to keep the bulk of the tube filled with the specific gas rather than atmospheric air.

1. Introduction and Theory

In this experiment, our goal was to build a DC glow discharge plasma source, and use this plasma source to breakdown gas into plasma. The breakdown of the gas occured within an enclosed acrylic tube, and was prompted by electric activity between an anode and cathode at opposing ends of the tube. We observed how the breakdown voltage and distance between the eletrodes correlate by setting a constant pressure, then finding the required voltage to create plasma for different distances between the anode and cathode. For this reason the anode of the circuit was movable within the tube. All of this data allowed us to generate a Paschen curve for a number of different gasses, namely, Nitrogen (N_2), Argon (Ar), and Carbon Dioxide (CO_2). These gasses were chosen due to each one sharing traits with another. Argon and Carbon Dioxide have a similar kinetic diameter,[1] and Nitrogen gas and Argon have similar first ionization energies.[2] Performing the process using each of these gasses allowed us to see how each gas affects the correlation between breakdown voltage, distance between the electrodes and gas pressure. These measurements determined the Paschen curve for each gas and allowed us to analyze the similarities and differences between each gas and what properties affect the Paschen curve.

In order to measure the effect different gasses have on the Paschen curve, we required Paschen's Law (Eq. 1). Paschen's Law returns the breakdown voltage V_b of a gas, given pressure p, the length of the electron gap d, and the secondary electron emission coefficient γ_{se}, as well as two values A and B representing the saturation ionization and the excitation and ionization energies of the gasses respectively.[3] The goal of the experimental procedure was to find A and B using a Paschen curve generated using measured breakdown voltages, and the values for p, d, and γ_{se} when those voltages are measured.

$$V_b = \frac{B\cdot p\cdot d}{ln(A\cdot p\cdot d) - ln\left(ln\left(1+\frac{1}{\gamma_{se}}\right)\right)}. \tag{1}$$

Paschen's Law is an equation that was fit to Paschen curves for various gasses (Fig. 1). Paschen's curve plots V_b as a function of the product of the pressure and gap length $p \cdot d$. The mean free path of an electron in a gas is the average distance between its collision with molecules.[4] This is inversely proportional to the pressure of the gas and is critical to this experiment as it defines how far the electron must travel in order to ionize another molecule. If an electron in the outer shell of a gas molecule is accelerated by an electric field and acquires enough energy to sufficiently surpass the molecule's first ionization energy, it will dislodge from the molecule in question. This electron will acquire further energy through further acceleration, and in turn ionize a different molecule.[3, 5]

This new ionized molecule will release an electron, leading to another collision, causing a chain reaction which leads to an avalanche breakdown, and creating an arc.[3] This reaction results in a number of molecules having a missing outer electron, resulting in the creation of cold plasma. Paschen's Law requires that the creation of free electrons is only achieved by impact ionization.[6] Thus a light source creating secondary electrons by the photoelectric effect has to be considered in experiments and its effects mitigated. For this reason, performing the experiment in a sufficiently dark space (i.e. a dark box or concealed beneath a curtain) may result in more accurate results.

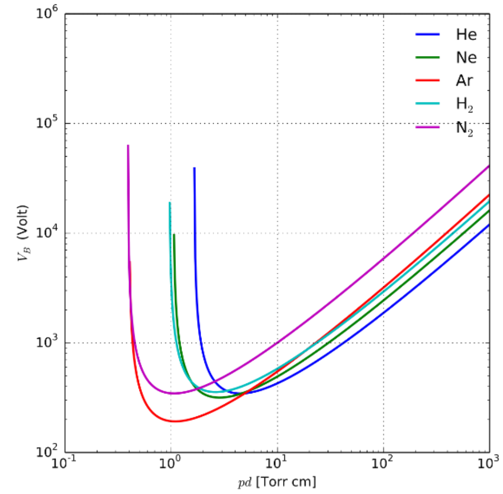


Figure 1: A plot of Paschen Curves for several gasses. Image reproduced from Ref. 6.

