A Literature Review on P-wave Velocities in Rock Under Compression[1]

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Abstract

P-waves are acoustic waves propagated through geological mediums. P-wave velocities are increasingly being used in natural resource industries. The purpose of this paper is to present a literature background on the behavior of P-wave velocities in typical geological settings. P-wave velocities can be represented mathematically using the elastic moduli of the geological material, the density of the material, and the acoustic impedance of the material. The impact of environmental factors that can change the acoustic wave velocities are also addressed, including external pressures, humidity and temperature.

1. Introduction

Unconfined compressive strength (UCS) testing has been used in mining and civil industries to test the strength of materials and base designs on such results. Such designs can include structural designs relating to building integrity or production shaft sizing and support system requirements for mining projects. The results from UCS testing is used for a variety of design purposes mainly pertaining to material strength. This type of material strength testing has been proven accurate for homogenous materials such as pure or compound metals. However, UCS results from testing heterogeneous materials such as rock and concrete have been known to have very high variation. The variation in results from UCS testing on heterogeneous material is considerable enough to make the results unreliable for design purposes, such that significant factor of safety's is used to compensate for such variability. This literature review attempts to explore the behavior of P-wave velocities in rock under compression for the purposes of utilizing P-wave velocities for more accurate compression strength testing.

This literature review covers some of the mathematical representations of P-wave velocities, the impact external pressure has on P-wave velocities and the impact humidity and temperature have on P-wave velocities propagated through rock.

2. P-waves and their velocities in geological mediums

Energy is constantly being released within geological domains in the form of seismic waves. Energy release can come in the form of tectonic shifts, microfractures, friction from molten material, and even energy transfer from non-geological sources. There are multiple types of seismic waves. There are P-waves, S-waves and surface waves. P-waves are compressional waves, also known as acoustic waves, in geological mediums. They are the fastest type of seismic wave. P-waves travel longitudinally from the source of

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energy release, causing particles to travel back and forth from the source. S-waves are transverse waves that cause particle motion at right angles from the direction of propagation. The other type of seismic waves are surface waves, which are unrelated to the purposes of this research (Robinson & Clark, 2017).

P-waves travel in geological mediums often between 2-12 km/s, while S-waves travel at a fraction of the speed of P-waves (Robinson & Clark, 2017). Seismic wave velocity is often a measurement of interest in mining and oil industries and research, particularly P-wave velocities. Seismic wave velocities are often measured by taking the distance of wave propagation to reach two sensors "d" and the time difference between the initial seismic wave readings of the two sensors "t" (Robinson & Clark, 2017). The seismic wave velocity is easily determined using Equation 1.

Equation 1: Seismic velocity distance over time equation (Elocity, 1997).

$$Vseismic = \frac{d}{t}$$

Often geological mediums are considered homogenous and isotropic by nature, particularly in seismic exploration. As a result, the physical characteristics of geological domains are identified by their acoustic impedance "*Z*", which is described by the density of the geological domain "*ρ*" and the P-wave velocity "*Vp*", as shown by Equation 2 (Robinson & Clark, 2017).

Equation 2: Acoustic impedance of homogenous and isotropic geological domains (Robinson & Clark, 2017)

$$Z = \rho(V_p)$$

It is known however that rock, in the clear majority of cases, is anisotropic and heterogenous, composed of faults, diverse mineral composition, water saturation, and microfractures.

Birches law dictates that the P-wave velocity in geological mediums is linearly dependent on the density of the material the wave is propagating through as detailed by Figure 3.

Equation 3: Birches law (Campbell & Heinz, 1992)

$$V_p = a(\overline{m}) + b\rho$$

Where "$a(\overline{m})$" is a mathematical function dependent on the mean atomic weight "$\overline{m}$" of the material under observation, "*b*" is a constant, "*ρ*" is the density of the material under observation, and "V_p" is the P-wave velocity of the material (Campbell & Heinz, 1992). Campbell et al. (1992) conducted experiments on KCl and NaCl samples to obtain P-wave velocity data as samples were compressed. The objective of the test was to alter the density of each sample through compression and record the velocity of P-waves propagated through the samples. Figure 1 shows the P-wave velocity VS sample density data recorded for this study.