Since we have limited experience with plasma physics, we felt this project would be a great opportunity for us to learn about something new. Plasma physics is such an interesting topic, that we were all motivated to dive deeper into. Plasma has a distinct appearance that most of us have been fascinated by since we saw plasma globes when we were young. Plasma physics also has many common uses, including fluorescent lights and arc welding. Choosing to research Paschen’s Law specifically, gave us a way to quantify what we were seeing, and tinker with the appearance and behaviour of the plasma, while still testing our analytical skills, and coming to conclusions. Paschen’s law is important in finding the voltage efficiency of fluorescent lamps and neon signs. It is also used in developing electrical insulation, as you would ideally use a gas with a very high breakdown voltage to prevent arcing in high voltage circuits, preventing fires and potential electrocutions.

2. Apparatus and Data

The device we used for our experiment was a DC glow source. In this case a discharge tube. This is a clear tube with a cathode at one end and an anode at the other. In our device, the anode was movable to allow for the changing in gap size between the electrodes. The basic instructions for such a device came from an article in the Journal of Electrostatics[7], however, they have been heavily adapted for our situation to make the device usable and safer (Fig.2).

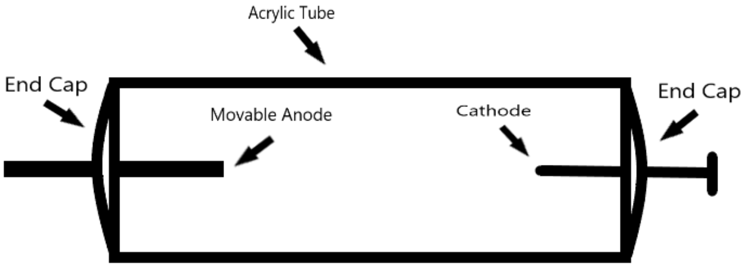


Figure 2: A diagram of our discharge tube.

The apparatus was constructed using a pre-built discharge tube, consisting of a clear tube with a cathode at one end and an anode at the other, which were kept movable in order to adjust gap distances throughout the experiment (Fig. 3). When the electrodes are hooked up to a sufficient voltage source, an arc of plasma forms between the two electrodes. Gas enters the system through the endcap located at the anode (Fig. 4). A Tee-connector was used to connect both the canister of the gas we were currently testing, and the vacuum gauge, which measured the pressure throughout the system. On the other endcap, located at the cathode, a vacuum pump is connected and pulls a vacuum throughout the system. A picture of the gas canister is shown in Fig. 5.

Figure 3: The discharge tube, electrodes, and connections used in the apparatus.

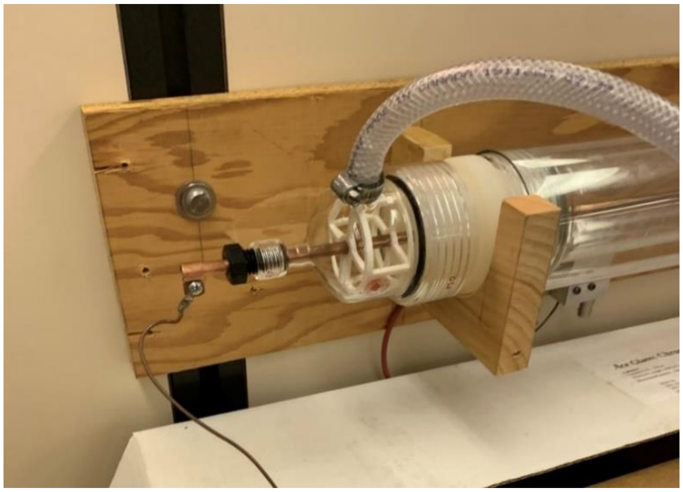


Figure 4: The grounded anode that would be adjusted to change the gap distance.

Figure 5: The gas canister along with gauges and connectors used to control the flow of gas into the tube.

In this experiment, we measured a few different values, each relating to the Paschen's Law equation. First was the pressure of the gas. This was measured using the vacuum gauge attached to the gas system. The pressure was stable and far below standard pressure, as the plasma arc will appear more easily at lower pressures. The pressure measurements were used to maintain a constant pressure throughout each trial, so that the measured data was easily compared. The gap distance is the second quantity that was measured. It served as the independent measurement in our system. The gap distance is defined as the distance between the end of the anode and the end of the cathode. It was measured using a ruler held up to the side of the wall of the tube. Multiple gap lengths were used for each gas, forming the x-axis of our plots. The final measurement taken was the minimum voltage at which a constant arc of plasma appeared. This acted as our dependent variable in this experiment. The voltage was adjusted using a dial on the high voltage power supply, to find the point at which any less voltage resulted in a significant voltage drop congruent with an arc. This minimum voltage was measured and recorded using the multimeter. This was done for several gap distances and changed in relation to said gap distance. This formed the y-axis of our plots. We also ensured that as many other physical properties of the apparatus remained as consistent as possible.

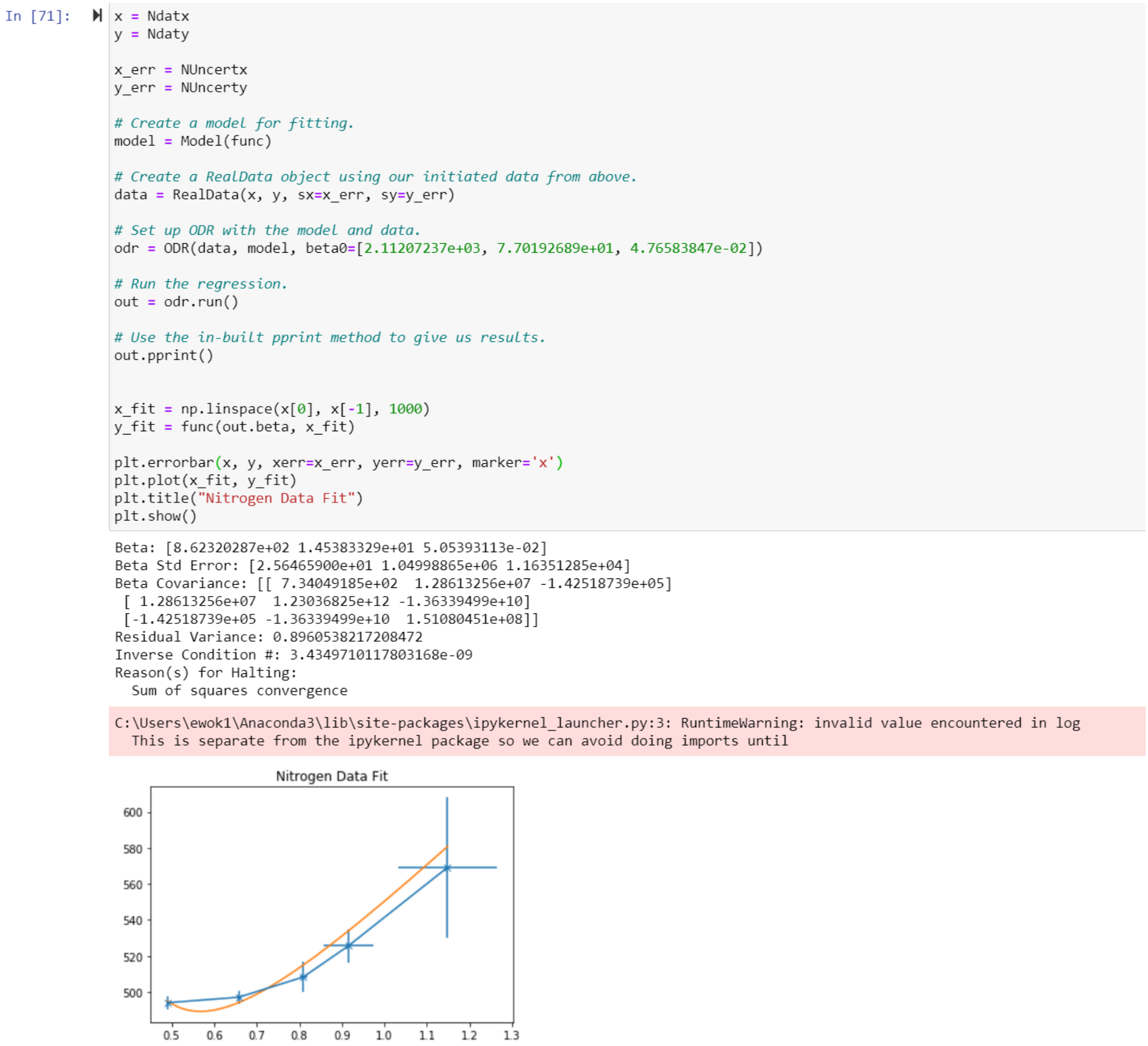
The data was collected over the course of several weeks and consisted of multiple trials at varying distances. We ideally would have liked to keep the pressure constant throughout the system, but due to our system not being completely airtight, gas leakage resulted in inconsistent pressure readings between measurements.