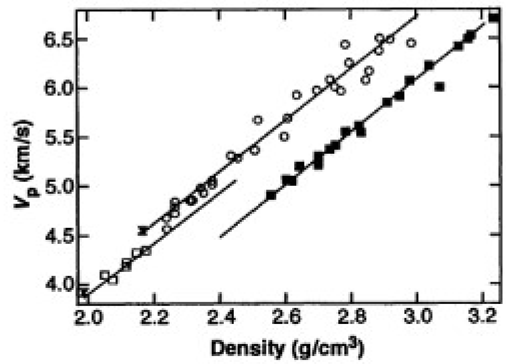

Figure 1: Campbell's et al. P-wave velocity VS sample density results. White circles represent NaCl samples, white squares represent KCl samples and black squares represent KCl samples with CsCl structure developed after the structural change of the KCl samples (Campbell & Heinz, 1992).

As can be seen from Campbell et al. (1992) study, the P-wave velocity is linearly dependent on the density of a sample. It is interesting to note that in this experiment, a structural change of the KCl samples occurred creating CsCl structure, which formed a very different linear relation for the P-wave velocity with respect to the density. It is strange however that a convergence of the data was not presented to show the non-sudden structural change that occurred. What is also interesting to note from this study is the P-wave velocity VS sample compression data shown in Figure 6. (Campbell & Heinz, 1992)

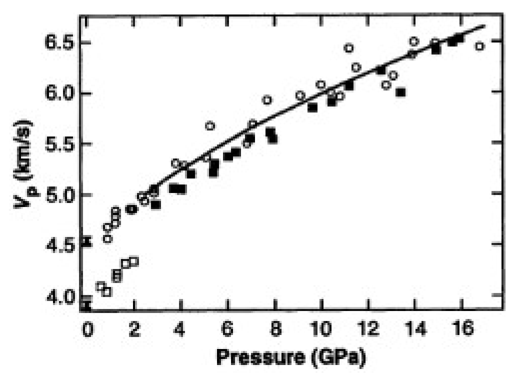

Figure 2: Campbell's et al. P-wave velocity VS external pressure results. White circles represent NaCl samples, white squares represent KCl samples and black squares represent KCl samples with CsCl structure developed after the structural change of the KCl samples (Campbell & Heinz, 1992).

As can be seen from the above figure, the P-wave velocity almost appears to follow a linear trend with respect with the external pressure applied to samples. There does however, appear to be a recognizable curved relation to the data. This non-linear feature can likely be explained from the closing of porous space in the samples.

P-wave velocities are also dependant on the elastic properties of the material the wave propagates through. In the case of isotropic elastic material, Hooke’s law can be used to describe the P-wave velocity for a material based on an elastic property “M”, the P-wave modulus, and ρ, the density of the material as shown in Equation 4. (Mavko, Mukerji, & Dvorkin, 2009)

Equation 4: Describing P-wave velocity in accordance to Hooke's law for a isotropic and linear elastic material (Mavko et al., 2009)

$$V_p = \sqrt{\frac{M}{\rho}}$$

The P-wave modulus, "M", can be further broken down into the expression seen in Equation 5.

Equation 5: P-wave modulus described by the bulk modulus and the shear modulus (Mavko et al., 2009)

$$M = K + \frac{4\mu}{3}$$

Where K is the bulk modulus (modulus of incompressibility) and μ is the shear modulus of the material. The equation describing P-wave velocity in accordance to Hooke's law for an isotropic and linearly elastic material can be described further by substituting for M, as shown in

Equation 6: P-wave velocity equation for isotropic and linearly elastic material according to Hooke's law after a P-wave modulus substitution for the bulk modulus and shear modulus.

$$V_p = \sqrt{\frac{K + \frac{4\mu}{3}}{\rho}}$$

It should be noted that these equations from Hooke's law hold true for material that is both isotropic and linearly elastic material. Most rock is considered heterogeneous, anisotropic and exhibits non-linear behavior with regards to the stress strain relation. For example, when rock is compressed uniaxially, the Young's modulus reading for the material will ramp up as the porous spaces in the rock are sufficiently closed before the rock begins behaving elastically, exhibiting a constant Young's modulus until the rock yields and plastically deforms.

3. Impact of external pressure on P-wave velocities

As discussed, the velocities of seismic waves are dependent on the elastic properties of the geological medium they are propagated through and the density of the material. The confining pressure applied on a rock sample can alter the elastic properties of the material, such as the bulk modulus, and the density of the material by squeezing the porous space in the rock and reducing the volume of the rock. The fluid saturation of rock increases the average density of the material by filling the porous space with fluid and increases or decreases pore pressure depending on the pore aperture size and the degree of saturation, all of which impacts seismic velocities in geological mediums.

G. Mavko (2017) presented that seismic waves are dependent on the effective pressure applied on materials. The effective pressure applied on a material is determined through the subtraction of the pore pressure from the confining pressure exerted on the sample, as shown in Equation 7.

Equation 7: Equation for the effective pressure applied on rock (Mavko, 2017)

$$P_{Effective} = P_{Confining} - P_{Pore}$$

According to G. Mavko (2017), seismic wave velocity dependence on the effective pressure can be modeled by normalizing the seismic wave velocity readings with respect to the maximum velocity readings obtained from a given test, and modeling the average trend with an exponential model. Figure 3 presents example data from G. Mavko's paper regarding the normalized P-wave velocity data with respect to the effective pressure applied to the material tested. In this case Mavko also presented how multiple exponential models could be developed to model the relationship while presenting the average relationship mathematically with an exponential model.

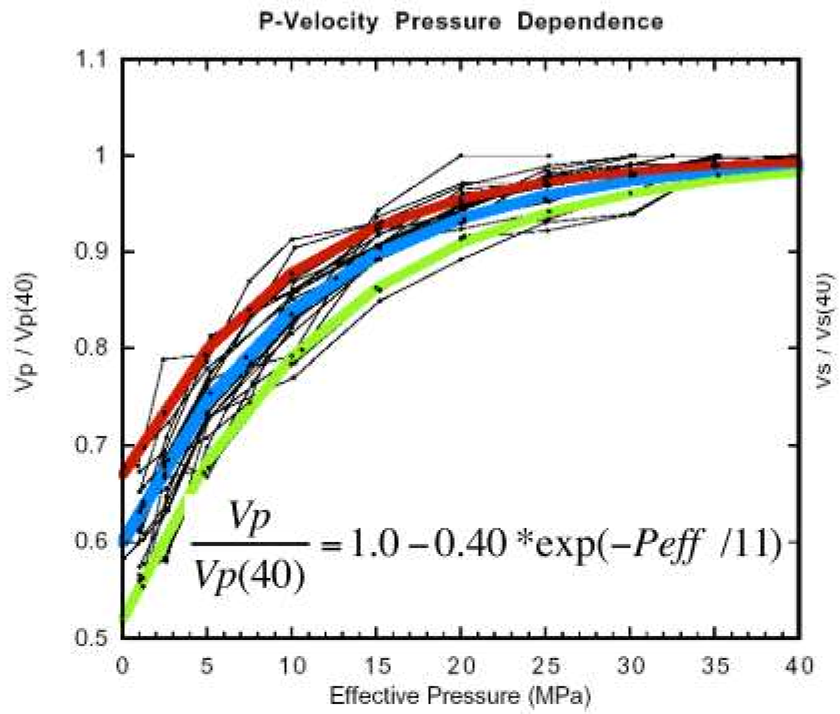

Figure 3: P-wave dependence on the effective pressure exerted on a rock samples for sample data. Three potential exponential models are drawn to show potential models to describe the relationship, while the average relationship is described mathematically with an exponential model. (Mavko, 2017)

As can be seen from Figure 3, a convergence effect occurs for the P-wave velocities propagated through the samples as the effective pressure increases. This convergence occurs to the point that nearly the same P-wave velocity is experienced in each sample once an effective pressure of 40 MPa is applied.