3. Analysis

The analysis performed was relatively simple but given that we had never worked with plasma before and we had some COVID-19 related obstacles, we felt that doing anything beyond testing the fundamental principles of Paschen's law would require more time and experience beyond our own. For these reasons, we decided to compare our gathered data to the expected values and curves from literature we could find. If the behaviour within the tube was congruent with expectation, we could gauge how effective our apparatus would be at performing other experiments, and where adjustments could be made to make the apparatus more functional for research purposes.

We decided that the easiest way to do this would be to gather multiple data sets for each gas, with multiple samples taken at varying gap distances, then taking the average values and their respective uncertainties, and performing a fit using python. The data would be fit to the Paschen's law equation, and the parameters A and B in the equation would be found through this process. We could then check the curves and all the parameters against the literature.

The gathered values for d, p and V_b were fed into a python code that used an orthogonal distance regression to output a graph containing the data and the fit along with error bars (Fig. 6). The error on the raw data was found in excel using standard deviation. The code also returned values for parameters A and B, as well as a residual, which was taken as the goodness of fit. The fit was performed directly to the Paschen's law equation, rather than any other experimental data, as we felt that the experimental data gathered by others was less effective as a baseline and more effective as a tool for later comparison. The orthogonal distance regression fit was chosen for its ubiquity, as it applies to a multitude of applications. This meant that there was plenty of information regarding performing the fit in python, and we had high hopes that it would be applicable to our analysis. Orthogonal distance regression extends least squares data fitting to scenarios with independent variables such as our A and B values.



```
In [71]:  x = Ndatx
          y = Ndaty

          x_err = NUncertx
          y_err = NUncerty

          # Create a model for fitting.
          model = Model(func)

          # Create a RealData object using our initiated data from above.
          data = RealData(x, y, sx=x_err, sy=y_err)

          # Set up ODR with the model and data.
          odr = ODR(data, model, beta0=[2.11207237e+03, 7.70192689e+01, 4.76583847e-02])

          # Run the regression.
          out = odr.run()

          # Use the in-built pprint method to give us results.
          out.pprint()


          x_fit = np.linspace(x[0], x[-1], 1000)
          y_fit = func(out.beta, x_fit)

          plt.errorbar(x, y, xerr=x_err, yerr=y_err, marker='x')
          plt.plot(x_fit, y_fit)
          plt.title("Nitrogen Data Fit")
          plt.show()
```

```
Beta: [8.62320287e+02 1.45383329e+01 5.05393113e-02]
Beta Std Error: [2.56465900e+01 1.04998865e+06 1.16351285e+04]
Beta Covariance: [[ 7.34049185e+02  1.28613256e+07 -1.42518739e+05]
 [ 1.28613256e+07  1.23036825e+12 -1.36339499e+10]
 [-1.42518739e+05 -1.36339499e+10  1.51080451e+08]]
Residual Variance: 0.8960538217208472
Inverse Condition #: 3.4349710117803168e-09
Reason(s) for Halting:
  Sum of squares convergence
```

```
C:\Users\ewok1\Anaconda3\lib\site-packages\ipykernel_launcher.py:3: RuntimeWarning: invalid value encountered in log
  This is separate from the ipykernel package so we can avoid doing imports until
```

Figure 6: A small piece of the code used to perform the orthogonal distance regression fit, along with its output

The graph in Fig. 6 shows the most accurate fit we were able to achieve, which was for nitrogen gas. This was the only fit that returned any result that was close to correct, as most of the other graphs looked sporadic, with CO_2 shown in Fig. 9 looking marginally worse, and Argon shown in Fig. 7 looking completely incongruent with the projected graph.