The effective pressure applied on a material is dependent on both the confining pressure and the pore pressure in the material. The confining pressure is the pressure applied externally to compress the material and reduce the material's volume. Pore pressure is an expanding compressive pressure applied by the fluid or gas that fills the porous space in the material. The pore pressure acting on porous material is discussed in further detail in the next section of this literature review addressing the impact of humidity and temperature on the P-wave velocity in material. Pore pressure is dependent on the porosity of the material tested, the pore diameter or crack aperture size, the percent saturation of the material, and the type of fluid or gas filling porous space in the material (Mavko, 2017) (Nakao, Nara, & Kubo, 2016).

The type of fluid or gas present in porous material filling the porous space influences the velocity of P-waves propagated through the material (Mavko et al., 2009). Such influential properties of the fluid or gas include the density of the fluid substance and the bulk modulus of the fluid (Mavko et al., 2009). As the bulk density for a penetrating fluid increases, the porous material's overall bulk density also increases (Mavko, 2017). Figure 4 presents how the bulk modulus of porous material is impacted as penetrating fluids with different densities and bulk moduli are used to penetrate the pore space of the porous material

(Mavko, 2017). As can be seen, oil increased the bulk density of the porous material as it is a denser and exhibits a higher bulk modulus than air. Water is denser than oil and has a higher bulk modulus than oil, therefore it is unsurprising that the bulk modulus of the porous material increased with the addition of water in comparison with the result for the addition of oil. It can also be seen in Figure 4 that the bulk modulus reacts differently with respect to the different sandstones tested, with the bulk modulus rapidly increasing and plateauing as the effective pressure on the Fontainebleau increases in comparison to the rate of increase the bulk modulus for the Beaver sandstone experiences. This difference is explained with the difference in pore density and pore diameter that forms the difference in porosity between the two porous materials (Nakao et al., 2016). This subject will be discussed later in this review.

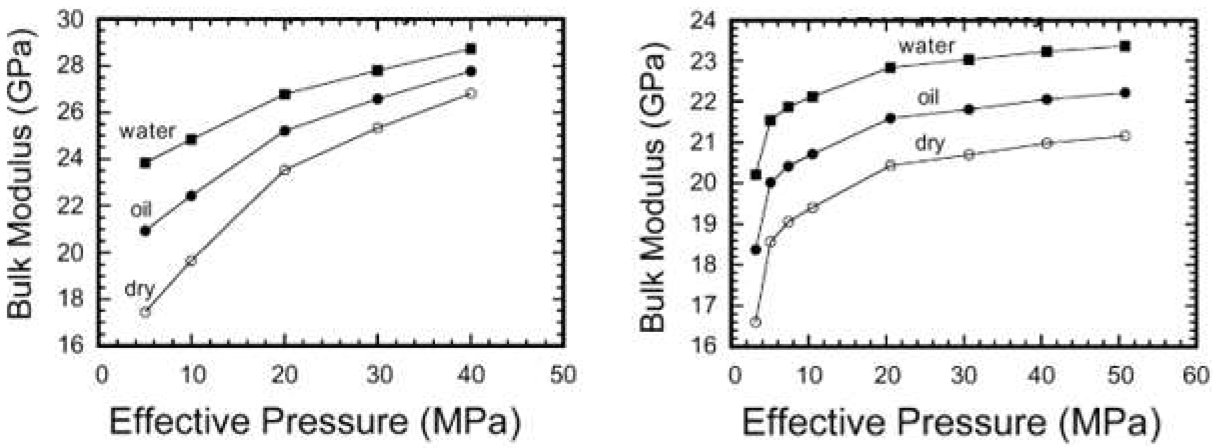

Figure 4: Bulk Modulus for porous sandstones VS the effective pressure applied. Different fluids with different densities and bulk moduli. Beaver sandstone, with a porosity of 6%, is on the left, while Fontainebleau sandstone, with a porosity of 15% is on the right (Mavko, 2017).

Filling the porous space of material with fluids denser than that of air and with a bulk modulus higher than that of air almost always further stiffens the porous material, therefore increasing the material's bulk modulus (Mavko, 2017). However, fluid penetration in rock does not always increase the seismic wave velocity in the material (Mavko, 2017). As can be recalled from Hook's law presented by Equation 6, the P-wave velocity in a material is dependant on the square root of the bulk modulus and dependant on the square root of the inverse density of the material (Mavko et al., 2009). So, although the bulk modulus and density of the porous material will increase from fluid penetration, the P-wave velocity may increase or decrease as a result.

Figure 5 shows the impact of fluid penetration on the P-wave velocity readings as the effective pressure is increased on the sandstone samples discussed regarding Figure 4 for varying fluids used to penetrate the porous space (Mavko, 2017). For both sandstone results, the P-wave velocity increases and slightly begins to plateau as the effective pressure is increased, the main difference between the samples being the rate of velocity increase. This difference it due to the similar reason explained for the same scenario as the bulk modulus results from Figure 4. The P-wave velocity also increases as a denser penetrating fluid is used. However, this was not the case for the Fontainebleau sandstone as the P-wave velocity was greater than when the porous space was filled. As explained with respect to Hook's law, the bulk modulus of the Fontainebleau sandstone certainly increases with the penetration of fluids, but the density of the material must have increased significantly as a result as well, which increased the acoustic impedance as described by Equation 2 (Mavko et al., 2009) (Mavko, 2017).

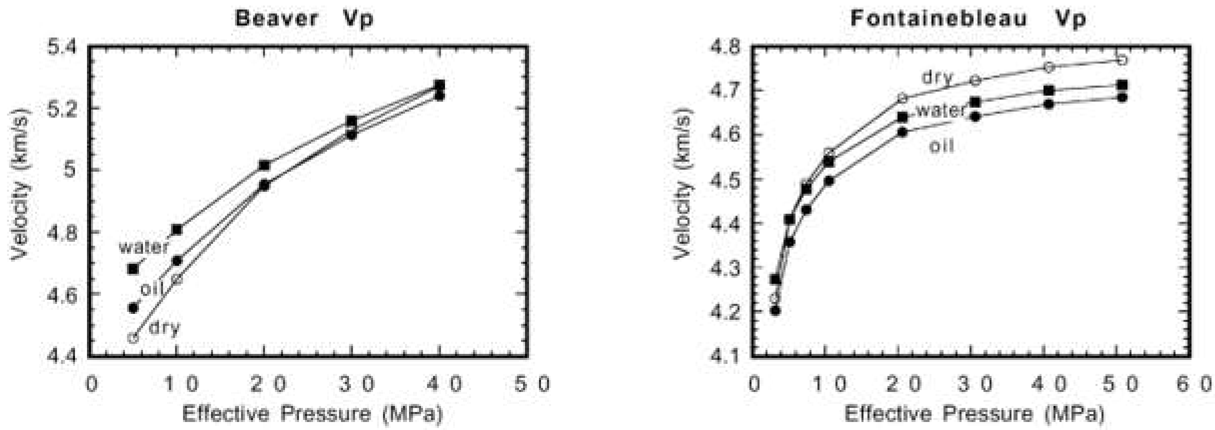

Figure 5: Velocity VS Effective pressure applied on Beaver sandstone (6% porosity on left) and Fontainebleau sandstone (15% porosity on right) for varying fluids used to penetrate the porous space of the materials (Mavko, 2017).

4. Impact of humidity and temperature on P-wave velocities through rock

The relative humidity can impact the velocities of acoustic waves that travel through rock. The impact relative humidity has on acoustic wave velocities in rock is impacted by the porosity of rock and the size of pores in rock.

One of the primary factors that impacts acoustic wave velocity in rocks is the crack density in rock. The existence of cracks in rock impacts the physical, strength and fracturing properties. Cracks impact the permeability of rock and increases anisotropies regarding strength and fracturing toughness. Relative environmental humidity has been known to increase the water content in rock, which impacts the strength and fracturing strength of rock, and as a result, the crack density of the rock as well. Reports have consistently supported this relation. An example of the impact on P-wave velocity and water content on a Berea sandstone core sample is shown in Figure 6.