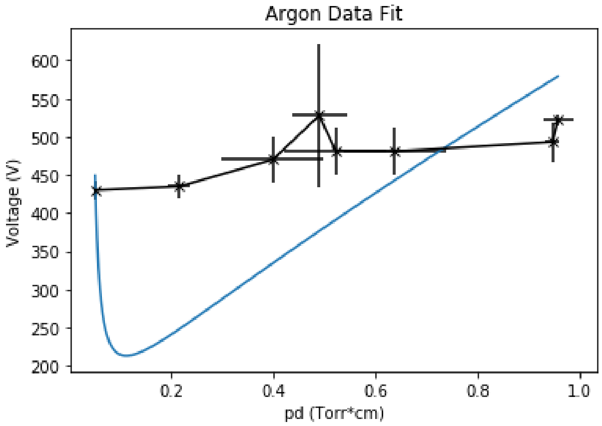


Figure 7: The Graph for Argon showing our data in black and the fit in blue.

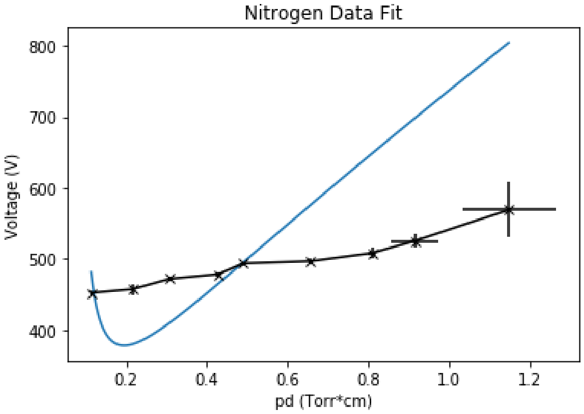


Figure 8:The Graph for Nitrogen showing our data in black and the fit in blue.

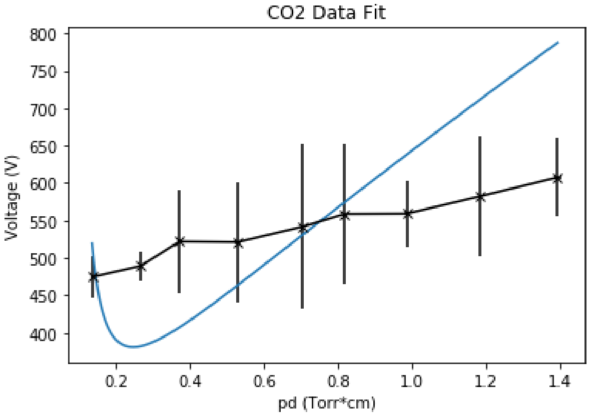


Figure 9:The Graph for CO_2 showing our data in black and the fit in blue.

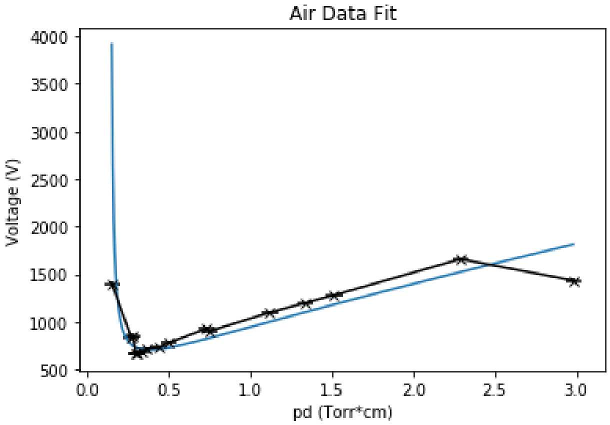


Figure 10: A graph showing data collected from an experiment at Princeton, with the data collected there in black and the fit in blue.

After gathering, graphing, and fitting the data, we calculated the percentage of each of these gasses that would be present in atmospheric air if the data sets were accurate. Our findings for nitrogen were almost correct, but as you'll see in Table 1, our other data didn't behave quite so well.

Table 1. Calculated and expected percentages of each gas in atmospheric air.

Gas	Percent of Air	Expected Percent (Dry Air)[8]
N_2	0.773	0.7808
CO_2	0.0304	0.0004
O_2	1.78e-07	0.2095
Ar	0.196	0.0093

The primary sources of error had to do with the structure of our apparatus. We spent a long time early in the building process determining how to make the system as airtight as possible, yet we routinely encountered unstable pressures, or inability to maintain pressures low enough for our experiment. This very likely has to do with the connections between each section of vacuum hose, and the parts of the apparatus, with one notable problem we found being our Tee-connector always seeming to have a small leak. Another significant source of error is that the pressure gauge we were using wasn't located at the location the plasma was being created, so we only knew the pressure at a part of the system that wasn't as pertinent to the experiment. These two primary error factors likely lead to our tests failing at lower pressures and gap distances. Our other sources of uncertainty came in the form of our measurement process, which involved slowly raising the voltage value, and recording a video of the multimeter in order to detect a voltage drop, indicative of an arc of plasma. The rate at which we increased the voltage changed each time, as we were manually turning the dial with our hands. This likely caused not only our reading of the multimeter to be more difficult, but the plasma to begin arcing at slightly different voltages, as sudden voltage changes could have a higher chance of arcing than a gradual voltage change. Our ruler, multimeter, and vacuum gauge also have a set uncertainty based on the accuracy of their measurement, leading to even greater uncertainty.

4. Results and Discussion

Our results overall were quite poor, giving us little insight into the inner workings of Paschen's law. Fig. 7, 8, and 9 show just how far off our data was from the actual fit. Table 1 also shows that our data for each gas is in violation of the accepted values that we should be finding. These results have led us to the conclusion that the apparatus is not functioning as intended. The most likely cause of this is the consistently shifting pressure as a result of leakage in the system, as well as the placement of our pressure gauge not being at the part of the system most pertinent to our data collection. However, this experiment wasn't devoid of purpose. Through this experiment we were able to design, create, and operate a functioning plasma discharge tube, and were able to experiment with how the different variables of each trial affected the breakdown voltage of the gas.

5. Conclusions

This experiment while ultimately lacking in scientific merit wasn't without its exciting moments. Being able to generate plasma using an apparatus designed by our group felt like a phenomenal accomplishment and being able to measure values using new equipment was consistently interesting. Expanding our knowledge of experimental design and lab procedure will be greatly beneficial to us whether our experiment was a success or not. In future we would like to work more with plasma and develop this experiment into one that operates more smoothly and can give us accurate data points. This will likely require rethinking the couplings of all the sections of the apparatus, as well as relocating the vacuum gauge to the center of the tube, which brings along a whole new set of challenges.

6. References

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7. Appendix

Below are tables of all the data gathered for each gas in the laboratory, with the average values of a gas shown directly below the raw data.

Table 2. Data gathered for CO_2

Carbon Dioxide	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
Gap Distance (cm	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure	Voltage (volts)	Pressure	Voltage (volts)
1.8	0.78	699.97	0.78	596.398	0.77	578.163	0.77	580.007	0.77	583.625
1.6	0.74	718.08	0.74	558.951	0.74	516.949	0.74	538.839	0.74	578.898
1.4	0.71	592.839	0.71	595.592	0.7	488.888	0.7	568.226	0.7	549.466
1.2	0.69	724.88	0.68	517.265	0.68	523.562	0.68	505.76	0.68	521.684
1	0.71	732.899	0.71	469.206	0.7	499.365	0.7	533.9	0.7	472.639
0.8	0.68	662.698	0.66	473.785	0.66	473.711	0.66	502.629	0.65	492.925
0.6	0.64	641.872	0.62	496.653	0.62	506.776	0.61	495.008	0.61	469.069
0.4	0.68	516.28	0.67	481.266	0.67	470.495	0.66	501.864	0.66	475.814
0.2	0.68	480.977	0.68	430.121	0.68	470.495	0.68	496.777	0.68	494.304