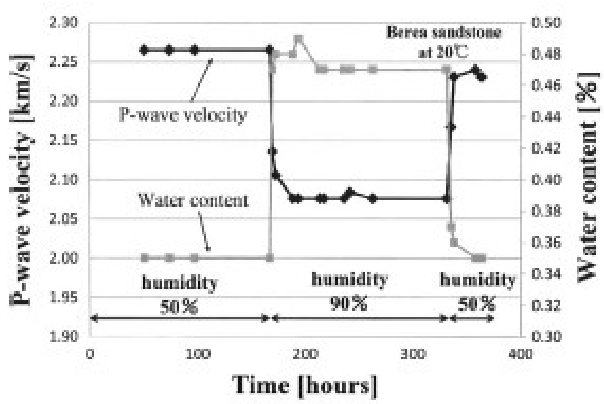

Figure 6: P-wave velocity and water content results for Berea sandstone core experiment with a constant environmental temperature of 20 ^{o}C (Nakao et al., 2016)

As can be seen the P-wave velocity in porous rock is dependant on the relative humidity. It is important to note that sample tested in Figure 6 is a very porous rock. It has been experimentally determined that

P-wave velocities in rocks with little porosity are not as sensitive to the effects of relative humidity. The temperature has a similar impact, in that as the temperature increases the P-wave velocity of samples decreases. It is particularly noted that rocks with large pores, despite being more porous than a rock with smaller pores, may experience less of a P-wave velocity change from the relative humidity. This inconsistency is explained with capillary condensation. When pores are small in diameter, water from the humidity condenses at the pore locations and immerses the pores in water, since the van der Waals force among polar water molecules in this confined space encourages condensation. A force that can be described as suction is produced via the occurrence of liquid building in the confined spaces and producing a capillary pressure due to the fluid medium separation of air and water via the capillary pathway. Capillary pressure assists in applying an internalized compressive force on porous material which assists in increasing grain contact points. The capillary pressure applied on a fluid in a confined space can be represented by the Young-Laplace equation as shown in Equation 8 (Nakao et al., 2016).

Equation 8: Young-Laplace equation for capillary pressure (Nakao et al., 2016)

$$p = \frac{2\sigma}{R_1}$$

Where p is the compressive pressure applied due to capillary condensation, σ is the surface tension of water and R_1 is the radius of curvature of the liquid surface. By looking at the Young-Laplace equation, it is recognized that, as more fluid is added to a pore space, the liquid radius of curvature increases and reduces the capillary pressure, which in turn reduces the internalized compressive stress that assists in creating grain contact points in the rock. As grain contact points reduce, the crack density of a given rock increases, therefore the P-wave velocity would decrease. This helps explain the large P-wave velocity change for porous rock with small pores, as their pores are small enough and there are typically many pores to experience significant capillary pressure to increase grain contact points. Therefore, when the humidity increased, the capillary pressure reduces, the grain contact points reduce, the crack density of the sandstones increases, and the P-wave velocity decreases significantly. The porous rock with large pores does not experience such radical change in the P-wave velocity because the pore diameter is significantly larger than the previously discussed case, and the curvature for the liquid in these porous spaces was already significant before the humidity increased during the experiments. Porous rock with large pores will still experience the same phenomena, but to less of an extent (Nakao et al., 2016).

The rate of acoustic velocity change in response to a change in the relative humidity is dependent on the gas permeability of a given rock sample (Nakao et al., 2016). The more permeable the sample the faster the velocity change occurs when the relative humidity is altered (Nakao et al., 2016). This is easily comprehensible as the permeability of a rock sample dictates the rate at which moisture equilibrium between the sample's porous space and the environment is reached.

As discussed earlier, P-wave velocities are dependant on the elastic properties of rock, including the Young's modulus. If a porous rock contains clay materials in its composition, the elastic properties of the material may be impacted due to water saturation. This could condition could contribute to the acoustic velocity dependence on the relative humidity (Nakao et al., 2016).

The acoustic waves also appear to attenuate more as the relative humidity increases. This makes sense as the crack density of material often increases as the relative humidity increases, thereby increasing the attenuating properties of the material (Nakao et al., 2016).

The relative humidity has a very minor impact on the acoustic wave velocity of low porosity rocks. Although a negative relation is still seen with the acoustic wave velocity and the relative humidity, the pores of the material are often few in quantity and therefore acoustic velocities for such rock is barely impacted. Any change in the acoustic velocity for material, such as granite, can be primarily attributed to the increase in microcrack aperture diameter enlargement from capillary condensation (Nakao et al., 2016).

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