Table 3. Average values and respective uncertainties for CO_2 data

Average Values			
Pressure (torr)	Voltage (volts)	Pressure Uncert	Voltage Uncert
1.3932	607.6326	0.009859006	52.10663977
1.184	582.3434	0.001	79.29779333
0.9856	559.0022	0.007668116	43.51942422
0.8184	558.6302	0.005366563	93.19352736
0.704	541.6018	0.005477226	110.0396558
0.5296	521.1496	0.008763561	80.10838957
0.372	521.8756	0.007348469	68.50860971
0.2672	489.1438	0.00334664	19.27237583
0.136	474.5348	0.001	26.99974206

Table 4. Data gathered for Nitrogen

Nitrogen	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
Gap Distance (cm	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)
0.2	0.57	450.232	0.57	450.327	0.57	453.05	0.56	454.285	0.56	455.11
0.4	0.54	466.71	0.54	465.284	0.54	451.079	0.54	452.371	0.54	453.116
0.6	0.52	470.27	0.52	476.676	0.51	472.69	0.51	470.086	0.51	471.186
0.8	0.54	477.431	0.54	476.055	0.53	477.21	0.53	479.852	0.53	481.582
1	0.49	491.193	0.49	500.106	0.49	490.794	0.49	494.698	0.49	493.285
1.2	0.55	493.598	0.55	498.028	0.55	502.812	0.55	495.501	0.54	495.384
1.4	0.6	526.924	0.62	533.859	0.67	510.815	0.69	532.677	0.69	523.896
1.6	0.51	495	0.5	512.769	0.5	513.986	0.51	514.256	0.51	505.584
1.8	0.55	539.189	0.59	583.586	0.67	631.008	0.69	541.596	0.69	550.452

Table 5. Average values and respective uncertainties for Nitrogen data

Average Values			
Pressure (torr)	Voltage (volts)	Pressure Uncert	Voltage Uncert
0.1132	452.6008	0.001095445	2.242530646
0.216	457.712	0.001	7.61487646
0.3084	472.1816	0.003286335	2.715468431
0.4272	478.426	0.00438178	2.240766275
0.49	494.0152	0.001	3.756267922
0.6576	497.0646	0.005366563	3.579361787
0.9156	525.6342	0.058230576	9.240427193
0.8096	508.319	0.008763561	8.246837939
1.1484	569.1662	0.115537007	38.86292767

Table 6. Data gathered for Argon

Argon	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
Gap Distance (cm	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)	Pressure (torr)	Voltage (volts)
0.1	0.53	452.633	0.51	419.263	0.51	425.083	0.51	427.236	0.5	426.205
0.4	0.6	452	0.52	426.402	0.5	426.402				
0.7	0.57	470								
0.9	0.58	482								
1	0.57	490	0.47	667.442	0.46	473.778	0.46	480.195		
1.1	0.58	481								
1.4	0.59	610								
1.6	0.6	484.446	0.6	539.306	0.59	477.118	0.59	483.302	0.58	482.066
1.8	0.56	510.2	0.53	527.604	0.53	527.037	0.52	522.155	0.52	520.938

Table 7. Average values and respective uncertainties for Argon data

Average Values			
Pressure (torr)	Voltage (volts)	Pressure Uncert	Voltage Uncert
0.0512	430.084	0.001095445	12.97805059
0.216	434.9346667	0.02116601	14.77901219
0.399	470	0.1	10
0.522	482	0.1	10
0.49	527.85375	0.053541261	93.29760468
0.638	481	0.1	10
0.826	610	0.1	10
0.9472	493.2476	0.01338656	25.89860762
0.9576	521.5868	0.029577018	7.005611729